

9



Areas of Parallelograms and Triangles

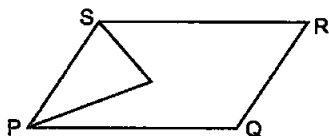
EXERCISE 9.1

Write the correct answer in each of the following:

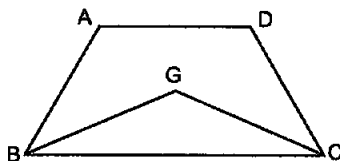
- The median of a triangle divides it into two
 - triangles of equal area.
 - congruent triangles
 - right triangles
 - isosceles triangles.

Sol. The median of a triangle divides it into two triangles of equal area. Hence, (a) is the correct answer.

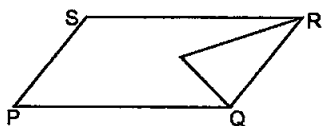
- In which of the following figures, you find two polygons on the same base and between the same parallels?



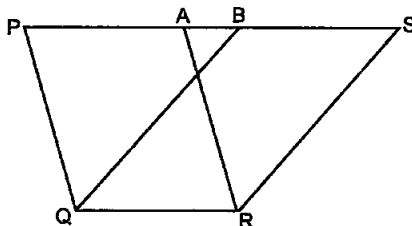
(a)



(b)



(c)

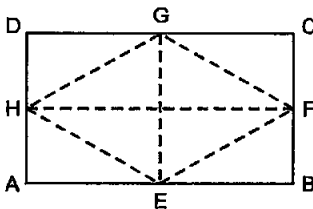


(d)

Sol. In figure (d), we find two polygons (parallelograms) on the same base and between the same parallels.

- The figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm is:
 - a rectangle of area 24 cm^2
 - a square of area 25 cm^2
 - a trapezium of area 24 cm^2
 - a rhombus of area 24 cm^2

Sol. ABCD is a rectangle and E, F, G and H are the mid-points of the sides AB, BC, CD and DA respectively. The figure obtained

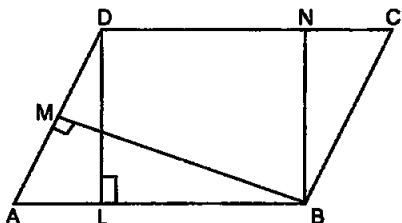


is a rhombus whose area = $\frac{1}{2} \times EG \times FH = \frac{1}{2} \times 6 \text{ cm} \times 8 \text{ cm} = 24 \text{ cm}^2$

Hence, (d) is the correct answer.

4. In the given figure, the area of parallelogram ABCD is:

- (a) $AB \times BM$ (b) $BC \times BN$ (c) $DC \times DL$ (d) $AD \times DL$

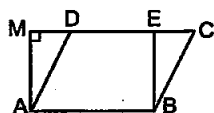


Sol. Area of a parallelogram = Base \times Corresponding altitude
 $= AB \times DL = DC \times DL$
 [$\because AB = DC$ (Opposite sides of a || gm)]

Hence, (c) is the correct answer.

5. In the given figure, if parallelogram ABCD and rectangle ABEM are of equal area then

- (a) perimeter of ABCD = perimeter of ABEM
 (b) perimeter of ABCD < perimeter of ABEM
 (c) perimeter of ABCD > perimeter of ABEM



- (d) perimeter of ABCD = $\frac{1}{2}$ (perimeter of ABEM)

Sol. If parallelogram ABCD and rectangle ABEM are of equal area, then perimeter of ABCD > perimeter of ABEM because of all the line segments to a given line from a point outside it, the perpendicular is the least. Hence, (c) is the correct answer.

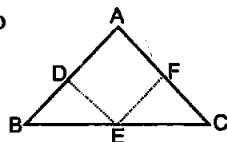
6. The mid-point of the sides of a triangle along with any of the vertices as the fourth point make a parallelogram of area equal to

- (a) $\frac{1}{2}$ ar(ΔABC) (b) $\frac{1}{3}$ ar(ΔABC)
 (c) $\frac{1}{4}$ ar(ΔABC) (d) ar(ΔABC)

Sol. Since median of a triangle divides it into two triangles of equal area

$$\therefore \text{ar}(\Delta ADE) = \text{ar}(\Delta BDE) \quad \dots(1)$$

$$\text{ar}(\Delta AEF) = \text{ar}(\Delta EFC) \quad \dots(2)$$



Since AE is the diagonal of a parallelogram ADEF. It divides it into two triangles of equal area.

$$\therefore \text{ar}(\triangle ADE) = \text{ar}(\triangle AFE) \quad \dots (3)$$

From (1), (2) and (3), we get

$$\therefore \text{ar}(\triangle ADE) = \text{ar}(\triangle BDE) = \text{ar}(\triangle AEF) = \text{ar}(\triangle EFC)$$

$$\text{Hence, ar}(\triangle ADEF) = \frac{1}{2} \text{ar}(\triangle ABC)$$

So, (a) is the correct answer.

7. Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is

(a) 1:2 (b) 1:1 (c) 2:1 (d) 3:1

Sol. We know that parallelograms on the same or equal bases and between the same parallels are equal in area.

So, the ratio of their areas is 1:1

Hence, (b) is the correct answer.

8. ABCD is a quadrilateral whose diagonal AC divides it in two parts, equal in area, then ABCD

(a) is a rectangle (b) is always a rhombus
(c) is a parallelogram (d) need not be any of (a), (b), or (c).

Sol. Since diagonal of a parallelogram divides it into two triangles of equal area and rectangle and a rhombus are also parallelograms. then ABCD need not be any of (a), (b) or (c).

Hence, (d) is the correct answer.

9. If a triangle and a parallelogram are on the same base and between the same parallels, then the ratio of the area of the triangle to the area of parallelogram is

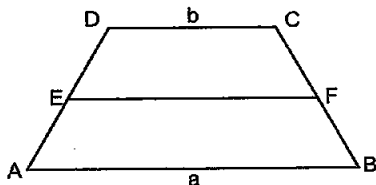
(a) 1:3 (b) 1:2 (c) 3:1 (d) 1:4

Sol. We know that a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram. Hence the ratio of the area of the triangle to the area of parallelogram is 1:2.

Hence, (b) is the correct answer.

10. ABCD is a trapezium with parallel sides $AB = a$ cm and $DC = b$ cm (see fig.) E and F are the mid-points of non parallel sides. The ratio of ar (ABFE) and ar (EFCD) is

(a) $a : b$
(b) $(3a + b) : (a + 3b)$
(c) $(a + 3b) : (3a + b)$
(d) $(2a + b) : (3a + b)$



Sol. ABCD is a trapezium in which $AB \parallel DC$. E and F are the mid-points of AD and BC, so

$$EF = \frac{1}{2}(a + b)$$

ABEF and EFCD are also trapeziums.

$$\text{ar(ABEF)} = \frac{1}{2} \left[\frac{1}{2}(a+b) + a \right] \times h = \frac{h}{4} (3a + b)$$

$$\text{ar(EFCD)} = \frac{1}{2} \left[b + \frac{1}{2}(a+b) \right] \times h = \frac{h}{4} (a + 3b)$$

$$\therefore \frac{\text{ar(ABEF)}}{\text{ar(EFCD)}} = \frac{\frac{h}{4}(3a+b)}{\frac{h}{4}(a+3b)} = \frac{(3a+b)}{(a+3b)}$$

So, the required ratio is $(3a + b) : (a + 3b)$.

Hence, (b) is the correct answer.

EXERCISE 9.2

Write True or False and justify your answer:

1. ABCD is a parallelogram and X is the mid-point of AB. If $\text{ar}(AXCD) = 24 \text{ cm}^2$, then $\text{ar}(\Delta ABC) = 24 \text{ cm}^2$.

Sol. We have ABCD is a parallelogram and X is the mid-point of AB.

Now, $\text{ar}(ABCD) = \text{ar}(AXCD) + \text{ar}(\Delta XBC)$... (1)

\therefore Diagonal AC of a parallelogram divides it into two triangles of equal area.

$\therefore \text{ar}(ABCD) = 2\text{ar}(\Delta ABC)$... (2)

Again X is the mid-point of AB, so

$$\text{ar}(\Delta CXB) = \frac{1}{2} \text{ar}(\Delta ABC) \quad \dots (3)$$

$[\because$ Median divides the triangle in two triangles of equal area]

$$\therefore 2\text{ar}(\Delta ABC) = 24 + \frac{1}{2} \text{ar}(\Delta ABC)$$

[Using (1), (2) and (3)]

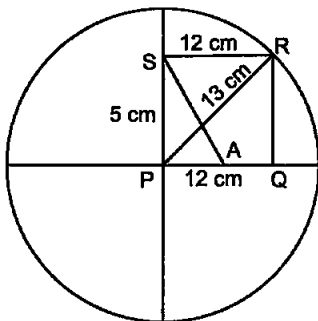
$$\therefore 2\text{ar}(\Delta ABC) - \frac{1}{2} \text{ar}(\Delta ABC) = 24$$

$$\Rightarrow \frac{3}{2} \text{ar}(\Delta ABC) = 24$$

$$\Rightarrow \text{ar}(\Delta ABC) = \frac{2 \times 24}{3} = 16 \text{ cm}^2$$

Hence, the given statement is false.

2. PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5 cm, then ar(ΔPAS) = 30 cm².



- Sol.** It is given that A is any point on PQ, therefore, PA < PQ.
It is given that A is any point on PQ, therefore PA < PQ.

$$\text{Now, ar}(\Delta PQR) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Now, ar}(\Delta PQR) = \frac{1}{2} \times PQ \times QR = \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$$

[\because PQRS is a rectangle
 $\therefore RQ = SP = 5 \text{ cm}$]

As PA < PQ (= 12 cm)

So ar(ΔPAS) < ar(ΔPQR)

or ar(ΔPAS) < 30 cm²

[\because ar(ΔPQR) = 30 cm²]

Hence, the given statement is false.

3. PQRS is a parallelogram whose area is 180 cm² and A is any point on the diagonal QS. The area of $\Delta ASR = 90 \text{ cm}^2$.

Sol. PQRS is a parallelogram.

We know that diagonal (QS) of a parallelogram divides parallelogram into two triangles of equal area, so

$$\begin{aligned} \therefore \text{ar}(\Delta QRS) &= \frac{1}{2} \text{ar}(\parallel \text{gm PQRS}) \\ &= \frac{1}{2} \times 180 = 90 \text{ cm}^2 \end{aligned}$$

\because A is any point on SQ

\therefore ar(ΔASR) < ar(ΔQRS)

Hence, ar(ΔASR) < 90 cm².

Hence, the given statement is false.

4. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Then, ar(ΔBDE) = $\frac{1}{4}$ ar(ΔABC).

Sol. ΔABC and ΔBDE are two equilateral triangles.

Let each side of triangle ABC be x.

Again, D is the mid-point of BC, so each side of triangle BDE is $\frac{x}{2}$.

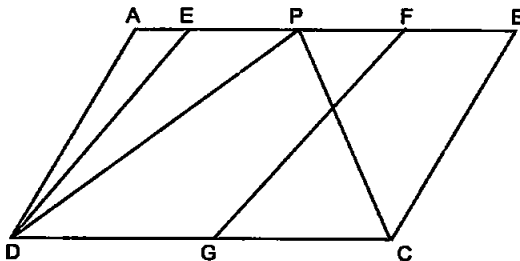
$$\text{Now, } \frac{\text{ar}(\triangle BDE)}{\text{ar}(\triangle ABC)} = \frac{\frac{\sqrt{3}}{4} \times \left(\frac{x}{2}\right)^2}{\frac{\sqrt{3}}{4} \times x^2} = \frac{x^2}{4x^2} = \frac{1}{4}$$

Hence, $\text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$

\therefore The given statement is true.

5. In the given figure, ABCD and EFGD are two parallelograms and G is the mid-point of CD. Then

$$\text{ar}(\triangle DPC) = \frac{1}{2} \text{ar}(\parallel \text{gm EFGD}).$$



Sol. As $\triangle DPC$ and $\parallel \text{gm ABCD}$ are on the same base DC and between the same parallels AB and DC, so

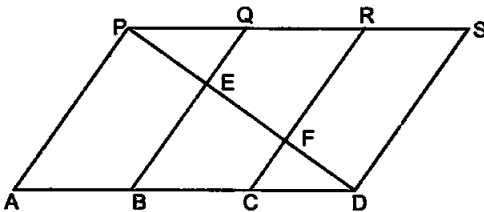
$$\text{ar}(\triangle DPC) = \frac{1}{2} \text{ar}(\parallel \text{gm. ABCD})$$

$$= \text{ar}(\parallel \text{gm EFGD}) \quad [\because G \text{ is the mid point of DC}]$$

Hence, the given statement is false.

EXERCISE 9.3

1. In given figure, PSDA is a parallelogram. Points Q and R are taken on PS such that $PQ=QR=RS$ and $PA \parallel QB \parallel RC$. Prove that $\text{ar}(\triangle PQE) = \text{ar}(\triangle CFD)$.



Sol. PSDA is a parallelogram. Points Q and R are taken on PS such that $PQ = QR = RS$ and $PA \parallel QB \parallel RC$.

We have to prove that $\text{ar}(\Delta PQE) = \text{ar}(\Delta CFD)$.

Now, $PS = AD$ [Opp. sides of a \parallel gm]

$$\therefore \frac{1}{3} PS = \frac{1}{3} AD \Rightarrow PQ = CD \quad \dots(1)$$

Again, $PS \parallel AD$ and QB cut them,

$$\therefore \angle PQE = \angle CBE \quad [\text{Alt. } \angle s] \dots(2)$$

Now, $QB \parallel RC$ and AD cuts them

$$\therefore \angle QBD = \angle RCD \quad [\text{Corres. } \angle s] \dots(3)$$

$$\text{so, } \angle PQE = \angle FCD \quad \dots(4)$$

[From (2) and (3), $\angle CBE$ and $\angle QBD$ are same and $\angle RCD$ and $\angle FCD$ are same]

Now, in ΔPQE and ΔCFD

$$\angle QPE = \angle CDF \quad [\text{Alt. } \angle s]$$

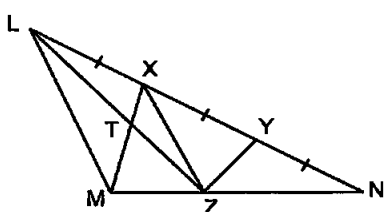
$$PQ = CD \quad [\text{From (1)}]$$

$$\text{and } \angle PQE = \angle FCD \quad [\text{From (4)}]$$

$$\therefore \Delta PQE \cong \Delta CFD \quad [\text{By ASA Congruence rule}]$$

Hence, $\text{ar}(\Delta PQE) = \text{ar}(\Delta CFD)$ [Congruent Δs are equal in area]

2. X and Y are points on the side LN of the triangle LMN such that $LX = XY = YN$. Through X, a line is drawn parallel to LM to meet MN at Z (See figure). Prove that $\text{ar}(\Delta LZY) = \text{ar}(\Delta MZYX)$.



Sol. We have to prove that $\text{ar}(\Delta LZY) = \text{ar}(\Delta MZYX)$

Since ΔLXZ and ΔXMZ are on the same base and between the same parallels LM and XZ , we have

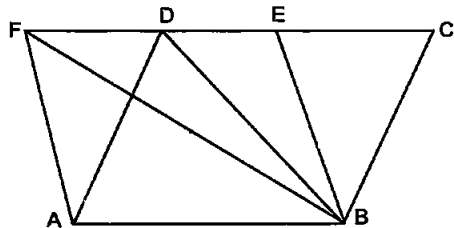
$$\text{ar}(\Delta LXZ) = \text{ar}(\Delta XMZ) \quad \dots(1)$$

Adding $\text{ar}(\Delta XYZ)$ to both sides of (1), we get

$$\text{ar}(\Delta LXZ) + \text{ar}(\Delta XYZ) = \text{ar}(\Delta XMZ) + \text{ar}(\Delta XYZ)$$

$$\Rightarrow \text{ar}(\Delta LZY) = \text{ar}(\Delta MZYX)$$

3. The area of the parallelogram ABCD is 90 cm^2 (See fig.). Find
 (i) $\text{ar}(\Delta BEF)$
 (ii) $\text{ar}(\Delta ABD)$
 (iii) $\text{ar}(\Delta BEF)$



Sol. (i) Since parallelograms on the same base and between the same parallels are equal in area, so we have

Hence $\text{ar}(\parallel \text{gm ABEF}) = \text{ar}(\parallel \text{gm ABCD})$
 $\text{ar}(\parallel \text{gm ABEF}) = \text{ar}(\parallel \text{gm ABCD}) = 90 \text{ cm}^2$

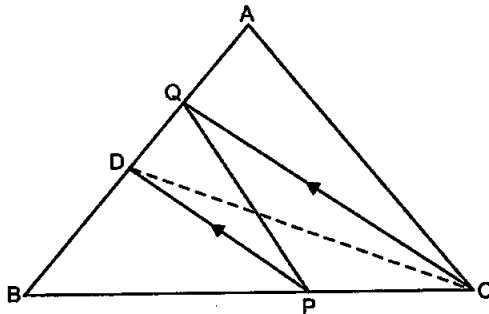
(ii) $\text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\parallel \text{gm ABCD})$

[\because A diagonal of a parallelogram divides the parallelogram in two triangles of equal area]

$= \frac{1}{2} \times 90 \text{ cm}^2 = 45 \text{ cm}^2.$

(iii) $\text{ar}(\triangle BEF) = \frac{1}{2} \text{ar}(\parallel \text{gm ABEF}) = \frac{1}{2} \times 90 \text{ cm}^2 = 45 \text{ cm}^2.$

4. In $\triangle ABC$, D is the mid-point of AB and P is any point on BC. If $CQ \parallel PD$ meets AB in Q (See fig.), then prove that $\text{ar}(\triangle BPQ) = \frac{1}{2} \text{ar}(\triangle ABC).$



Sol. D is the mid-point of AB and P is any point on BC of $\triangle ABC$. $CQ \parallel PD$ meets AB in Q, we have to prove that $\text{ar}(\triangle BPQ) = \frac{1}{2} \text{ar}(\triangle ABC).$
 Join CD. Since median of a triangle divides it into two triangles of equal area, so we have

$\text{ar}(\triangle BCD) = \frac{1}{2} \text{ar}(\triangle ABC) \quad \dots(1)$

Since triangles on the same base and between the same parallels are equal in area, so we have

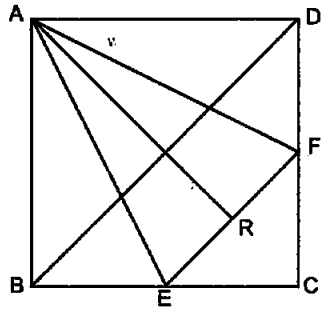
$\text{ar}(\triangle DPQ) = \text{ar}(\triangle DPC) \quad \dots(2)$

[\because Triangle DPQ and DPC are on the same base DP and between the same parallels DP and CQ]

$\text{ar}(\triangle DPQ) + \text{ar}(\triangle DPB) = \text{ar}(\triangle DPC) + \text{ar}(\triangle DPB)$

Hence, $\text{ar}(\triangle BPQ) = \text{ar}(\triangle BCD) = \frac{1}{2} \text{ar}(\triangle ABC) \quad \text{[Using (1)]}$

5. ABCD is a square. E and F are respectively the mid-points of BC and CD. If R is the mid-point of EF (See fig.) prove that $\text{ar}(\triangle AER) = \text{ar}(\triangle AFR)$.



Sol. ABCD is a square. E and F are respectively the mid-points of BC and CD. If R is the mid-point of EF, we have to prove that $\text{ar}(\triangle AER) = \text{ar}(\triangle AFR)$.

In $\triangle ABE$ and $\triangle ADF$, we have

$$\begin{aligned} AB &= AD \\ \angle ABE &= \angle ADF \\ BE &= DF \end{aligned}$$

[Sides of a square are equal]

[Each 90°]

[\because E is the mid-point of BC and F is

the mid-point of CD. Also $\frac{1}{2} BC = \frac{1}{2} CD$]

$\text{ar}(\triangle ABE) \cong \text{ar}(\triangle ADF)$ [By SAS Congruence rule]

$\therefore AE = AF$ (CPCT)...(1)

Now, in $\triangle AER$ and $\triangle AFR$, we have

$AE = AF$ [From (1)]

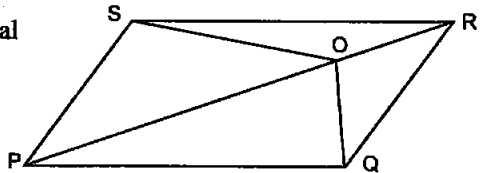
$ER = FR$ [\because R is mid-point of EF]

and $AR = AR$ [Common side]

$\therefore \triangle AER \cong \triangle AFR$ [By SSS rule of congruence]

Hence, $\text{ar}(\triangle AER) = \text{ar}(\triangle AFR)$ [\because Congruent triangles are equal in area]

6. In the given figure, O is any point on the diagonal PR of a parallelogram PQRS. Prove that $\text{ar}(\triangle PSO) = \text{ar}(\triangle PQO)$

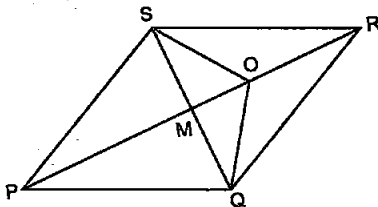


Sol. Join SQ, bisects the diagonal PR at M.

Since diagonals of a parallelogram bisect each other, so $SM = MQ$. Therefore, PM is a median of $\triangle PQS$.

$\text{ar}(\triangle PSM) = \text{ar}(\triangle PQM)$... (1)

[\because Median divides a triangle into two triangles of equal area]



Again, as OM is the median of triangle ΔOSQ , so

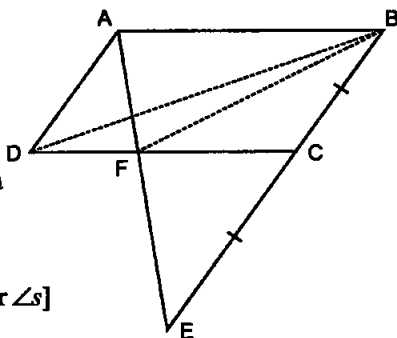
$$\text{ar}(\Delta OSM) = \text{ar}(\Delta OQM) \quad \dots(2)$$

Adding (1) and (2), we get

$$\begin{aligned} \text{ar}(\Delta PSM) + \text{ar}(\Delta OSM) &= \text{ar}(\Delta PQM) + \text{ar}(\Delta OQM) \\ \Rightarrow \text{ar}(\Delta PSO) &= \text{ar}(\Delta PQO) \end{aligned}$$

Hence, proved.

7. ABCD is a parallelogram in which BC is produced to E such that CE = BC in the given figure. AE intersects CD at F.



If $\text{ar}(\Delta DFB) = 3 \text{ cm}^2$, find the area of the parallelogram ABCD.

Sol. In ΔADF and ΔEFC , we have

$$\angle DAF = \angle CEF \quad [\text{Alt. interior } \angle s]$$

$$AD = CE$$

$$\angle ADF = \angle FCE$$

$$\therefore \Delta ADF \cong \Delta ECF$$

$$\therefore DF = CF$$

As BF is median of ΔBCD ,

$$\therefore \text{ar}(\Delta BDF) = \frac{1}{2} \text{ar}(\Delta BCD) \quad \dots(1)$$

[\because Median divides a triangle into two triangles of equal area]

Now, if a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangles is equal to half the area of the parallelogram.

$$\therefore \text{ar}(\Delta BCD) = \frac{1}{2} \text{ar}(\parallel \text{gm ABCD}) \quad \dots(2)$$

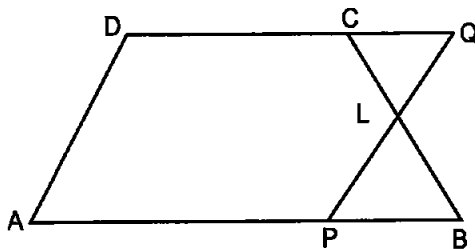
$$\therefore \text{By (1), we have } \text{ar}(\Delta BDF) = \frac{1}{2} \left\{ \frac{1}{2} \text{ar}(\parallel \text{gm ABCD}) \right\}$$

$$\Rightarrow 3 \text{ cm}^2 = \frac{1}{4} \text{ar}(\parallel \text{gm ABCD})$$

$$\Rightarrow \text{ar}(\parallel \text{gm ABCD}) = 12 \text{ cm}^2$$

Hence, the area of the parallelogram ABCD is 12 cm^2 .

8. In trapezium ABCD, $AB \parallel DC$ and L is the mid-point of BC. Through L, a line $PQ \parallel AD$ has been drawn which meets AB in P and DC produced in Q (See figure). Prove that $\text{ar}(ABCD) = \text{ar}(APQD)$.



Sol. As $AB \parallel DC$, so $AB \parallel DQ$

In $\triangle CLQ$ and $\triangle BLP$, we have

$$\therefore \angle QCL = \angle LBP$$

[Alt. \angle s]

$$CL = LB$$

[\because L is the mid-point of BC]

$$\angle CLQ = \angle BLP$$

[Vertically opposite \angle s]

$$\therefore \triangle CLQ \cong \triangle BLP$$

[By ASA Congruence rule]

$$\Rightarrow \text{ar}(\triangle CLQ) = \text{ar}(\triangle BLP) \dots (1) \text{ [Congruent } \Delta\text{s are equal in area]}$$

Adding $\text{ar}(\triangle APLCD)$ to both sides of (1), we get

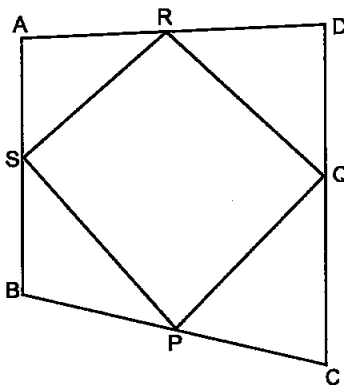
$$\text{ar}(\triangle CLQ) + \text{ar}(\triangle APLCD) = \text{ar}(\triangle BLP) + \text{ar}(\triangle APLCD)$$

$$\Rightarrow \text{ar}(APQD) = \text{ar}(ABCD)$$

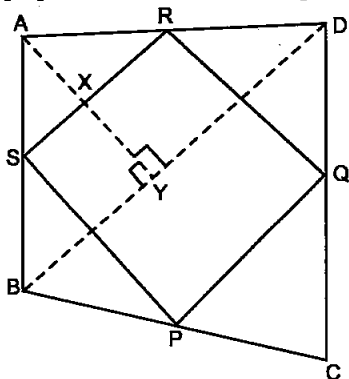
Hence, $\text{ar}(ABCD) = \text{ar}(APQD)$

9. If the mid-points of the sides of a quadrilateral are joined in order, prove that the area of the parallelogram so formed will be half of the area of the given quadrilateral. (figure)

[Hint: Join BD and draw perpendicular from A on BD].



Sol.



Given: A quadrilateral ABCD in which the mid-points of the sides of it are joined in order to form parallelogram PQRS.

To prove: $\text{ar}(\parallel\text{gm PQRS}) = \frac{1}{2} \text{ar}(\square ABCD)$

Construction: Join BD and draw perpendicular from A on BD which intersect SR and BD at X and Y respectively.

Proof: In $\triangle ABD$, S and R are the mid-points of sides AB and AD respectively.

\therefore SR \parallel BD
 \Rightarrow SX \parallel BY
 \Rightarrow X is the mid-point of AY [Converse of mid-point theorem]
 \Rightarrow AX = XY ... (1)
 [\because S is the mid-point of AB and SX \parallel BY]

And, SR = $\frac{1}{2}$ BD ... (2) [\because Mid-point theorem]

Now, $\text{ar}(\triangle ABD) = \frac{1}{2} \times \text{BD} \times \text{AY}$

and $\text{ar}(\triangle ASR) = \frac{1}{2} \times \text{SR} \times \text{AX}$

$\Rightarrow \text{ar}(\triangle ASR) = \frac{1}{2} \times \left(\frac{1}{2} \text{BD}\right) \times \left(\frac{1}{2} \text{AY}\right)$ [Using (1) and (2)]

$\Rightarrow \text{ar}(\triangle ASR) = \frac{1}{4} \times \left(\frac{1}{2} \times \text{BD} \times \text{AY}\right)$

$\Rightarrow \text{ar}(\triangle ASR) = \frac{1}{4} \text{ar}(\triangle ABD)$... (3)

Similarly,

$\text{ar}(\triangle CPQ) = \frac{1}{4} \text{ar}(\triangle CBD)$... (4)

$\text{ar}(\triangle BPS) = \frac{1}{4} \text{ar}(\triangle BAC)$... (5)

$\text{ar}(\triangle DRQ) = \frac{1}{4} \text{ar}(\triangle DAC)$... (6)

Adding (3), (4), (5) and (6), we get

$$\begin{aligned}
 \text{ar}(\triangle ASR) + \text{ar}(\triangle CPQ) + \text{ar}(\triangle BPS) + \text{ar}(\triangle DRQ) \\
 = \frac{1}{4} \text{ar}(\triangle ABD) + \frac{1}{4} \text{ar}(\triangle CBD) + \frac{1}{4} \text{ar}(\triangle BAC) \\
 + \frac{1}{4} \text{ar}(\triangle DAC)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{4} [\text{ar}(\triangle ABD) + \text{ar}(\triangle CBD) \\
 &\quad + \text{ar}(\triangle BAC) + \text{ar}(\triangle DAC)] \\
 &= \frac{1}{4} [\text{ar}(\square ABCD) + \text{ar}(\square ABCD)] \\
 &= \frac{1}{4} \times 2 \text{ar}(\square ABCD) \\
 &= \frac{1}{2} \text{ar}(\square ABCD) \\
 \therefore \text{ar}(\triangle ASR) + \text{ar}(\triangle CPQ) + \text{ar}(\triangle BPS) + \text{ar}(\triangle DRQ) \\
 &= \frac{1}{2} \text{ar}(\square ABCD) \\
 \Rightarrow \text{ar}(\square ABCD) - \text{ar}(\parallel \text{gm PQRS}) &= \frac{1}{2} \text{ar}(\square ABCD) \\
 \Rightarrow \text{ar}(\parallel \text{gm PQRS}) &= \text{ar}(\square ABCD) - \frac{1}{2} \text{ar}(\square ABCD) \\
 \Rightarrow \text{ar}(\parallel \text{gm PQRS}) &= \frac{1}{2} \text{ar}(\square ABCD) \\
 \text{Hence, Proved}
 \end{aligned}$$

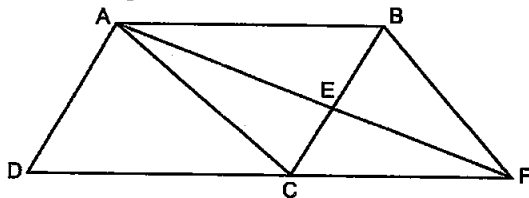
EXERCISE 9.4

1. A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that $\text{ar}(\triangle ADF) = \text{ar}(\square ABFC)$.

Sol. Given: ABCD is a parallelogram. A point E is taken on the side BC. AE and DC are produced to meet at F.

To prove: $\text{ar}(\triangle ADF) = \text{ar}(\square ABFC)$

Proof: Since ABCD is a parallelogram and diagonal AC divides it into two triangles of equal area, we have



$$\text{ar}(\triangle ADC) = \text{ar}(\triangle ABC) \quad \dots(1)$$

As $DC \parallel AB$, so $CF \parallel AB$

Since triangles on the same base and between the same parallels are equal in area, so we have

$$\text{ar}(\triangle ACF) = \text{ar}(\triangle BCF) \quad \dots(2)$$

Adding (1) and (2), we get

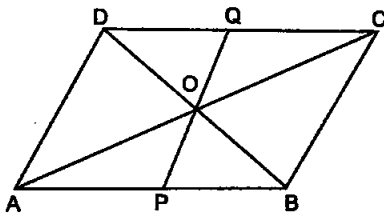
$$\text{ar}(\triangle ADC) + \text{ar}(\triangle ACF) = \text{ar}(\triangle ABC) + \text{ar}(\triangle BCF)$$

$$\Rightarrow \text{ar}(\triangle ADF) = \text{ar}(\triangle BFC)$$

Hence, proved.

2. The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. Show that PQ divides the parallelogram into two parts of equal area.

Sol. \therefore AC is a diagonal of the \parallel gm ABCD



$$\therefore \text{ar}(\triangle ACD) = \frac{1}{2} \text{ar}(\parallel \text{ gm ABCD}) \quad \dots(1)$$

Now, in $\triangle AOP$ and $\triangle COQ$

$$AO = CO \quad [\because \text{Diagonals of a } \parallel \text{ gm bisect each other}]$$

$$\angle AOP = \angle COQ \quad [\text{Vert. Opp. } \angle s]$$

$$\angle OAP = \angle OCQ \quad [\text{Alt. } \angle s; AB \parallel CD]$$

$$\therefore \triangle AOP \cong \triangle COQ \quad [\text{By ASA Cong. rule}]$$

$$\text{Hence, ar}(\triangle AOP) = \text{ar}(\triangle COQ) \quad [\text{Cong. area axiom}] \quad \dots(2)$$

Adding ar (quad. AOQD) to both sides of (2), we get

$$\text{ar}(\text{quad. AOQD}) + \text{ar}(\triangle AOP) = \text{ar}(\text{quad. AOQD}) + \text{ar}(\triangle COQ)$$

$$\Rightarrow \text{ar}(\text{quad. APQD}) = \text{ar}(\triangle ACD)$$

$$\text{But, ar}(\triangle ACD) = \frac{1}{2} \text{ar}(\parallel \text{ gm ABCD}) \quad [\text{From (1)}]$$

$$\text{Hence, ar}(\text{quad. APQD}) = \frac{1}{2} \text{ar}(\parallel \text{ gm ABCD}).$$

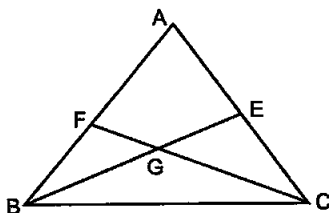
3. The medians BE and CF of a triangle ABC intersect at G. Prove that the area of $\triangle GBC$ = area of the quadrilateral AFGE.

Sol. BE and CF are medians of a triangle ABC intersect at G. We have to prove that the ar ($\triangle GBC$) = area of the quadrilateral AFGE.

Since median (CF) divides a triangle into two triangles of equal area, so we have

$$\text{ar}(\triangle BCF) = \text{ar}(\triangle ACF)$$

$$\Rightarrow \text{ar}(\triangle GBF) + \text{ar}(\triangle GBC) = \text{ar}(\text{AFGE}) + \text{ar}(\triangle GCE) \quad \dots(1)$$



Since median (BE) divides a triangle into two triangles of equal area, so we have

$$\Rightarrow \text{ar}(\triangle GBF) + \text{ar}(\triangle FGE) = \text{ar}(\triangle GCE) + \text{ar}(\triangle GBC) \quad \dots(2)$$

Subtracting (2) from (1), we get

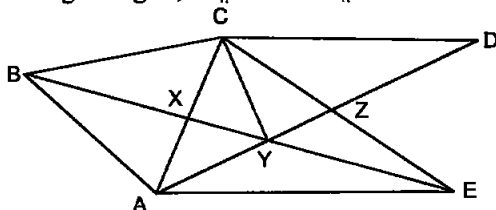
$$\text{ar}(\triangle GBC) - \text{ar}(\triangle FGE) = \text{ar}(\triangle AFGE) - \text{ar}(\triangle GBC)$$

$$\Rightarrow \text{ar}(\triangle GBC) + \text{ar}(\triangle GBC) = \text{ar}(\triangle FGE) + \text{ar}(\triangle FGE)$$

$$\Rightarrow 2 \text{ar}(\triangle GBC) = 2 \text{ar}(\triangle FGE)$$

Hence, $\text{ar}(\triangle GBC) = \text{ar}(\triangle FGE)$

4. In the given figure, $CD \parallel AE$ and $CY \parallel BA$. Prove that $\text{ar}(\triangle CBX) = \text{ar}(\triangle AXY)$.



Sol. $CD \parallel AE$ and $CY \parallel BA$. We have to prove that $\text{ar}(\triangle CBX) = \text{ar}(\triangle AXY)$.

Since triangles on the same base and between the same parallels are equal in area, so we have

$$\text{ar}(\triangle ABC) = \text{ar}(\triangle ABE)$$

$$\Rightarrow \text{ar}(\triangle CBX) + \text{ar}(\triangle ABX) = \text{ar}(\triangle ABX) + \text{ar}(\triangle AXY)$$

Hence, $\text{ar}(\triangle CBX) = \text{ar}(\triangle AXY)$

[Cancelling $\text{ar}(\triangle ABX)$ from both sides]

5. ABCD is a trapezium in which $AB \parallel DC$, $DC = 30$ cm and $AB = 50$ cm. If X and Y are respectively the mid-points of AD and BC, prove that

$$\text{ar}(\triangle DCYX) = \frac{7}{9} \text{ar}(\triangle XYBA)$$

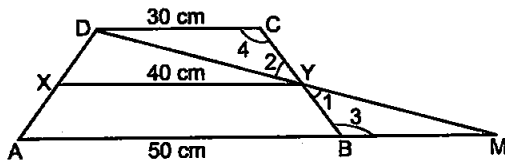
Sol. In $\triangle MBY$ and $\triangle DCY$, we have

$$\angle 1 = \angle 2$$

[Vertically opposite \angle s]

$$\angle 3 = \angle 4$$

[$\because AB \parallel DC$ and alt. \angle s are equal]



$BY = CY$ $[\because Y \text{ is the mid-point of } BC]$
 $\therefore \triangle MBY \cong \triangle DCY$ $[\text{By ASA Cong. rule}]$
 So, $MB = DC = 30 \text{ cm}$ $[\text{CPCT}]$
 Now, $AM = AB + BM = 50 \text{ cm} + 30 \text{ cm} = 80 \text{ cm}$

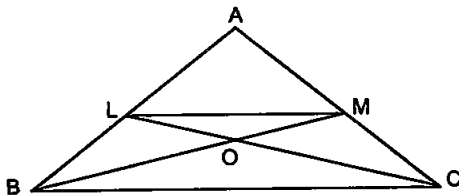
In $\triangle ADM$, we have $XY = \frac{1}{2} AM = \frac{1}{2} \times 80 \text{ cm} = 40 \text{ cm}$

As $AB \parallel XY \parallel DC$ and X and Y are the mid-points of AD and BC , so height of trapezium $DCYX$ and $XYBA$ are equal. Let the equal height be $h \text{ cm}$.

$$\frac{\text{ar}(\text{DCYX})}{\text{ar}(\text{XYBA})} = \frac{\frac{1}{2}(30 + 40) \times h}{\frac{1}{2} \times (40 + 50) \times h} = \frac{70}{90} = \frac{7}{9}$$

Hence, $\text{ar}(\text{DCYX}) = \frac{7}{9} \text{ar}(\text{XYBA})$

6. In $\triangle ABC$, if L and M are the points on AB and AC , respectively such that $LM \parallel BC$. Prove that $\text{ar}(\triangle LOB) = \text{ar}(\triangle MOC)$.



Sol. Since triangles on the same base and between the same parallels are equal in area, so we have

$$\therefore \text{ar}(\triangle LBM) = \text{ar}(\triangle LCM)$$

$[\triangle LBM \text{ and } \triangle LCM \text{ are on the same base } LM \text{ and between the same parallels } LM \text{ and } BC]$

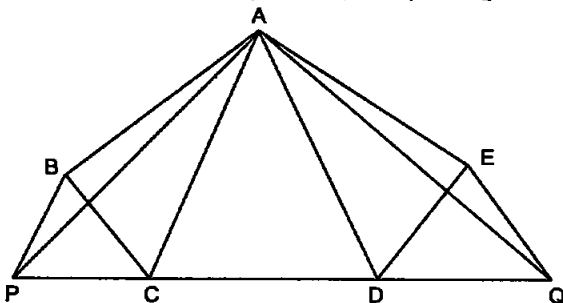
$$\therefore \text{ar}(\triangle LBM) = \text{ar}(\triangle LCM)$$

$$\Rightarrow \text{ar}(\triangle LOM) + \text{ar}(\triangle LOB) = \text{ar}(\triangle LOM) + \text{ar}(\triangle MOC)$$

Hence, $\text{ar}(\triangle LOB) = \text{ar}(\triangle MOC)$

$[\text{Cancelling } \text{ar}(\triangle LOM) \text{ from both sides}]$

7. In the given figure, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that $\text{ar}(\text{ABCDE}) = \text{ar}(\Delta \text{APQ})$.



Sol. $BP \parallel AC$ and $AD \parallel EQ$,

Since triangles on the same base and between the same parallels are equal in area

$$\text{ar}(\Delta ABC) = \text{ar}(\Delta APC) \quad \dots(1)$$

and $\text{ar}(\Delta ADE) = \text{ar}(\Delta ADQ) \quad \dots(2)$

Adding (1) and (2), we get

$$\text{ar}(\Delta ABC) + \text{ar}(\Delta ADE) = \text{ar}(\Delta APC) + \text{ar}(\Delta ADQ)$$

Adding $\text{ar}(\Delta ACD)$ to both sides, we get

$$\text{ar}(\Delta ABC) + \text{ar}(\Delta ADE) + \text{ar}(\Delta ACD) = \text{ar}(\Delta APC) + \text{ar}(\Delta ADQ) + \text{ar}(\Delta ACD)$$

Hence, $\text{ar}(\text{ABCDE}) = \text{ar}(\Delta \text{APQ})$

8. If the medians of a ΔABC intersect at G, show that

$$\text{ar}(\text{AGB}) = \text{ar}(\text{AGC}) = \text{ar}(\text{BGC}) = \frac{1}{3} \text{ar}(\text{ABC}).$$

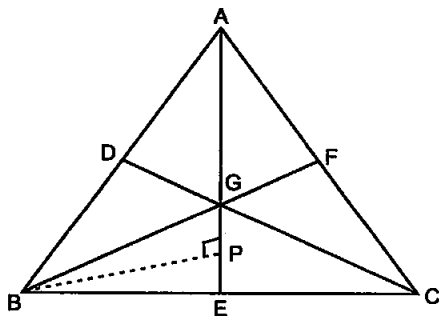
Sol. Given : Medians AE, BF and CD of ΔABC intersect at G.

To prove:

$$\begin{aligned} \text{ar}(\Delta \text{AGB}) &= \text{ar}(\Delta \text{AGC}) \\ &= \text{ar}(\Delta \text{BGC}) \\ &= \frac{1}{3} \text{ar}(\Delta \text{ABC}) \end{aligned}$$

Construction: Draw $BP \perp EG$.

Proof: $AG = \frac{2}{3} AE$



[\because Centroid divides the median in the ratio 2 : 1]

Now, $\text{ar}(\Delta \text{AGB}) = \frac{1}{2} \times AG \times BP$

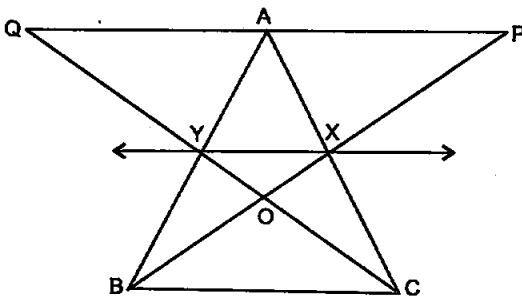
$$\begin{aligned}
 &= \frac{1}{2} \times \frac{2}{3} AE \times BP \\
 &= \frac{2}{3} \times \frac{1}{2} \times AE \times BP \\
 &= \frac{2}{3} \text{ar}(\triangle ABE) \\
 &= \frac{2}{3} \times \frac{1}{2} \text{ar}(\triangle ABC) \quad [\because \text{Median divides a triangle into} \\
 &\hspace{15em} \text{two triangles equal in area}] \\
 &= \frac{1}{3} \text{ar}(\triangle ABC)
 \end{aligned}$$

Similarly, $\text{ar}(\triangle AGC) = \text{ar}(\triangle BGC) = \frac{1}{3} \text{ar}(\triangle ABC)$

$$\therefore \text{ar}(\triangle AGB) = \text{ar}(\triangle AGC) = \text{ar}(\triangle BGC) = \frac{1}{3} \text{ar}(\triangle ABC)$$

Hence, proved.

9. In given figure, X and Y are the mid-points of AC and AB respectively, OP \parallel BC and CYQ and BXP are straight lines. Prove that $\text{ar}(\triangle ABP) = \text{ar}(\triangle ACQ)$.



Sol. In triangle ABC, X and Y are the mid-points of AB and AC.

$$\therefore XY \parallel BC \quad [\text{By mid-point theorem}]$$

Since triangles on the same base (BC) and between the same parallels (XY \parallel BC) are equal in area

$$\therefore \text{ar}(\triangle BYC) = \text{ar}(\triangle BXC) \quad \dots(1)$$

Subtracting $\text{ar}(\triangle BOC)$ from both sides, we get

$$\text{ar}(\triangle BYC) - \text{ar}(\triangle BOC) = \text{ar}(\triangle BXC) - \text{ar}(\triangle BOC)$$

$$\Rightarrow \text{ar}(\triangle BOY) = \text{ar}(\triangle COX) \quad \dots(2)$$

Adding $\text{ar}(\triangle XOY)$ to both sides of (2), we get

$$\text{ar}(\triangle BOY) + \text{ar}(\triangle XOY) = \text{ar}(\triangle COX) + \text{ar}(\triangle XOY)$$

$$\Rightarrow \text{ar}(\triangle BXY) = \text{ar}(\triangle CXY) \quad \dots(3)$$

Now, quad. XYAP and XYQA are on the same base XY and between the same parallels XY and PQ.

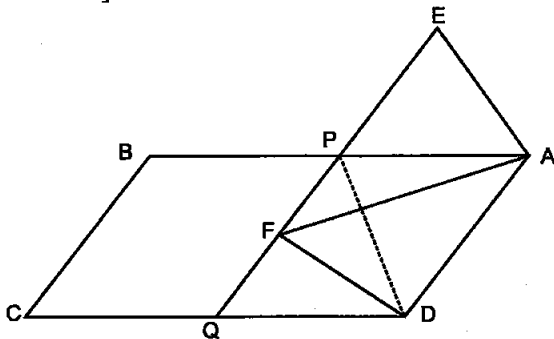
$$\therefore \text{ar}(\triangle XYAP) = \text{ar}(\triangle XYQA) \quad \dots(4)$$

Adding (3) and (4), we get

$$\text{ar}(\triangle BXY) + \text{ar}(\triangle XYAP) = \text{ar}(\triangle CXY) + \text{ar}(\triangle XYQA)$$

Hence, $\text{ar}(\triangle ABP) = \text{ar}(\triangle ACQ)$

10. In the given figure, ABCD and AEFD are two parallelograms. Prove that $\text{ar}(\triangle PEA) = \text{ar}(\triangle QFD)$.
 [Hint: Join PD]



Sol. ABCD and AEFD are two parallelograms.

We have to prove that $\text{ar}(\triangle PEA) = \text{ar}(\triangle QFD)$. Join PD.

In $\triangle PEA$ and $\triangle QFD$, we have

$$\angle APE = \angle DQF \quad [\because \text{Corresp. } \angle s \text{ are equal as } AB \parallel CD]$$

$$\angle AEP = \angle DFQ \quad [\because \text{Corresp. } \angle s \text{ are equal as } AE \parallel DF]$$

$$AE = DF \quad [\because \text{Opposite sides of a } \parallel \text{gm are equal}]$$

$$\therefore \triangle PEA \cong \triangle QFD \quad [\text{By AAS Cong. rule}]$$

Hence, $\text{ar}(\triangle PEA) = \text{ar}(\triangle QFD)$.

