9 Areas of Parallelograms and Triangles

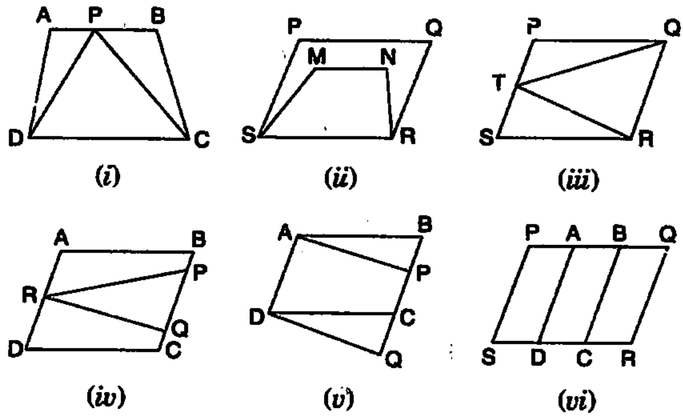
Lesson at a Glance

1. Congruent figures have the same shape and the same size.
2. Congruent figures have equal areas.
3. Figures having equal areas may or may not be congruent.
4. If the figures have a common base and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base, then the figures are said to be on the same base and between the same parallels.
5. Parallelograms on the same base (or equal bases) and between the same parallels are equal in area.
6. Triangles on the same base (or equal bases) and between the same parallels are equal in area.
7. If a triangle and a parallelogram are on the same base (or equal bases) and between the same parallels, then the area of the triangle is half of the area of the parallelogram.
8. Each median of a triangle divides it into two triangles of equal areas.
9. Area of a triangle is half the product of its base and the corresponding altitude.
10. Area of a parallelogram is the product of its base and the corresponding altitude.
11. The area of a trapezium is half of the product of sum of two parallel sides and the perpendicular distance between them.
12. The area of a rhombus is half of the product of the lengths of its diagonals.
13. The diagonals of a parallelogram divide it into four triangles of equal areas.

TEXTBOOK QUESTIONS SOLVED

Exercise 9.1 *(Page – 155)*

1. ***Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.***



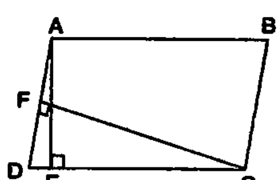
Sol. (*i*) Trapezium ABCD and triangle PDC lie on the same base CD and between the same parallels AB and CD.

(*iii*) Parallelogram PQRS and triangle TQR lie on the same base QR and between the same parallels PS and QR.

(*v*) Trapezium ABQD and APCD lie on the same base AD and between the same parallels AD and BQ.

Exercise 9.2 *(Pages – 159-160)*

1. *In figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.*



Sol. We have AD × CF = AE × CD ...(*i*)

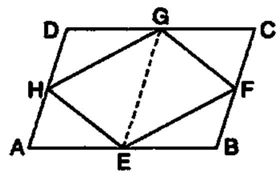
But AB = CD = 16 cm [Opposite sides of a parallelogram]

$\therefore$ AD × 10 = 8 × 16 $\Rightarrow$ AD = 12.8 cm.

2. *If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that*

$$ar(EFGH) = \frac{1}{2} ar(ABCD).$$

Sol. Construction: Join E and G.



Proof: In parallelogram ABCD,

AB = CD and AB ∥ CD.

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow$ DG = AE [$\because$ E and G are mid-points of AB and CD]

We have DG = AE and DG ∥ AE.

$\therefore$ AEGD is a parallelogram.

Now parallelogram AEGD and triangle HEG are on the same base and between the same parallels.

$$\therefore \quad ar(HEG) = \frac{1}{2} ar(AEGD) \quad ...(i)$$

Similarly, we can show that

$$ar(FGE) = \frac{1}{2} ar(EBCG) \quad ...(ii)$$

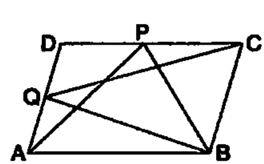
From (*i*) and (*ii*), we get

$$ar(HEG) + ar(FGE) = \frac{1}{2} [ar(AEGD) + ar(EBCG)]$$

$$\Rightarrow \quad ar(EFGH) = \frac{1}{2} ar(ABCD).$$

3. *P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar(APB) = ar(BQC).*

Sol. Triangle PAB and parallelogram ABCD are on the same base AB and between the same parallels AB and DC.



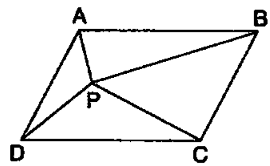
$\therefore \quad ar(APB) = \frac{1}{2} ar(ABCD)$ ...(*i*)

Similarly, triangle QBC and parallelogram ABCD are on the same base BC and between the same parallels AD and BC.

$\therefore \quad ar(QBC) = \frac{1}{2} ar(ABCD)$ ...(*ii*)

Hence, $ar(APB) = ar(QBC)$. [From (*i*) and (*ii*)]

4. *In Figure, P is a point in the interior of a parallelogram ABCD. Show that*

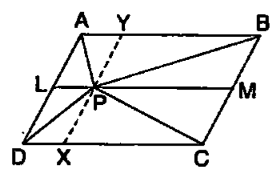


(*i*) $ar(APB) + ar(PCD) = \frac{1}{2} ar(ABCD)$

(*ii*) $ar(APD) + ar(PBC) = ar(APB) + ar(PCD)$

[**Hint:** *Through P, draw a line parallel to AB.*]

Sol. (*i*) **Construction:** Through P draw LM || AB, meeting AD at L and BC at M.



Proof: AD || BC

[Opposite side of a parallelogram]

$\Rightarrow$ AL || BM ...(*i*)

Also AB || LM ...(*ii*) [Construction]

$\Rightarrow$ ABML is a parallelogram. [From (*i*), (*ii*)]

Now parallelogram ABML and triangle ABP are on the same base AB and between the same parallels AB and LM.

$\therefore \quad ar(APB) = \frac{1}{2} ar(ABML)$ ...(*iii*)

Similarly we can show that

$ar(PDC) = \frac{1}{2} ar(LMCD)$ ...(*iv*)

Adding (*iii*) and (*iv*), we get

$$ar(APB) + ar(PDC) = \frac{1}{2} ar(ABML) + \frac{1}{2} ar(LMCD)$$

$$= \frac{1}{2}[ar(\text{ABML}) + ar(\text{LMCD})]$$

$$\Rightarrow \quad ar(\text{APB}) + ar(\text{PDC}) = \frac{1}{2}ar(\text{ABCD})$$

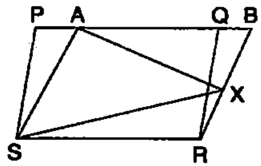
(*ii*) Similarly, by drawing XY || AD, through P, we can show that

$$ar(\text{APD}) + ar(\text{BPC}) = \frac{1}{2}ar(\text{ABCD})$$

$$\Rightarrow \quad ar(\text{APD}) + ar(\text{BPC}) = ar(\text{APB}) + ar(\text{PDC}).$$

[From result of part (*i*)]

5. *In figure PQRS and ABRS are parallelograms and X is any point on side BR. Show that*

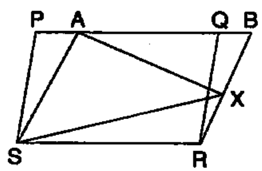


(*i*) *ar(PQRS) = ar(ABRS)*

(*ii*) $ar(AXS) = \frac{1}{2}ar(PQRS)$.

Sol. (*i*) As parallelograms PQRS and ABRS are on the same base SR and between the same parallels SR and PB.

Hence, *ar*(PQRS) = *ar*(ABRS).



(*ii*) Triangle XAS and parallelogram ABRS are on the same base AS and between the same parallels AS and BR.

$$\therefore \quad ar(\text{AXS}) = \frac{1}{2}ar(\text{ABRS})$$

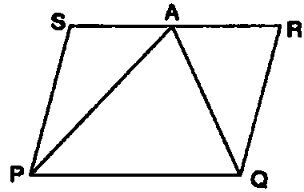
But *ar*(ABRS) = *ar*(PQRS) [From result of part (*i*)]

$$\therefore \quad ar(\text{AXS}) = \frac{1}{2}ar(\text{PQRS}).$$

6. *A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?*

Sol. The field is divided into three parts as: triangle PSA; triangle PAQ; triangle ARQ.

As triangle PAQ and parallelogram PQRS are on the same base PQ and between the same parallels PQ and RS.



$$\therefore \quad ar(PAQ) = \frac{1}{2} ar(PQRS) \quad ...(i)$$

From above result, we can say that

$$ar(PSA) + ar(QAR) = \frac{1}{2} ar(PQRS) \quad ...(ii)$$

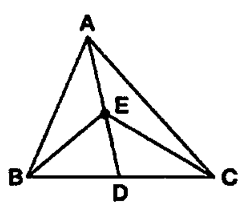
From (*i*) and (*ii*), farmer has two options:

Option I: Wheat in *ar*(PAQ) and pulses in *ar*(PSA) + *ar*(QAR).

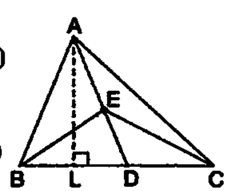
Option II: Pulses in *ar*(PAQ) and wheat ir *ar*(PSA) + *ar*(QAR).

Exercise 9.3 *(Pages – 162-164)*

1. *In figure, E is any point on median AD of a ΔABC. Show that $ar(ABE) = ar(ACE)$.*



Sol. Construction: Draw AL $\perp$ BC, which meets BC at L.



Proof: $ar(ABD) = \frac{1}{2} \times BD \times AL \quad ...(i)$

$ar(ACD) = \frac{1}{2} \times DC \times AL \quad ...(ii)$

Also, $BD = DC$ ...(*iii*) [$\because$ AD is median]

From (*i*), (*ii*), (*iii*), we get

$$ar(ABD) = ar(ACD) \quad ...(iv)$$

Similarly, in triangle BEC, ED is the median, proceeding as above,

we have

$$\therefore \quad ar(BED) = ar(CED) \quad ...(v)$$

From (*iv*) and (*v*), we get

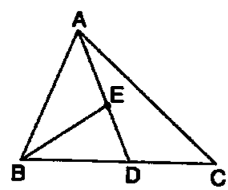
$$ar(ABD) - ar(BED) = ar(ACD) - ar(CED)$$

$$\therefore \quad ar(ABE) = ar(ACE).$$

2. *In a triangle ABC, E is the mid-point of median AD. Show that*

$$ar(BED) = \frac{1}{4} ar(ABC).$$

Sol. AD is median of triangle ABC.



$$\Rightarrow \quad ar(ABD) = ar(ADC)$$

[Median of a triangle divides the triangle into two parts of equal area.]

$$\Rightarrow \quad ar(ABD) = \frac{1}{2} ar(ABC) \quad ...(i)$$

Also, BE is median of triangle ABD.

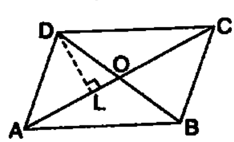
$$\Rightarrow \quad ar(BED) = \frac{1}{2} ar(ABD) \quad \text{[Reason same as above]}$$

$$\Rightarrow \quad ar(BED) = \frac{1}{2} \times \frac{1}{2} ar(ABC)$$

$$\Rightarrow \quad ar(BED) = \frac{1}{4} ar(ABC).$$

3. *Show that the diagonals of a parallelogram divide it into four triangles of equal area.*

Sol. Construction: Draw DL perpendicular to AC.



Proof: We know that diagonals of a parallelogram bisect each other.

i.e., OA = OC and OB = OD ...(*i*)

$$ar(AOD) = \frac{1}{2} \times AO \times DL \quad ...(ii)$$

$ar(DOC) = \frac{1}{2} \times OC \times DL$ ...(*iii*)

From (*i*), (*ii*), (*iii*), we get

$ar(AOD) = ar(DOC)$ ...(*iv*)

Similarly, by drawing perpendiculars from C and B on BD and AC respectively, we can show that

$ar(DOC) = ar(COB)$ ...(*v*)

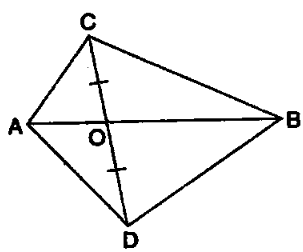
and $ar(BOC) = ar(AOB)$ ...(*vi*)

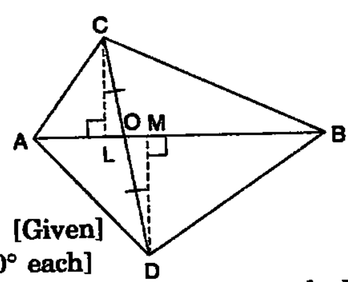
From (*iv*), (*v*) and (*vi*), we get

$ar(AOB) = ar(BOC) = ar(COD) = ar(AOD)$.

4. ***In figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that***

$ar(ABC) = ar(ABD)$.





Sol. Construction: Draw perpendiculars CL and DM on AB.

Proof: Consider triangles CLO and DMO.

$OC = OD$ ...(*i*) [Given]

$\angle CLO = \angle DMO$ ...(*ii*) [90° each]

$\angle COL = \angle DOM$ ...(*iii*) [Vertically opposite angles]

From (*i*), (*ii*) and (*iii*),

$\Delta CLO \cong \Delta DMO$ [AAS]

$\therefore$ $CL = DM$ ...(*iv*) [CPCT]

Now, $ar(ABC) = \frac{1}{2} \times AB \times CL$ ...(*v*)

and $ar(ADB) = \frac{1}{2} \times AB \times DM$ ...(*vi*)

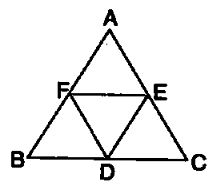
$\therefore$ ar(ABC) = ar(ABD). [From (*iv*), (*v*), (*vi*)]

5. *D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that*

(*i*) *BDEF is a parallelogram* (*ii*) *ar(DEF)* = $\frac{1}{4}$ *ar(ABC)*

(*iii*) *ar(BDEF)* = $\frac{1}{2}$ *ar(ABC).*

Sol. (*i*) **Proof:** Since E and F are respectively the mid-points of AC and AB of Δ ABC.



$\therefore$ EF ∥ BC and EF = $\frac{1}{2}$ BC [Mid-point theorem]

Also, BD = $\frac{1}{2}$ BC [$\because$ D is mid-point of BC]

$\therefore$ EF ∥ BD and EF = BD.

$\Rightarrow$ BDEF is a parallelogram. [In a quadrilateral, if a pair of opposite sides is equal and parallel, then it is a parallelogram.]

(*ii*) DF is diagonal.

$\therefore$ ar(BFD) = ar(DEF) ...(*i*) [Diagonal of a parallelogram divides it into two triangles of equal area.]

Similarly, we can show that

ar(DEF) = ar(CDE) = ar(AFE) ...(*ii*)

From equations (*i*) and (*ii*),

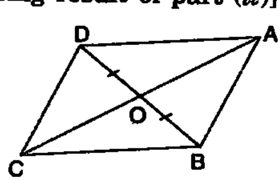
$$ar(ABC) = 4ar(DEF) \Rightarrow ar(DEF) = \frac{1}{4}ar(ABC).$$

(*iii*) ar(BDEF) = ar(BDF) + ar(DEF) = 2ar(DEF) [From (*i*)]

$$= 2 \times \frac{1}{4}ar(ABC) = \frac{1}{2}ar(ABC).$$

[Using result of part (*ii*)]

6. *In figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:*



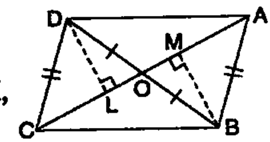
(*i*) *ar(DOC) = ar(AOB)*

(*ii*) *ar(DCB) = ar(ACB)*

(*iii*) *DA || CB or ABCD is a parallelogram.*

[**Hint:** *From D and B, draw perpendiculars to AC.*]

Sol. (*i*) **Construction:** Draw DL and BM perpendiculars to AC.



Consider triangles DOL and BOM,

OB = OD [Given]

∠DLO = ∠BMO [Construction]

∠DOL = ∠BOM [Vertically opposite angles]

∴ ΔDLO ≅ ΔBMO [AAS]

∴ DL = BM ...(*i*) [CPCT]

Consider triangles CLD and AMB,

DL = BM [Proved above] [From (*i*)]

DC = AB [Given]

∠DLC = ∠BMA [90° each]

∴ ΔDCL ≅ ΔBAM [RHS]

∴ ∠DCL = ∠BAM ...(*ii*) [CPCT]

Consider triangles COD and AOB,

OD = OB [Given]

∠COD = ∠AOB [Vertically opposite angles]

∠DCO = ∠BAO [From (*ii*)]

∴ ΔCOD ≅ ΔAOB [AAS]

⇒ *ar*(COD) = *ar*(AOB)

(*ii*) From result of part (*i*),

ar(AOB) = *ar*(COD)

⇒ *ar*(AOB) + *ar*(BOC) = *ar*(COD) + *ar*(BOC)

⇒ *ar*(ABC) = *ar*(DCB)

(*iii*) AB = CD and ∠DCO = ∠BAO

[Given and from result (*ii*)]

⇒ AB = CD and AB || CD.

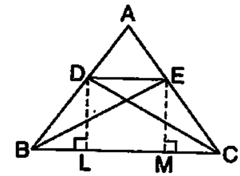
Therefore, ABCD is a parallelogram.

[In a quadrilateral, if a pair of opposite sides is equal and parallel, then it is a parallelogram.]

$\therefore$ AD || BC.

7. *D and E are points on sides AB and AC respectively of ΔABC such that ar(DBC) = ar(EBC). Prove that DE || BC.*

Sol. Construction: Draw DL and EM perpendicular to BC.



Proof: $ar(DBC) = ar(EBC)$ [Given]

$\Rightarrow \frac{1}{2} \times BC \times DL = \frac{1}{2} \times BC \times EM$

$\Rightarrow$ DL = EM

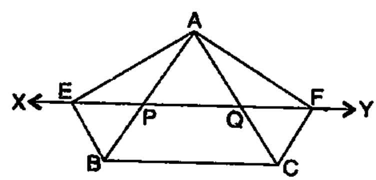
As distance between two given lines BC and DE is same at different points.

$\therefore$ DE || BC.

8. *XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that*

ar(ABE) = ar(ACF)

Sol. Triangle ABE and parallelogram BCQE are on the same base BE and between the same parallels BE and AC.



$\therefore$ $ar(ABE) = \frac{1}{2} ar(BCQE)$ ...(*i*)

Similarly, we can show that

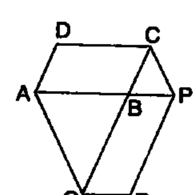
$ar(ACF) = \frac{1}{2} ar(BCFP)$ ...(*ii*)

and $ar(BCQE) = ar(BCFP)$ ...(*iii*)

From (*i*), (*ii*), (*iii*), we get

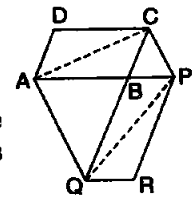
$ar(ABE) = ar(ACF)$.

9. *The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see figure). Show that ar(ABCD) = ar(PBQR).*



[**Hint:** *Join AC and PQ. Now compare ar(ACQ) and ar(APQ).*]

Sol. Construction: Draw diagonals AC and PQ.



Proof: AQ || CP

Triangles ACQ and APQ are on the same base AQ and between the same parallels AQ and PC.

$\therefore \quad ar(ACQ) = ar(APQ)$

$\Rightarrow \quad ar(ACB) + ar(ABQ) = ar(ABQ) + ar(PBQ)$

$\Rightarrow \quad ar(ACB) = ar(PBQ)$ ...(*i*)

Also, $\quad ar(ACB) = \frac{1}{2} ar(ABCD)$ ...(*ii*)

[Diagonals of a parallelogram divide the parallelogram into two parts of equal area.]

and $\quad ar(PBQ) = \frac{1}{2} ar(PBQR)$ ...(*iii*) [Reason same as above]

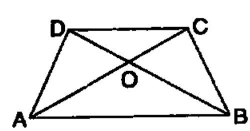
From (*i*), (*ii*), (*iii*), we get

$$\frac{1}{2} ar(ABCD) = \frac{1}{2} ar(PBQR)$$

$\Rightarrow \quad ar(ABCD) = ar(PBQR).$

10. *Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar(AOD) = ar(BOC).*

Sol. Triangles DAB and CAB are on the same base AB and between the same parallels AB and DC.

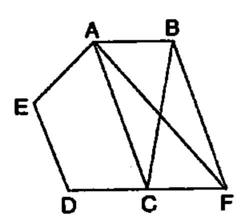


$\therefore \quad ar(DAB) = ar(CAB)$

$\Rightarrow \quad ar(DOA) + ar(OAB) = ar(OAB) + ar(COB)$

$\Rightarrow \quad ar(AOD) = ar(BOC).$

11. *In figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that*



(*i*) *ar(ACB) = ar(ACF)*

(*ii*) *ar(AEDF) = ar(ABCDE)*

Sol. Triangles ABC and AFC are on the same base AC and between the same parallels AC and BF.

(i) $\therefore$ $ar(ACB) = ar(ACF)$.

(ii) Adding $ar(AEDC)$ to both sides, we get

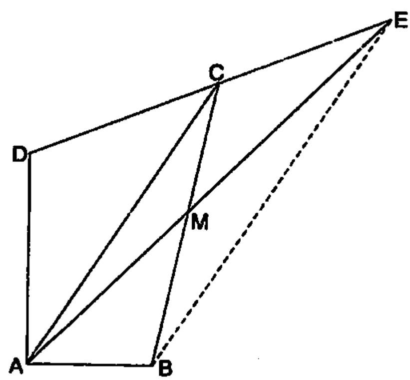
$$ar(ACB) + ar(AEDC) = ar(ACF) + ar(AEDC)$$

$\Rightarrow$ $ar(ABCDE) = ar(AFCDE)$

or $ar(ABCDE) = ar(AEDF)$

12. *A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.*

Sol. Let the villager's plot be quadrilateral ABCD. Let the corner ABM be taken over to construct the Health Centre in a way that AM produced meets the side DC produced at E and is such that diagonal AC is parallel to BE as shown in the adjoining figure.



Now, ΔACE and ΔACB are on the same base AC and between the same parallels AC and BE.

Therefore, $ar(ACE) = ar(ACB)$

$\Rightarrow$ $ar(ACM) + ar(CEM) = ar(ACM) + ar(ABM)$

$$ar(CEM) = ar(ABM) \quad ...(i)$$

Thus, the land CEM is provided to the villager according to the condition.

Equation (i) shows that the area of the Health Centre is equal to the area of the land provided to the villager.

Add *ar*(ADCM) to both sides of equation (*i*), to get

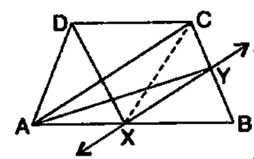
$$ar(\text{CEM}) + ar(\text{ADCM}) = ar(\text{ABM}) + ar(\text{ADCM})$$

$\Rightarrow$ $ar(\text{ADE}) = ar(\text{ABCD})$.

13. *ABCD is a trapezium with AB ∥ DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar(ADX) = ar(ACY).*

[Hint: *Join CX.*]

Sol. Join CX. Triangles ADX and ACX are on the same base AX and between the same parallels AB and CD.



$\therefore$ $ar(\text{ADX}) = ar(\text{ACX})$ ...(*i*)

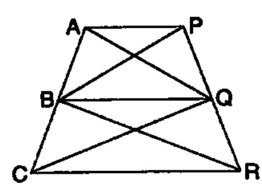
Triangles ACX and ACY are on the same base AC and between the same parallels AC and XY.

$\therefore$ $ar(\text{ACX}) = ar(\text{ACY})$ ...(*ii*)

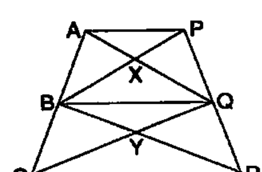
From (*i*) and (*ii*), we get

$$ar(\text{ADX}) = ar(\text{ACY}).$$

14. *In figure, AP ∥ BQ ∥ CR. Prove that ar(AQC) = ar(PBR).*



Sol. Triangles ABQ and PBQ are in the same base BQ and between the same parallels AP and BQ.



$\therefore$ $ar(\text{ABQ}) = ar(\text{PBQ})$

$\Rightarrow$ $ar(\text{AXB}) + ar(\text{BXQ}) = ar(\text{BXQ}) + ar(\text{PXQ})$

$\Rightarrow$ $ar(\text{AXB}) = ar(\text{PXQ})$ ...(*i*)

Similarly, we can show that

$ar(\text{BYC}) = ar(\text{QYR})$ ...(*ii*)

Adding (*i*) and (*ii*), we get

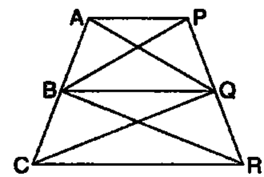
$$ar(\text{AXB}) + ar(\text{BYC}) = ar(\text{PXQ}) + ar(\text{QYR})$$

Adding $ar(BXQY)$ to both sides, we get

$$ar(AXB) + ar(BYC) + ar(BXQY) = ar(PXQ) + ar(QYR) + ar(BXQY)$$

$$\Rightarrow \quad ar(AQC) = ar(PBR).$$

Alternative Method:



ΔBAQ and ΔBPQ are on the same base BQ and between same parallels AP and BQ.

$\therefore \quad ar(BAQ) = ar(BPQ) \quad ...(i)$

Also, ΔBCQ and ΔBRQ are on the same base BQ and between the same parallels CR and BQ.

$\therefore \quad ar(BCQ) = ar(BRQ) \quad ...(ii)$

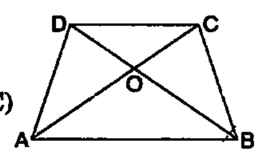
Adding equations (*i*) and (*ii*), we have

$$ar(BAQ) + ar(BCQ) = ar(BPQ) + ar(BRQ)$$

$$\Rightarrow \quad ar(AQC) = ar(PBR).$$

15. *Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar(AOD) = ar(BOC). Prove that ABCD is a trapezium.*

Sol. $ar(AOD) = ar(BOC)$ [Given]



Adding $ar(AOB)$ to both sides, we get

$ar(AOD) + ar(AOB) = ar(AOB) + ar(BOC)$

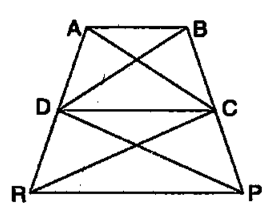
$\Rightarrow \quad ar(ADB) = ar(ACB) \quad ...(i)$

Triangles ADB and ACB are on the same base AB and as their areas are equal, so these must be in the same parallels.

$\therefore \quad AB \parallel CD.$

Hence, ABCD is a trapezium.

16. *In figure, ar(DRC) = ar(DPC) and ar(BDP) = ar(ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.*



Sol. $ar(DRC) = ar(DPC)$ [Given]

As triangles DRC and DPC are on the same base DC and their areas are same, hence these must be in the same parallels.

∴ DC || RP. Hence, DCPR is a trapezium.

Now, $ar(BDP) = ar(ARC)$ [Given]

Also, $ar(DPC) = ar(DRC)$ [Given]

∴ $ar(BDP) - ar(DPC) = ar(ARC) - ar(DRC)$

⇒ $ar(BDC) = ar(ADC)$

Now, triangles ADC and BDC are on the same base DC and their areas are same. Hence, they must be between the same parallels.

∴ AB || DC. Hence, ABCD is a trapezium.

Exercise 9.4 *(Pages – 164-166)*

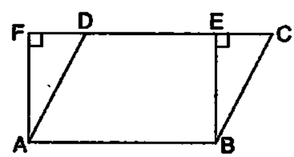
1. *Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.*

Sol. Here, AB = CD = EF. ...(*i*)

Also in triangle BEC,

BC > BE

[Side opposite to greater angle is larger]



Similarly, AD > AF

[Reason same as above]

∴ BC + AD > BE + AF

⇒ BC + AD + AB + DC > BE + AF + AB + EF

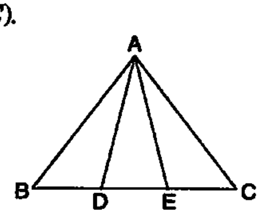
[From (*i*)]

⇒ Perimeter of parallelogram ABCD > Perimeter of rectangle ABEF.

2. *In figure, D and E are two points on BC such that BD = DE = EC. Show that*

ar(ABD) = ar(ADE) = ar(AEC).

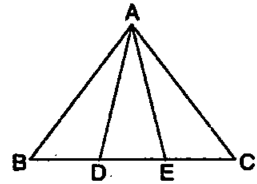
Can you now answer the question that you have in the 'Introduction' of this chapter in NCERT, whether the field of Budhia has been actually divided into three parts of equal areas?



[Remark: *Note that by taking BD = DE = EC, the triangle ABC is divided into three triangles ABD, ADE and*

AEC of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into n triangles of equal areas.]

Sol. In triangle ABE, BD = DE [Given]



$\therefore$ AD is median.

$\therefore$ $ar(ABD) = ar(ADE)$ ...(*i*)

[Median of a triangle divides the triangle into two parts of equal area]

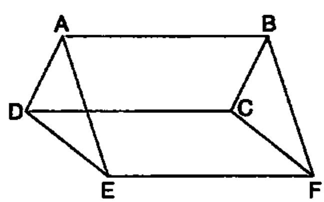
Similarly, $ar(ADE) = ar(AEC)$ ...(*ii*)

From (*i*) and (*ii*), we have

$$ar(ABD) = ar(ADE) = ar(AEC).$$

Hence, we answer the question in the 'Introduction' of this chapter in NCERT that Budhia has actually divided her field into three parts of equal areas.

3. *In figure, ABCD, DCFE and ABFE are parallelograms. Show that ar(ADE) = ar(BCF).*



Sol. As ABCD, DCEF and ABFE are parallelograms.

$\therefore$ AD = BC, DE = CF and AE = BF. ...(*i*)

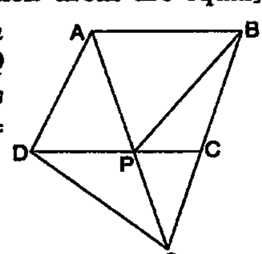
$\therefore$ $\Delta ADE \cong \Delta BCF$. [SSS] [From (*i*)]

$\Rightarrow$ $ar(ADE) = ar(BCF)$.

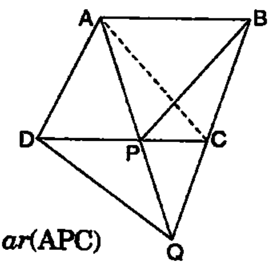
[If triangles are congruent, their areas are equal]

4. *In figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersects DC at P, show that ar(BPC) = ar(DPQ).*

[**Hint:** *Join AC.*]



Sol. Construction: Join AC.



Proof: Triangles ADQ and ADC are on the same base AD and between the same parallels AD and BQ.

$\therefore$ $ar(ADQ) = ar(ADC)$

$\Rightarrow$ $ar(APD) + ar(DPQ) = ar(ADP) + ar(APC)$

$\Rightarrow$ $ar(DPQ) = ar(APC)$ ...(*i*)

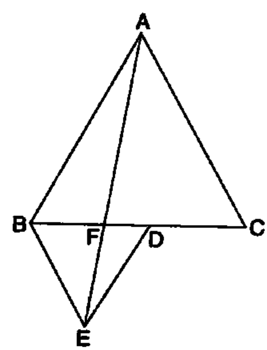
Triangles APC and BPC are on the same base PC and between the same parallels PC and AB.

$\therefore$ $ar(APC) = ar(BPC)$ ...(*ii*)

From (*i*) and (*ii*), we get

$ar(DPQ) = ar(BPC)$.

5. *In figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that*



(*i*) $ar(BDE) = \frac{1}{4} ar(ABC)$

(*ii*) $ar(BDE) = \frac{1}{2} ar(BAE)$

(*iii*) $ar(ABC) = 2\, ar(BEC)$

(*iv*) $ar(BFE) = ar(AFD)$

(*v*) $ar(BFE) = 2ar(FED)$

(*vi*) $ar(FED) = \frac{1}{8} ar(AFC)$

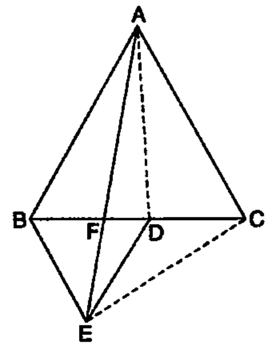
[**Hint:** *Join EC and AD. Show that BE || AC and DE || AB, etc.*]

Sol. Let $AB = BC = AC = x$, then $BD = BE = ED = \frac{x}{2}$

(*i*) $ar(ABC) = \frac{\sqrt{3}}{4}x^2$

and $ar(BDE) = \frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2 = \frac{\sqrt{3}}{16}x^2$

$\Rightarrow \quad ar(BDE) = \frac{1}{4} ar(ABC)$



(*ii*) $\angle ACB = 60°$ and $\angle DBE = 60°$
or $\angle CBE = 60°$

$\Rightarrow$ BE || AC

[∵ Alternate angles are equal.]

∴ $ar(BAE) = ar(BCE)$ ...(*i*)

[Triangles on the same base and between the same parallels are equal in areas.]

Since DE is median of ΔBEC.

∴ $ar(BED) = \frac{1}{2} ar(BEC)$

[Median divides triangle into two parts of equal areas]

$\Rightarrow \quad ar(BDE) = \frac{1}{2} ar(BAE).$ [From (*i*)]

(*iii*) $ar(BCE) = 2\, ar(BDE)$

[Median divides triangle into two parts of equal areas.]

$= 2 \times \frac{1}{4} ar(ABC)$ [From result of part (*i*)]

$\Rightarrow \quad 2\, ar(BCE) = ar(ABC).$

(*iv*) Proceeding as result of part (*ii*), we get ED || AB.

∴ $ar(ADE) = ar(BED)$

[Triangles on the same base and between the same parallels are equal in area.]

$\Rightarrow \quad ar(ADE) - ar(EFD) = ar(BED) - ar(EFD)$

$\Rightarrow \quad ar(AFD) = ar(BEF).$

(*v*) Draw EL ⊥ BD. Also AD ⊥ BC

Then $\quad EL = \sqrt{\left(\frac{x}{2}\right)^2 - \left(\frac{x}{4}\right)^2} = \frac{\sqrt{3}}{4}x$

Also $\quad AD = \frac{\sqrt{3}}{2}x$

$$ar(\Delta AFD) = \frac{1}{2} \cdot FD \cdot AD$$

$$ar(\Delta FED) = \frac{1}{2} FD \cdot EL$$

$$\therefore \quad \frac{ar(\Delta AFD)}{ar(\Delta FED)} = \frac{AD}{EL} = \frac{\sqrt{3}}{2}x \times \frac{4}{\sqrt{3}x} = 2$$

$$\Rightarrow \quad ar(\Delta AFD) = 2ar(\Delta FED)$$

$$\Rightarrow \quad ar(\Delta BFE) = 2ar(\Delta FED) \qquad \text{[From result } (iv)]$$

(*vi*) From result (*v*),

$$ar(\Delta BFE) = 2ar(\Delta FED)$$

$$\Rightarrow \quad BF = 2FD$$

$$\therefore \quad FC = FD + DC = FD + BD = FD + 3FD = 4FD$$

$$\therefore \quad ar(\Delta FED) = \frac{1}{2} \cdot FD \cdot EL = \frac{1}{2} \cdot FD \cdot \frac{\sqrt{3}}{4}x$$

[From result (*v*)]

$$= \frac{\sqrt{3}}{8} FD \cdot x \qquad \ldots(i)$$

$$ar(\Delta AFC) = \frac{1}{2} \cdot FC \cdot AD = \frac{1}{2} \cdot 4FD \cdot \frac{\sqrt{3}}{2}x$$

$$= \sqrt{3} \cdot FD \cdot x$$

$$\therefore \quad = 8\left[\frac{\sqrt{3}}{8} FD \cdot x\right] \qquad \text{[From } (i)]$$

$$= 8 \text{ or } (\Delta FED)$$

$$\therefore \quad ar(\Delta FED) = \frac{1}{8} ar(\Delta AFC).$$

6. *Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that*

ar(APB) × ar(CPD) = ar(APD) × ar(BPC).

[**Hint:** *From A and C, draw perpendiculars to BD.*]

Sol. $ar(APB) \times ar(CPD)$

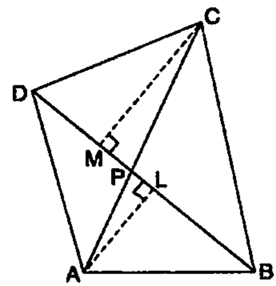
$$= \left(\frac{1}{2} \times BP \times AL\right)\left(\frac{1}{2} \times PD \times CM\right)$$

$$= \frac{1}{4} \times BP \times DP \times AL \times CM \quad ...(i)$$

And $ar(APD) \times ar(BPC)$

$$= \left(\frac{1}{2} \times PD \times AL\right)\left(\frac{1}{2} \times BP \times CM\right)$$

$$= \frac{1}{4} \times BP \times DP \times AL \times CM \quad ...(ii)$$



From (*i*) and (*ii*), we get

$$ar(APB) \times ar(CPD) = ar(APD) \times ar(BPC).$$

7. *P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that*

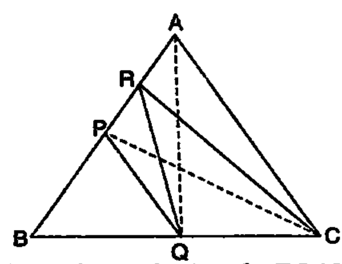
(*i*) $ar(PRQ) = \frac{1}{2} ar(ARC)$ (*ii*) $ar(RQC) = \frac{3}{8} ar(ABC)$

(*iii*) $ar(PBQ) = ar(ARC)$

Sol. (*i*) **Construction:** Join A and Q; P and C.

Proof: QP is median of triangle BQA.

$\therefore$ $ar(BQP) = ar(PQA)$ ...(*i*)



Similarly, $ar(QPR)$

$= ar(RQA)$ [QR is median of triangle PQA]

$\Rightarrow$ $ar(PQA) = 2\,ar(QPR)$ [From above result]

$\therefore$ $2\,ar(QPR) = ar(BQP)$ [From (*i*)]

$= \frac{1}{2} ar(BPC)$ [PQ is median]

$= \frac{1}{2} ar(APC)$ [CP is median]

$= ar(ARC)$ [RC is median]

$$\Rightarrow \quad ar(QPR) = \frac{1}{2} ar(ARC).$$

(ii) $ar(ARC) = \frac{1}{2} ar(CAP)$ [∵ CR is median of ΔACP]

$$= \frac{1}{2}\left\{\frac{1}{2} ar(ABC)\right\} \quad \text{[CP is median]}$$

As RQ is median of ΔBRC

$$\therefore \quad ar(RQC) = \frac{1}{2} ar(RBC) = \frac{1}{2}\{ar(ABC) - ar(ARC)\}$$

$$= \frac{1}{2}\left\{ar(ABC) - \frac{1}{4} ar(ABC)\right\} = \frac{3}{8} ar(ABC).$$

(iii) $ar(ARC) = \frac{1}{2} ar(CAP)$ ...(ii) [CR is median]

And $ar(CAP) = ar(CPB)$ ...(iii) [CP is median]

From equations (ii) and (iii), we have

$$ar(ARC) = \frac{1}{2} ar(CPB)$$

Further, $ar(PBQ) = \frac{1}{2} ar(PBC)$ [PQ is median]

$\therefore \quad ar(ARC) = ar(PBQ).$

8. *In figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that:*

(i) $\Delta MBC \cong \Delta ABD$

(ii) $ar(BYXD) = 2ar(MBC)$

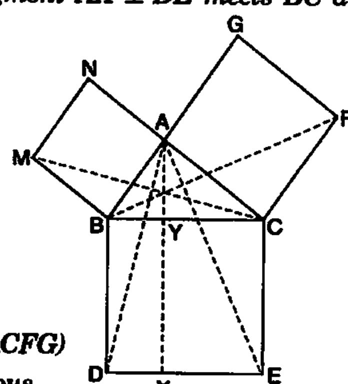
(iii) $ar(BYXD) = ar(ABMN)$

(iv) $\Delta FCB \cong \Delta ACE$

(v) $ar(CYXE) = 2ar(FCB)$

(vi) $ar(CYXE) = ar(ACFG)$

(vii) $ar(BCED) = ar(ABMN) + ar(ACFG)$



[**Note:** *Result (vii) is the famous Theorem of Pythagoras. You shall learn a simpler proof of this theorem in Class X.*]

Sol. (*i*) Consider triangles MBC and ABD,

$\angle MBA = \angle CBD$ [Each 90°]

Adding $\angle ABC$ to both sides, we have

$\angle MBA + \angle ABC = \angle CBD + \angle ABC$

$\Rightarrow$ $\angle MBC = \angle ABD$

$MB = BA$ [Sides of a square]

and $BC = BD$ [Sides of a square]

$\therefore$ $\Delta MBC \cong \Delta ABD$. [SAS]

(*ii*) Triangle ABD and rectangle BDXY are on the same base BD and between the same parallels BD and AX.

$\therefore$ $ar(ABD) = \frac{1}{2} ar(BDXY)$

As $\Delta ABD \cong \Delta MBC$ [Using result of part (*i*)]

$\Rightarrow$ $ar(\Delta ABD) = ar(\Delta MBC)$

$\therefore$ $ar(MBC) = \frac{1}{2} ar(BDXY)$

$\Rightarrow$ $ar(BDXY) = 2\,ar(MBC)$.

(*iii*) Triangle MBC and square MBAN are on the same base MB and between the same parallels MB and NC.

$\therefore$ $2\,ar(MBC) = ar(MBAN)$

$\therefore$ $ar(MBAN) = ar(BDXY)$.

[Using result from part (*ii*)]

(*iv*) $\angle ACF = \angle BCE$ [90° each]

Adding $\angle ACB$ to both sides, we get

$\angle ACF + \angle ACB = \angle BCE + \angle ACB$

$\Rightarrow$ $\angle BCF = \angle ACE$

Consider triangles BCF and ACE,

$\angle BCF = \angle ACE$ [Proved above]

$AC = CF$ [Sides of a square]

and BC = CE [Sides of a square]

$\therefore$ $\Delta FCB \cong \Delta ACE$. [SAS]

(*v*) Since rectangle CYXE and ΔACE are on the same base CE and between the same parallels CE and AX.

$\therefore$ $ar(CYXE) = 2ar(ACE)$

Now, use the result from part (*iv*), we get

$ar(ACE) = ar(FCB)$

[Congruent triangles have equal areas]

Hence, $ar(CYXE) = 2ar(FCB)$

(*vi*) As triangle FCB and square ACFG are on the same base CF and between the same parallels CF and BG,

$\therefore$ $ar(ACFG) = 2ar(FCB)$

Comparing this result and the result from part (*v*), we get

$ar(CYXE) = ar(ACFG)$

(*vii*) Adding the results from parts (*iii*) and (*vi*), we get

$ar(BYXD) + ar(CYXE) = ar(ABMN) + ar(ACFG)$

$\Rightarrow$ $ar(BCED) = ar(ABMN) + ar(ACFG)$.

□□