8 Quadrilaterals

Lesson at a Glance

1. There are four sides of a quadrilateral.
2. The sum of the angles of a quadrilateral is 360°. This property is called *Angle Sum Property of a Quadrilateral.*
3. **Types of quadrilaterals:** Trapezium, parallelogram, rectangle, rhombus, square.
4. One pair of opposite sides of a trapezium is parallel.
5. **Properties of parallelogram:**
   (*i*) Opposite sides are equal and parallel.
   (*ii*) Opposite angles are equal.
   (*iii*) Diagonals bisect each other.
   (*iv*) Each of diagonals divides the parallelogram into two congruent triangles.
6. **Properties of a rectangle:**
   (*i*) Opposite sides are equal and parallel.
   (*ii*) Each of angles is of 90°.
   (*iii*) Diagonals are of equal lengths.
   (*iv*) Diagonals bisect each other.
   (*v*) Each of diagonals divides the rectangle into two congruent triangles.
7. A kite is not a parallelogram.
8. **Properties of a rhombus:**
   (*i*) All the sides are of equal lengths.
   (*ii*) Opposite angles are equal.
   (*iii*) Diagonals bisect each other at right angles.
   (*iv*) Each diagonal divides the rhombus into two congruent triangles.
9. **Properties of a square:**
   (*i*) All the sides are of equal lengths.
   (*ii*) Each of angle is of 90°.

(*iii*) Diagonals are of equal lengths.

(*iv*) Diagonals bisect each other at right angles.

(*v*) Each diagonal divides the square into two congruent triangles.

10. A parallelogram is a trapezium.
11. A square is a rectangle as well as a rhombus.
12. The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and half of it.

TEXTBOOK QUESTIONS SOLVED

Exercise 8.1 *(Pages – 146-147)*

1. *The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.*

Sol. Let the angles be $3x$, $5x$, $9x$ and $13x$.

Then $3x + 5x + 9x + 13x = 360°$

[Sum of angles of a quadrilateral is 360°.]

$\Rightarrow \quad 30x = 360° \Rightarrow x = 12°$

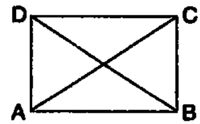
$\therefore$ Angles are 36°, 60°, 108° and 156°.

2. *If the diagonals of a parallelogram are equal, then show that it is a rectangle.*

Sol. Consider triangles DAB and CBA,

AD = BC [Opposite sides of a parallelogram]

AB is common.



AC = BD [Given]

$\therefore$ $\Delta DAB \cong \Delta CBA$ [SSS]

$\Rightarrow$ $\angle DAB = \angle CBA$ ...(*i*) [CPCT]

As ABCD is a parallelogram. AD || BC and AB is transversal.

$\therefore$ $\angle DAB + \angle CBA = 180°$ [Sum of interior angles on the same side of transversal is 180°.]

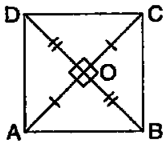
$\Rightarrow \quad 2\angle DAB = 180°$ [From (*i*)]

$\Rightarrow \quad \angle DAB = 90°$

As in a parallelogram, $\angle DAB = 90°$. Hence, the parallelogram is a rectangle.

3. *Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.*

Sol. Consider triangles AOB and COD,



$AO = OC$ [Given]

$OB = OD$ [Given]

$\angle AOB = \angle COD$ [90° each]

$\therefore \quad \Delta AOB \cong \Delta COD$ [SAS]

$\Rightarrow \quad AB = CD$ ...(*i*)

Similarly, we can show that BC = DA ...(*ii*)

Consider triangles AOB and BOC,

$AO = OC$ [Given]

BO is common.

and $\angle AOB = \angle BOC$ [90° each]

$\therefore \quad \Delta AOB \cong \Delta COB$ [SAS]

$\Rightarrow \quad AB = BC$ ...(*iii*)

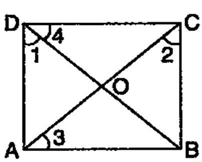
From (*i*), (*ii*) and (*iii*), we get

$AB = BC = CD = DA$

Hence, ABCD is a rhombus.

4. *Show that the diagonals of a square are equal and bisect each other at right angles.*

Sol. Consider triangles DAB and CBA,



$AD = BC$ [Sides of a square]

AB is common.

$\angle DAB = \angle CBA$ [90° each]

$\therefore \quad \Delta DAB \cong \Delta CBA$ [SAS]

$\Rightarrow \quad BD = AC$ [CPCT]

and $\quad \angle 1 = \angle 2$ ...(*i*) [CPCT]

Proving as above we can show $\angle 3 = \angle 4$. ...(*ii*)

Also, $\quad \angle 2 = \angle 3$

[$\because$ AB = BC, angles opposite to equal sides are equal.]

$\therefore \quad \angle 1 = \angle 4$ [From (*i*), (*ii*), (*iii*)]

Consider triangles AOD and COD,

$AD = DC$ [Sides of a square]

OD is common.

$\angle 1 = \angle 4$ [Proved above]

$\therefore \quad \Delta AOD \cong \Delta COD$ [SAS]

$\therefore \quad OA = OC$ ...(*iv*) [CPCT]

Similarly, we can show that

$$OB = OD$$

and $\angle AOD = \angle COD$ ...(*v*) [CPCT]

Also, $\angle AOD + \angle COD = 180°$ [Linear pair]

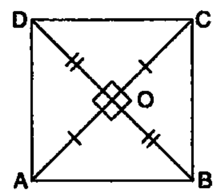
$\Rightarrow \quad 2\angle AOD = 180°$ [Using (*v*)]

$\Rightarrow \quad \angle AOD = 90°$ ...(*vi*)

Hence, diagonals are equal and bisect each other at right angles.

5. *Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.*

Sol. Consider triangles AOB and COD,



$AO = OC$ [Given]

$OB = OD$ [Given]

$\angle AOB = \angle COD$ [90° each]

$\therefore \quad \Delta AOB \cong \Delta COD$ [SAS]

$\Rightarrow \quad AB = CD$ ...(*i*)

Similarly, we can show that BC = DA ...(*ii*)

Consider triangles AOB and BOC,

$AO = OC$ [Given]

BO is common.

and $\angle AOB = \angle BOC$ [90° each]

$\therefore \quad \Delta AOB \cong \Delta COB$ [SAS]

$\Rightarrow \quad AB = BC$ ...(*iii*)

From (*i*), (*ii*) and (*iii*), we get

$$AB = BC = CD = DA$$

Hence, ABCD is a rhombus.

Further, consider ΔDAB and ΔCBA

$AD = BC$ [Proved rhombus]

AB is common

$BD = AC$ [Given]

$\therefore \quad \Delta DAB \cong \Delta CBA$ [SSS]

$\Rightarrow \quad \angle DAB = \angle CBA$ ...(*i*)

Also, as AD || BC (opposite sides of a rhombus) and AB is transversal.

$\therefore$ $\angle DAB + \angle CBA = 180°$ (Sum of interior angles on the same side of transversal is 180°.)

$\Rightarrow$ $2\angle DAB = 180°$ [From (*i*)]

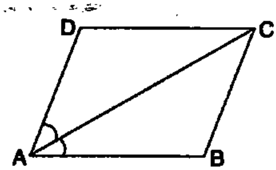
$\Rightarrow$ $\angle DAB = 90°$.

As in a rhombus one angle is 90°. Hence rhombus is a square.

6. *Diagonal AC of a parallelogram ABCD bisects ∠A (see figure). Show that*

(*i*) *it bisects ∠C also,*

(*ii*) *ABCD is a rhombus.*



Sol. (*i*) Consider triangles ABC and ADC,

AB = CD [Opposite sides of parallelogram]

AC is common.

AD = BC [Opposite sides of parallelogram]

$\therefore$ $\Delta DAC \cong \Delta BCA$ [SSS]

$\Rightarrow$ $\angle DAC = \angle BCA$ ...(*i*) [CPCT]

$\Rightarrow$ $\angle DCA = \angle BAC$ ...(*ii*) (CPCT)

Also, $\angle DAC = \angle BAC$ ...(*iii*) [Given]

$\Rightarrow$ $\angle DCA = \angle BCA$ [From (*i*), (*ii*), (*iii*)]

$\therefore$ AC bisects ∠C also.

(*ii*) In parallelogram $\angle DAB = \angle DCB$,

[Opposite angles of a parallelogram are equal.]

$\Rightarrow$ $\frac{1}{2}\angle DAB = \frac{1}{2}\angle DCB$.

$\Rightarrow$ $\angle DAC = \angle DCA$ [$\because$ AC is bisector of ∠A and ∠C.]

$\therefore$ CD = AD [Sides opposite to equal angles are equal.]

In parallelogram, as adjacent sides are equal, hence ABCD is a rhombus.

7. *ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.*

Sol. Consider triangles ADC and ABC,

AD = AB [Sides of a rhombus]

AC is common.

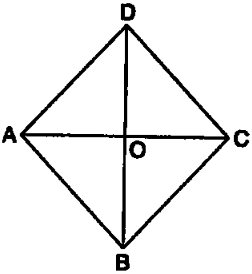
CD = CB [Sides of a rhombus]

∴ Δ ADC ≅ Δ ABC [SSS]

⇒ ∠DAC = ∠BAC ...(*i*) [CPCT]

and ∠DCA = ∠BCA ...(*ii*) [CPCT]

Hence AC bisects ∠A and ∠C.

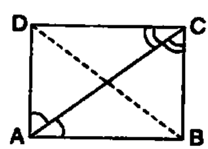


Similarly, by taking triangles BAD and BCD, we can show that BD bisects ∠B and ∠D.

8. *ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that:*

(*i*) *ABCD is a square*

(*ii*) *diagonal BD bisects ∠B as well as ∠D.*



Sol. (*i*) Consider triangles ADC and ABC,

∠DAC = ∠BAC [AC is bisector of ∠A]

∠DCA = ∠BCA [AC is bisector of ∠C]

AC is common.

∴ Δ ADC ≅ ΔABC [ASA]

AD = AB. [CPCT]

As in rectangle ABCD, adjacent sides are equal. Hence ABCD is a square.

(*ii*) Consider triangles DAB and BCD,

AB = BC = CD = DA [Sides of a square]

BD is common.

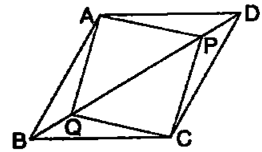
∴ Δ DAB ≅ Δ DCB [SSS]

∴ ∠ADB = ∠CDB [CPCT]

and $\angle ABD = \angle CBD$ [CPCT]

$\therefore$ BD bisects $\angle B$ and $\angle D$. [Using above results]

9. *In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see figure). Show that:*



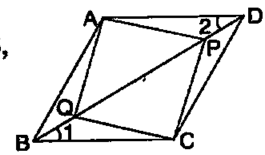
(*i*) $\Delta APD \cong \Delta CQB$

(*ii*) $AP = CQ$

(*iii*) $\Delta AQB \cong \Delta CPD$

(*iv*) $AQ = CP$

(*v*) *APCQ is a parallelogram.*



Sol. (*i*) Consider triangles APD and CQB,

AD || BC and BD is transversal.

$\therefore \quad \angle 1 = \angle 2$ [Alternate angles]

$AD = BC$ [Opposite sides of a parallelogram]

$DP = BQ$ [Given]

$\therefore \quad \Delta APD \cong \Delta CQB$. [SAS]

(*ii*) $AP = CQ$ [CPCT] [From result (*i*)]

(*iii*) Consider triangles AQB and CPD,

$AB = CD$ [Opposite sides of a parallelogram]

$\angle ABQ = \angle CDP$ [Alternate interior angles as AB || CD and BD is transversal]

$BQ = DP$ [Given]

$\therefore \quad \Delta AQB \cong \Delta CPD$ [SAS]

(*iv*) From result (*iii*),

$\Delta AQB \cong \Delta CPD$

$\therefore \quad AQ = CP$ [Corresponding sides]

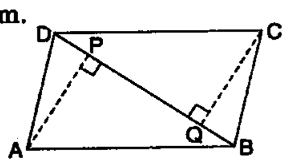
(*v*) In quadrilateral APCQ,

$AP = CQ$ [From result (*ii*)]

$AQ = CP$ [From result (*iv*)]

Thus, opposite sides of quadrilateral APCQ are equal.

Hence, APCQ is a parallelogram.



10. *ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see figure). Show that*

(i) $\Delta APB \cong \Delta CQD$

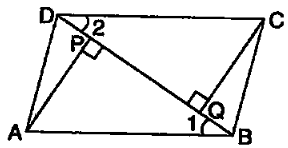
(ii) $AP = CQ$.

Sol. Consider triangles APB and CQD,

$\angle 1 = \angle 2$ [Alternate angles, AB || CD, BD is transversa]

$\angle APB = \angle DQC$ [90° each]

AB = CD [Opposite sides of a parallelogram]



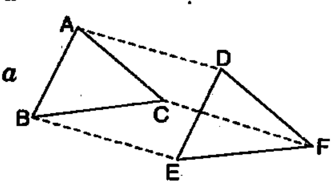
(i) ∴ $\Delta APB \cong \Delta CQD$ [AAS]

(ii) AP = CQ. [CPCT]

11. *In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see figure). Show that*

(i) *quadrilateral ABED is a parallelogram*

(ii) *quadrilateral BEFC is a parallelogram*



(iii) *AD || CF and AD = CF*

(iv) *quadrilateral ACFD is a parallelogram*

(v) *AC = DF*

(vi) *$\Delta ABC \cong \Delta DEF$.*

Sol. (i) Consider quadrilateral ABED,

AB = DE and AB || DE [Given]

⇒ ABED is a parallelogram. [In a quadrilateral if a pair of opposite sides is equal and parallel, then it is a parallelogram.]

(ii) Consider quadrilateral BEFC,

BC = EF and BC || EF [Given]

⇒ BEFC is a parallelogram. [Reason same as above]

(iii) From result (i), ABED is a parallelogram.

AD || BE and AD = BE

From result (ii), BEFC is a parallelogram.

BE || CF and BE = CF

$\Rightarrow$ AD || CF and AD = CF. [From results (*i*) and (*ii*)]

(*iv*) As AD || CF and AD = CF [From result (*iii*)]

$\therefore$ Quadrilateral ACFD is a parallelogram.

(*v*) AC = DF. [$\because$ ACFD is a parallelogram, result (*iv*)]

(*vi*) Consider triangles ABC and DEF,

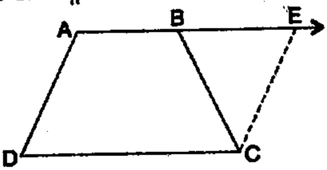
AB = DE [Given]

BC = EF [Given]

AC = DF [From result (*v*)]

$\therefore$ $\Delta ABC \cong \Delta DEF$. [SSS]

12. *ABCD is a trapezium in which AB || CD and AD = BC (see figure). Show that*



(*i*) $\angle A = \angle B$

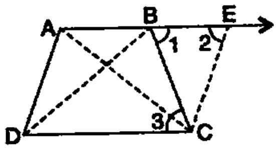
(*ii*) $\angle C = \angle D$

(*iii*) $\Delta ABC \cong \Delta BAD$

(*iv*) *diagonal AC = diagonal BD*

[**Hint:** *Extend AB and draw a line through C parallel to DA intersecting AB produced at E.*]

Sol. (*i*) **Construction:** Draw CE || AD, meeting AB produced at E.



Proof: AB || CD [Given]

and AD || CE [Construction]

$\therefore$ AECD is a parallelogram.

$\Rightarrow$ AD = CE [Opposite sides of a arallelogram]

Also, AD = BC [Given]

$\therefore$ CE = BC.

$\Rightarrow$ $\angle 1 = \angle 2$ ...(*i*)

[Angles opposite to equal sides are equal]

Also, $\angle D = \angle 2$ ...(*ii*)

[Opposite angles of a parallelogram]

and $\angle 1 = \angle 3$ ...(*iii*) [Alternate angles]

$\Rightarrow$ $\angle D = \angle 3$ [From equations (*i*), (*ii*), (*iii*)]

$\Rightarrow$ $\angle D = \angle C$

As AB || CD and AD, BC are transversals.

$\therefore$ $\angle A + \angle D = 180°$ ...(*iv*) [Sum of interior angles on the same side of transversal is 180°.]

$\angle B + \angle C = 180°$ ...(*v*) [Reason same as above]

Also, $\angle C = \angle D$ ...(*vi*) [Proved above]

$\therefore$ $\angle A = \angle B$ [From equations (*iv*), (*v*), (*vi*)]

(*iii*) **Construction:** Draw AC and BD.

Proof: Consider triangles DAB and CBA.

AD = BC [Given]

AB is common.

$\angle DAB = \angle CBA$ [Proved above]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [SAS]

(*iv*) AC = BD. [CPCT]

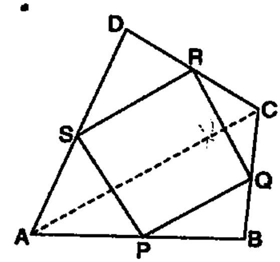
Exercise 8.2 *(Pages – 150-151)*

1. *ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see figure). AC is a diagonal. Show that:*

(*i*) *SR* $\parallel$ *AC and SR* $= \frac{1}{2}$*AC*

(*ii*) *PQ = SR*

(*iii*) *PQRS is a parallelogram.*



Sol. (*i*) Consider triangle ACD,

S and R are mid-points of sides AD and DC respectively.

$\therefore$ SR $\parallel$ AC and SR $= \frac{1}{2}$AC ...(*i*)

[Line segment joining mid-points of two sides of a triangle is parallel to the third and half of it.]

(*ii*) Consider triangle ABC, P and Q are mid-points of sides AB and BC respectively.

$\therefore$ PQ $\parallel$ AC and PQ $= \frac{1}{2}$AC ...(*ii*)

[Reason same as above]

From (*i*) and (*ii*),

$SR \parallel AC$ and $PQ \parallel AC \Rightarrow SR \parallel PQ$ ...(*iii*)

and $SR = \frac{1}{2}AC$ and $PQ = \frac{1}{2}AC \Rightarrow SR = PQ$. ...(*iv*)

(*iii*) $SR \parallel PQ$ and $SR = PQ$. [From (*iii*) and (*iv*)]

$\Rightarrow$ PQRS is a parallelogram.

2. *ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.*

Sol. First prove that PQRS is a parallelogram.

(*i*) Consider triangle ACD,

S and R are mid-points of sides AD and DC respectively.

$\therefore$ $SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(*i*)

[Line segment joining mid-points of two sides of a triangle is parallel to the third and half of it.]

(*ii*) Consider triangle ABC, P and Q are mid-points of sides AB and BC respectively.

$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(*ii*)

[Reason same as above]

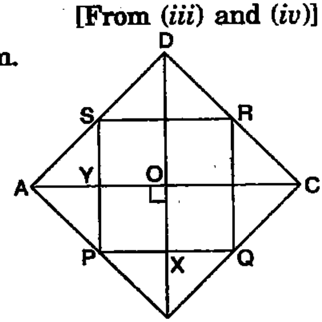
From (*i*) and (*ii*),

$SR \parallel AC$ and $PQ \parallel AC \Rightarrow SR \parallel PQ$ ...(*iii*)

and $SR = \frac{1}{2}AC$ and $PQ = \frac{1}{2}AC \Rightarrow SR = PQ$. ...(*iv*)

(*iii*) $SR \parallel PQ$ and $SR = PQ$. [From (*iii*) and (*iv*)]

$\Rightarrow$ PQRS is a parallelogram.



As $PX \parallel YO$ and $PY \parallel OX$, PXOY is a parallelogram.

$\Rightarrow$ $\angle YPX = \angle YOX = 90°$

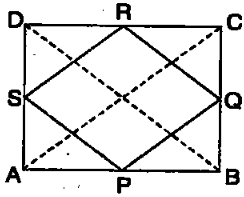
[$\because$ Diagonals of a rhombus bisect each other and are at right angles.]

As in parallelogram PQRS,
$\angle SPQ$ is 90°.

$\therefore$ PQRS is a rectangle.

3. *ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.*

Sol. Construction: Join AC and BD.



As ABCD is a rectangle.

$\therefore$ AC = BD ...(*i*)

Consider ΔABC, P and Q are mid-points of sides AB and BC respectively.

$$\therefore \quad PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC \quad ...(ii)$$

Similarly, consider ΔADC, S and R are mid-points of sides AD and DC respectively.

$$\therefore \quad SR \parallel AC \text{ and } SR = \frac{1}{2}AC \quad ...(iii)$$

From (*ii*) and (*iii*),

$$PQ = SR = \frac{1}{2}AC \quad ...(iv)$$

Similarly, we can show

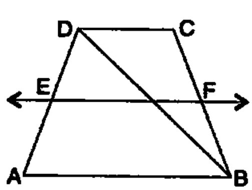
$$PS = QR = \frac{1}{2}BD \quad ...(v)$$

From (*i*), (*iv*) and (*v*), we have

$$PQ = QR = RS = SP$$

$\therefore$ PQRS is a rhombus.

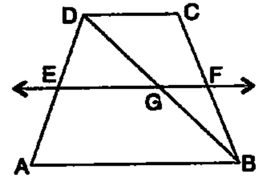
4. *ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see figure). Show that F is the mid-point of BC.*



Sol. Consider ΔADB, AB || EF $\Rightarrow$ AB || EG.

$\Rightarrow$ G is mid-point of BD. ...(*i*)

[$\because$ A line drawn through mid-point of one side, parallel to other bisects the third side.]



Consider triangle BCD,

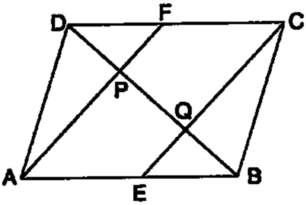
AB $\parallel$ CD and EF $\parallel$ AB

$\Rightarrow$ EF $\parallel$ CD $\Rightarrow$ GF $\parallel$ CD

$\Rightarrow$ F is mid-point of BC.

[Reason same as above]

5. *In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see figure). Show that the line segments AF and EC trisect the diagonal BD.*



Sol. AB = CD $\Rightarrow$ $\frac{1}{2}$AB = $\frac{1}{2}$CD

$\Rightarrow$ AE = CF

As AE = CF and AE $\parallel$ CF [$\because$ AB $\parallel$ CD]

$\Rightarrow$ AECF is a parallelogram.

$\Rightarrow$ AP $\parallel$ CE ...(*i*)

Consider triangle ABP,

E is mid-point of AB and EQ $\parallel$ AP [From (*i*)]

$\Rightarrow$ Q is mid-point of BP [A line segment drawn through mid-point of one side of a triangle and parallel to other, bisects the third side.]

BQ = PQ ...(*ii*)

Similarly, by considering triangle DCQ and proceeding as above, we can show that

DP = PQ ...(*iii*)

$\Rightarrow$ BQ = PQ = DP [From (*ii*) and (*iii*)]

$\Rightarrow$ P and Q trisect BD.

6. *Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.*

Sol. (*i*) Consider triangle ACD,

S and R are mid-points of sides AD and DC respectively.

$\therefore$ SR $\parallel$ AC and SR $= \frac{1}{2}$AC ...(*i*)

[Line segment joining mid-points of two sides of a triangle is parallel to the third and half of it.]

(*ii*) Consider triangle ABC, P and Q are mid-points of sides AB and BC respectively.

$\therefore$ PQ $\parallel$ AC and PQ $= \frac{1}{2}$AC ...(*ii*)

[Reason same as above]

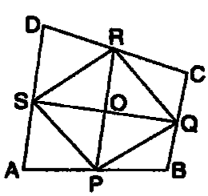
From (*i*) and (*ii*),

SR $\parallel$ AC and PQ $\parallel$ AC $\Rightarrow$ SR $\parallel$ PQ ...(*iii*)

and SR $= \frac{1}{2}$AC and PQ $= \frac{1}{2}$AC $\Rightarrow$ SR = PQ. ...(*iv*)

(*iii*) SR $\parallel$ PQ and SR = PQ. [From (*iii*) and (*iv*)]

$\Rightarrow$ PQRS is a parallelogram.



We know that diagonals of a parallelogram bisect each other, *i.e.*, OP = OR and OQ = OS.

Hence, line segments joining mid-points of opposite sides of a quadrilateral bisect each other.

7. *ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that*

(*i*) *D is the mid-point of AC*

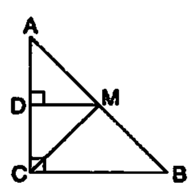
(*ii*) *MD* $\perp$ *AC*

(*iii*) *CM = MA =* $\frac{1}{2}$*AB.*

Sol. (*i*) MD $\parallel$ BC, meets AC at D.

$\therefore$ D is mid-point of AC.

[A line through the mid-point of a side of a triangle parallel to other bisects and third side.]



(ii) MD || BC and AC is transversal.

$\therefore$ $\angle ADM = \angle ACB$ [Corresponding angles]

$\Rightarrow$ $\angle ADM = 90°$ [$\because$ $\angle ACB = 90°$]

$\Rightarrow$ $MD \perp AC$.

(iii) Consider triangles ADM and CDM,

AD = DC [From result (i)]

MD is common.

$\angle ADM = \angle CDM$ [90° each] [From result (ii)]

$\therefore$ $\Delta ADM \cong \Delta CDM$ [SAS]

$$\therefore \quad MA = CM = \frac{1}{2}AB.$$

[$\because$ M is mid-point of AB)]