

## Lesson at a Glance

1. A circle is the collection of those points in a plane, whose distances from a fixed point are equal.
2. The centre of a circle is a fixed point.
3. Radius of a circle is the line segment joining the centre and any point on the circle.
4. A circle divides its plane into three parts:  
(i) interior of the circle (ii) The circle and (iii) Exterior of the circle.
5. The circular region is the combination of the circle and interior of the circle.
6. Diameter of a circle is double of its radius.
7. All radii of a circle have the same length.
8. All diameters of a circle have the same length.
9. A chord of a circle is the line segment joining any two distinct points lying on the circle.
10. Diameter of a circle is the longest chord.
11. Circumference of a circle is its whole length.
12. A segment of a circle is the region between a chord and either of its arcs.
13. The arc is a piece of the circle.
14. A diameter of a circle divides it into two equal arcs.
15. If a circle is divided into two unequal arcs, then the longer one and the shorter one are called major arc and minor arc respectively.
16. A sector of a circle is the region between an arc and two corresponding radii.
17. Equal chords of circles subtend equal angles at the centre.
18. Any chord of a circle is bisected by the perpendicular from the centre.

19. The line drawn through the centre of a circle to bisect any chord is perpendicular to that chord.
20. There is one and only one circle passing through three given non-collinear points.
21. Circumcircle of a triangle is a unique circle passing through all the three vertices of the triangle.
22. The radius of a circumcircle is called the circumradius of the triangle.
23. The centre of a circumcircle is called the circumcentre of the triangle.
24. The distance of a line from a point is the length of the perpendicular drawn from the point to the line.
25. Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).
26. The chords equidistant from the centre of a circle (or from the centres of congruent circles) are equal in length.
27. The corresponding arcs of equal chords of a circle are congruent.
28. The chords of a circle corresponding to congruent arcs are equal in length.
29. Equal arcs (or congruent arcs) of a circle subtend equal angles at the centre.
30. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
31. Angles in the same segment of a circle are equal.
32. The points lying on same circle are called concyclic.
33. A quadrilateral is said to be cyclic quadrilateral, if all the four vertices lie on the same circle.
34. The sum of either pair of opposite angles of a cyclic quadrilateral is  $180^\circ$ .
35. An exterior angle of a cyclic quadrilateral is equal to its opposite interior angle.
36. Angle in a semicircle is a right angle.

## TEXTBOOK QUESTIONS SOLVED

### Exercise 10.1 (Page – 171)

**1. Fill in the blanks:**

- (i) The centre of a circle lies in \_\_\_\_\_ of the circle.  
(exterior / interior).
- (ii) A point, whose distance from the centre of a circle is greater than its radius lies in \_\_\_\_\_ of the circle.  
(exterior / interior).
- (iii) The longest chord of a circle is a \_\_\_\_\_ of the circle.
- (iv) An arc is a \_\_\_\_\_ when its ends are the ends of a diameter.
- (v) Segment of a circle is the region between an arc and \_\_\_\_\_ of the circle.
- (vi) A circle divides the plane, on which it lies, in \_\_\_\_\_ parts.

**Sol.** (i) interior                      (ii) exterior                      (iii) diameter  
(iv) semicircle                      (v) the chord                      (vi) three.

**2. Write True or False: Give reasons for your answers.**

- (i) Line segment joining the centre to any point on the circle is a radius of the circle.
- (ii) A circle has only finite number of equal chords.
- (iii) If a circle is divided into three equal arcs, each is major arc.
- (iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.
- (v) Sector is the region between the chord and its corresponding arc.
- (vi) A circle is a plane figure.

**Sol.** (i) True

**Reason:** Each point on the circumference of a circle is equidistant from a fixed point and this distance is called radius of the circle.

(ii) False

**Reason:** Infinitely many chords can be drawn in a circle.

(iii) False

**Reason:** Major arc has larger length than that of minor arc. Any arc is neither major nor minor among three equal arcs.

(iv) True

**Reason:** A diameter is the longest chord of a circle, which is double of its radius.

(v) False

**Reason:** A sector is the region among the arc and two corresponding radii.

(vi) True.

**Reason:** A circle is a two-dimensional figure.

### Exercise 10.2 (Page – 173)

1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

**Sol.** Consider, triangles OAB and PQR,

$$OA = OB = PQ = PR$$

[Radii of congruent circles]

$$AB = QR$$

[Given]

$$\therefore \triangle OAB \cong \triangle PQR$$

[SSS]

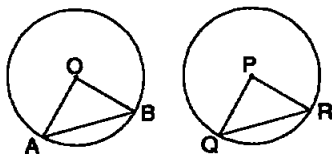
$$\therefore \angle AOB = \angle QPR$$

[CPCT]

2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

**Sol.** Consider, triangles OAB and PQR,

$$OA = OB = PQ = QR \quad [\text{Radii of congruent circles}]$$



$$\angle AOB = \angle QPR$$

[Given]

$$\therefore \triangle AOB \cong \triangle QPR$$

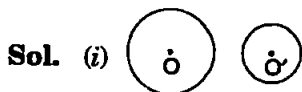
[SAS]

$$\Rightarrow AB = QR$$

[CPCT]

### Exercise 10.3 (Page – 176)

1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?



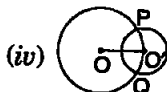
No point is common.



One point P is common.



One point P is common.



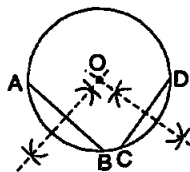
Two points P and Q are common.

Hence, the maximum number of common points is two, which is in case (iv).

2. Suppose you are given a circle. Give a construction to find its centre.

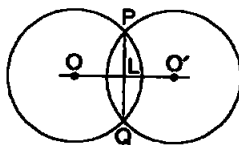
Sol. Steps to find centre of the circle:

- (i) Two non-parallel chords AB and CD of a circle are drawn.
- (ii) Perpendicular bisectors of AB and CD are drawn.
- (iii) Let these bisectors meet at O. Then O is the required centre of the circle.



3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Sol. Let two circles with centres O and O' intersect each other at P and Q. Thus PQ is the common chord as shown in the adjoining figure. Let us draw perpendicular OL on PQ,



then OL bisects PQ at L,

[Perpendicular from centre of a circle to the chord bisects the chord]

i.e.,  $\angle OLP = 90^\circ$  and  $PL = QL$  ... (i)

L and O' are joined

Then O'L is perpendicular to PQ ... (ii)

[Line segment joining the centre of the circle to mid-point of the chord is perpendicular to the chord]

From (i) and (ii), we have

$$\angle OLP + \angle O'LP = 90^\circ + 90^\circ = 180^\circ$$

$\Rightarrow \angle OLP$  and  $\angle O'LP$  form a linear pair.

Hence, OLO' is a straight line with  $PL = QL$ .

Hence centres of the two circles lie on the perpendicular bisector of the common chord.

### Exercise 10.4 (Page – 179)

1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

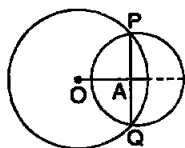
**Sol.** We know that line joining the centres is perpendicular bisector of the common chord.

The common chord passes through the centre of the smaller circle.

$$\therefore \angle PAO = 90^\circ, OA = 4 \text{ cm and } OP = 5 \text{ cm.}$$

$\therefore$  Applying Pythagoras theorem, we have

$$\begin{aligned} PA &= \sqrt{(5)^2 - (4)^2} \text{ cm} \\ &= \sqrt{25 - 16} \text{ cm} \\ &= \sqrt{9} \text{ cm} = 3 \text{ cm.} \end{aligned}$$



Further,  $PQ = 2PA = 2 \times 3 = 6 \text{ cm.}$

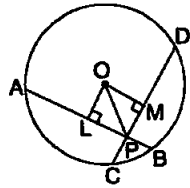
2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

**Sol. Construction:** Draw OL and OM perpendiculars to chords AB and CD respectively. Join OP.

**To prove:**  $AP = DP$  and  $PB = CP$ .

**Proof:** Consider triangles OLP and OMP,

$$OL = OM \quad \text{[Equal chords AB and CD are equidistant from the centre of the circle]}$$



OP is common.

$$\angle OLP = \angle OMP$$

[90° each]

$$\therefore \triangle OLP \cong \triangle OMP$$

[RHS]

$$\therefore LP = PM$$

...(i) [CPCT]

$$\text{Also, } AL = LB$$

...(ii)

[Perpendicular from centre to the chord bisects the chord]

$$CM = DM$$

...(iii) [Reason same as above]

$$AL + LP = DM + MP$$

[From (i), (ii), (iii)]

$$AP = DP$$

...(iv)

$$\text{Now, } AB = CD$$

$$\Rightarrow AP + PB = CP + PD$$

$$\Rightarrow AP + PB = CP + AP$$

[From (iv)]

$$\Rightarrow PB = CP.$$

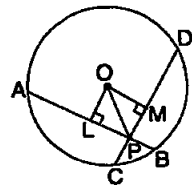
3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

**Sol. Construction:** Draw OL and OM perpendiculars to chords AB and CD respectively. Join OP.

**To prove:**  $\angle OPL = \angle OPM$

**Proof:** Consider triangles OLP and OMP,

$$OL = OM \quad \text{[Equal chords AB and CD are equidistant from the centre of the circle]}$$



OP is common.

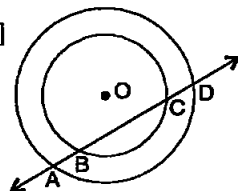
$$\angle OLP = \angle OMP$$

[90° each]

$$\triangle OLP \cong \triangle OMP \quad \text{[RHS]}$$

$$\Rightarrow \angle OPL = \angle OPM. \quad \text{[CPCT]}$$

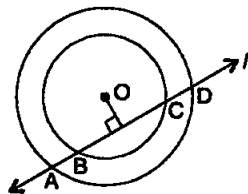
4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that  $AB = CD$  (see figure).



**Sol. Construction:** Draw perpendicular  $OL$ , from centre  $O$ , to the line  $l$ .

**Proof:**  $AD$  is the chord of a bigger circle and  $OL \perp AD$ .

$$\therefore AL = DL \quad \dots(i)$$



[Perpendicular from centre of the circle to the chord bisects the chord]

Also,  $BC$  is the chord of a smaller circle and  $OL \perp BC$ .

$$BL = CL \quad \dots(ii) \text{ [Reason same as above]}$$

$$\Rightarrow AL - BL = DL - CL \quad \text{[From (i) and (ii)]}$$

$$\Rightarrow AB = CD.$$

5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

**Sol.**  $\angle MOS = \angle ROS$

[Angles subtended by equal chords are equal]

$$OM = OR \quad \text{[Radii]}$$

$OT$  is common.

$$\therefore \triangle OMT \cong \triangle ORT. \quad \text{[SAS]}$$

$$\therefore MT = TR \quad \dots(i)$$

$$\angle OTM = \angle OTR = 90^\circ$$

Let  $OT = x$

In right-angled triangle  $OTM$ ,

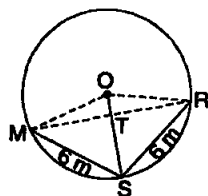
$$MT = \sqrt{25 - x^2} \quad \dots(ii)$$

In right-angled triangle  $MTS$ ,

$$MT = \sqrt{36 - (5 - x)^2} \quad \dots(iii)$$

From (ii) and (iii), we get

$$\sqrt{25 - x^2} = \sqrt{36 - 25 - x^2 + 10x}$$





$$\Rightarrow 25 - x^2 = 11 - x^2 + 10x$$

$$\Rightarrow 10x = 14 \quad \Rightarrow x = 1.4$$

Substituting this value of  $x$  in (ii), we get

$$MT = \sqrt{25 - (1.4)^2} = \sqrt{25 - 1.96} = \sqrt{23.04} = 4.8 \text{ m.}$$

From (i),  $MR = 2MT = 2 \times 4.8 \text{ m} = 9.6 \text{ m.}$

6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

**Sol.** Let Ankur, Syed and David are sitting at A, S and D respectively and so, DAS is an equilateral triangle, as if arc are equal then corresponding chords are equal.

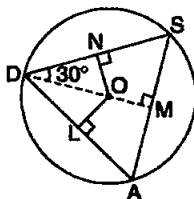
$$\therefore \angle ADS = 60^\circ.$$

$$\text{Also, } \angle NDO = \frac{1}{2} \angle ADS = 30^\circ$$

$$\frac{DN}{OD} = \cos 30^\circ$$

$$\Rightarrow \frac{DN}{20} = \frac{\sqrt{3}}{2} \Rightarrow DN = 10 \times 1.73 = 17.3 \text{ m}$$

$$\therefore DS = 2DN = 2 \times 17.3 = 34.6 \text{ m.}$$

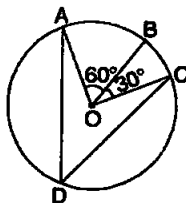
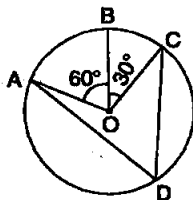


### Exercise 10.5 (Pages – 184-186)

1. In the adjoining figure, A, B and C are three points on a circle with centre O such that  $\angle BOC = 30^\circ$  and  $\angle AOB = 60^\circ$ . If D is a point on the circle other than the arc ABC, find  $\angle ADC$ .

**Sol.**  $\angle AOC = 60^\circ + 30^\circ = 90^\circ$

$$\angle ADC = \frac{1}{2} \angle AOC$$



[Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.]

$$\angle ADC = \frac{1}{2} \times 90^\circ = 45^\circ.$$

2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

**Sol.** We have,  $OA = OB = AB$  [Given]

$\therefore \triangle OAB$  is equilateral triangle.

$$\angle AOB = 60^\circ$$

$$\angle APB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 60^\circ = 30^\circ.$$

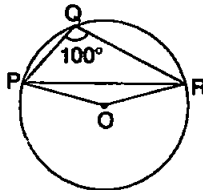
Also  $APBQ$  is a cyclic quadrilateral.

$$\therefore \angle P + \angle Q = 180^\circ$$

[Sum of opposite angles of a cyclic quadrilateral is  $180^\circ$ .]

$$\Rightarrow 30^\circ + \angle Q = 180^\circ \Rightarrow \angle Q = 150^\circ.$$

3. In the figure given below,  $\angle PQR = 100^\circ$ , where  $P, Q$  and  $R$  are points on a circle with centre  $O$ . Find  $\angle OPR$ .



**Sol.** Let  $\angle OPR = x$ , then  $\angle ORP = x$

and  $\angle POR = 180^\circ - 2x$ .

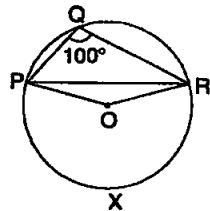
$\therefore$  Angle formed by arc  $PXR$  at the centre

$$= 360^\circ - (180^\circ - 2x)$$

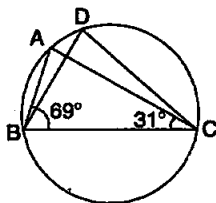
$$= 180^\circ + 2x.$$

Also,  $\angle PQR = \frac{1}{2} (180^\circ + 2x)$

$$\Rightarrow 100^\circ = 90^\circ + x \Rightarrow x = 10^\circ.$$



4. In figure,  $\angle ABC = 69^\circ$ ,  $\angle ACB = 31^\circ$ , find  $\angle BDC$ .



**Sol.** In triangle ABC,

$$\angle A + 69^\circ + 31^\circ = 180^\circ$$

[Sum of angles of a triangle is  $180^\circ$ ]

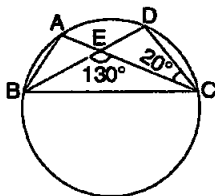
$$\Rightarrow \angle A = 180^\circ - 100^\circ = 80^\circ$$

$$\text{Also, } \angle D = \angle A = 80^\circ$$

[Angles in the same segment of a circle]

$$\text{i.e., } \angle BDC = 80^\circ.$$

5. In figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that  $\angle BEC = 130^\circ$  and  $\angle ECD = 20^\circ$ . Find  $\angle BAC$ .



**Sol.** Given  $\angle BEC = 130^\circ$ ,  $\angle ECD = 20^\circ$ .

$$\angle DEC + \angle BEC = 180^\circ$$

$$\therefore \angle DEC = 180^\circ - 130^\circ = 50^\circ.$$

In triangle DEC,

$$\angle D + 50^\circ + 20^\circ = 180^\circ$$

[Sum of angles of a triangle is  $180^\circ$ ]

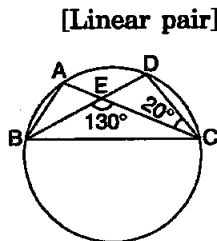
$$\Rightarrow \angle D = 110^\circ \quad \dots(i)$$

$$\text{Also, } \angle BAC = \angle D$$

[Angles in the same segment of a circle are equal]

$$\therefore \angle BAC = 110^\circ.$$

[From (i)]



6.  $ABCD$  is a cyclic quadrilateral whose diagonals intersect at a point  $E$ . If  $\angle DBC = 70^\circ$ ,  $\angle BAC$  is  $30^\circ$ , find  $\angle BCD$ . Further, if  $AB = BC$ , find  $\angle ECD$ .

**Sol.**  $\angle CDB = \angle BAC = 30^\circ$  ... (i)

[Angles in the same segment]

In triangle  $BCD$ ,

$$\angle CBD + \angle BCD + \angle CDB = 180^\circ$$

[Sum of angles of a triangle is 180]

$$70^\circ + \angle BCD + 30^\circ = 180^\circ$$

$$\Rightarrow \angle BCD = 80^\circ \quad \dots(ii)$$

Now, in  $\triangle ABC$ ,

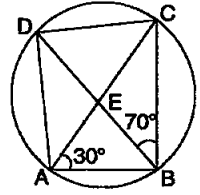
$$\text{if } AB = BC, \text{ then } \angle BCA = \angle BAC = 30^\circ \quad \dots(iii)$$

[Angles opposite to equal sides are equal]

Now,  $\angle BCD = \angle BCA + \angle ACD$

$$\Rightarrow 80^\circ = 30^\circ + \angle ECD \quad [\because \angle ACD = \angle ECD]$$

$$\Rightarrow \angle ECD = 50^\circ. \quad [\text{From (ii) and (iii)}]$$

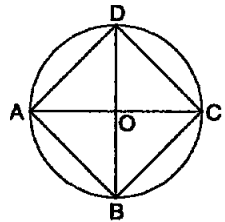


7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

**Sol.** As  $AC$  and  $BD$  are the diagonals of a cyclic quadrilateral.

$\therefore \angle ADC, \angle BAD, \angle ABC$  and  $\angle BCD$  are angles in a semicircle. Hence, each angle is  $90^\circ$ .

As in a quadrilateral each angle is  $90^\circ$ , hence quadrilateral is a rectangle.



8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

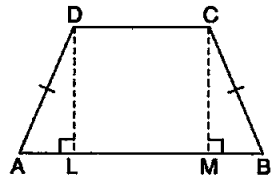
**Sol. Construction:** Draw  $DL$  and  $CM$  perpendiculars to  $AB$ .

**Proof:** In  $\triangle DLA$  and  $\triangle CMB$ ,

$$DL = CM$$

[Distance between parallel lines]

$$AD = BC \quad [\text{Given}]$$



$$\begin{aligned} \angle DLA &= \angle CMB && [90^\circ \text{ each}] \text{ [Construction]} \\ \therefore \triangle DLA &\cong \triangle CMB && \text{[RHS]} \\ \therefore \angle DAL &= \angle CBM && \dots(i) \text{ [CPCT]} \end{aligned}$$

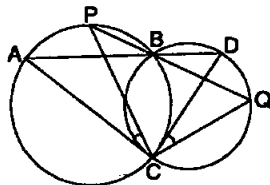
Now,  $AB \parallel CD$  and  $AD$  is transversal

$$\begin{aligned} \therefore \angle CDA + \angle DAL &= 180^\circ \\ \Rightarrow \angle CDA + \angle CBM &= 180^\circ && \text{[From (i)]} \\ \Rightarrow \angle CDB + \angle CBA &= 180^\circ && [\because \angle CBM = \angle CBA] \end{aligned}$$

As sum of opposite angles of a quadrilateral is  $180^\circ$ , then it is cyclic.

Hence,  $ABCD$  is a cyclic quadrilateral.

9. Two circles intersect at two points  $B$  and  $C$ . Through  $B$ , two line segments  $ABD$  and  $PBQ$  are drawn to intersect the circles at  $A, D$  and  $P, Q$  respectively (see figure). Prove that  $\angle ACP = \angle QCD$ .



**Sol.**

$$\begin{aligned} \angle ACP &= \angle ABP && \dots(i) \\ & \text{[Angles in the same segment of a circle are equal]} \\ \angle QCD &= \angle QBD && \dots(ii) \text{ [Reason same as above]} \\ \angle ABP &= \angle QBD && \dots(iii) \text{ [Vertically opposite angles]} \end{aligned}$$

From (i), (ii) and (iii), we get

$$\angle ACP = \angle QCD.$$

10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

**Sol. Construction:** Join  $AD$ .

**Proof:** Let circle with  $AB$  as diameter meets  $BC$  at  $D$ .

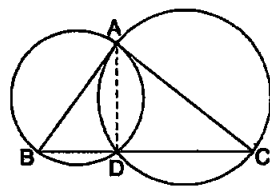
Then  $\angle ADB = 90^\circ$ . [Angle in a semicircle]

Now  $\angle ADB + \angle ADC = 180^\circ$  [Linear pair]

$$\therefore \angle ADC = 90^\circ$$

As we know angle in a semicircle is  $90^\circ$ , therefore, a circle with  $AC$  as diameter passes through  $D$ .

Hence both the circles meet the third side at  $D$ .

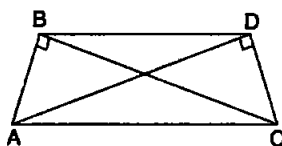


11.  $ABC$  and  $ADC$  are two right triangles with common hypotenuse  $AC$ . Prove that  $\angle CAD = \angle CBD$ .

**Sol.**  $\angle ABC = \angle ADC = 90^\circ$

$\therefore$   $ACDB$  is a cyclic quadrilateral.

[As if a line segment subtends equal angles at two other points on the same side of the segment, then the four points are concyclic.]



$\therefore \angle CAD = \angle CBD$ .

[Angles in the same segment of a circle are equal.]

12. Prove that a cyclic parallelogram is a rectangle.

**Sol.**  $ABCD$  is a cyclic parallelogram.

$\therefore \angle A + \angle C = 180^\circ$  ... (i)

[Sum of opposite angles of a cyclic quadrilateral is  $180^\circ$ .]

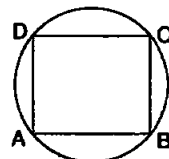
Also,  $\angle A = \angle C$  ... (ii)

[Opposite angles of a parallelogram]

From (i) and (ii), we have

$$2\angle A = 180^\circ \Rightarrow \angle A = 90^\circ$$

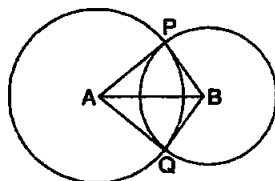
As in a parallelogram one angle is  $90^\circ$ , hence it is a rectangle.



### Exercise 10.6 (Pages – 186-187)

1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

**Sol.** Consider triangles  $APB$  and  $AQB$ ,  
 $AP = AQ$  [Radii]



$$PB = QB$$

[Radii]

AB is common

$$\therefore \triangle APB \cong \triangle AQB$$

[SSS]

$$\therefore \angle APB = \angle AQB.$$

[CPCT]

2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

**Sol.** Let OL is drawn perpendicular to AB and LO is produced to M meeting CD at M, then  $LM \perp CD$ .

$$[\because AB \parallel CD]$$

$$\therefore \angle ALM = 90^\circ; \angle OMC = 90^\circ$$

$$AL = \frac{1}{2} AB = \frac{5}{2} \text{ cm};$$

$$CM = \frac{1}{2} CD = \frac{11}{2} \text{ cm}.$$

Let  $OL = x$ ,  $OA = r$ . Then  $OM = 6 - x$ .

Now, in right-angled triangle OLA,

$$OA^2 = AL^2 + OL^2 = \left(\frac{5}{2}\right)^2 + x^2$$

$$\Rightarrow r^2 = \frac{25}{4} + x^2 \quad \dots(i)$$

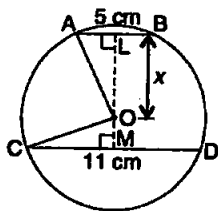
In right-angled triangle OML,

$$OC^2 = CM^2 + OM^2 = \left(\frac{11}{2}\right)^2 + (6 - x)^2$$

$$\Rightarrow r^2 = \frac{121}{4} + 36 - 12x + x^2 \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{25}{4} + x^2 = \frac{121}{4} + 36 - 12x + x^2$$



$$\Rightarrow 12x = \frac{121}{4} + 36 - \frac{25}{4} = 60 \Rightarrow x = 5$$

Substituting  $x = 5$  in (i), we get

$$r^2 = \frac{25}{4} + (5)^2 = \frac{25}{4} + 25 = \frac{125}{4}$$

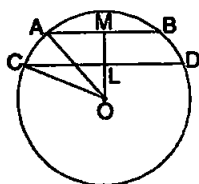
$$\Rightarrow r = \frac{5\sqrt{5}}{2} \text{ cm}$$

Hence, radius of the circle is  $\frac{5\sqrt{5}}{2}$  cm.

3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

**Sol.** Let  $AB = 6$  cm and  $CD = 8$  cm are two parallel chords of a circle with centre at  $O$ .

We draw  $OM$  perpendicular to  $AB$  meeting  $AB$  at  $M$  and  $CD$  at  $L$ . Then  $OL$  is also perpendicular to  $CD$ .



[ $\therefore AB \parallel CD$ ]

$$AM = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3 \text{ cm}$$

$$\text{and } CL = \frac{1}{2} CD = \frac{1}{2} \times 8 = 4 \text{ cm.}$$

Let  $OL = x$ .

In right-angled triangle  $OMA$ ,

$$OA^2 = AM^2 + OM^2 = (3)^2 + (4)^2 = 25 \Rightarrow OA = 5 \text{ cm.}$$

As  $OC = OA \therefore OC = 5$  cm.

In right-angled triangle  $OLC$ ,

$$OC^2 = CL^2 + OL^2 \therefore (5)^2 = (4)^2 + x^2$$

$$\Rightarrow x^2 = 25 - 16 = 9 \therefore x = 3$$

Therefore, distance of other chord is 3 cm.



4. Let the vertex of an angle  $ABC$  be located outside a circle and let the sides of the angle intersect equal chords  $AD$  and  $CE$  with the circle. Prove that  $\angle ABC$  is equal to half the difference of the angles subtended by the chords  $AC$  and  $DE$  at the centre.

**Sol. Construction:** Join  $E$  and  $A$ .

**Proof:**  $\angle AOC = 2\angle AEC$  ... (i)

[Angle subtended by an arc at the centre is double the angle

subtended by it on the remaining part of the circle.]

$$\angle DOE = 2\angle DAE$$

... (ii)

[Reason same as above]

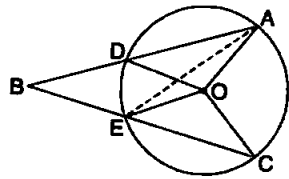
Also  $\angle AEC = \angle ABC + \angle BAE$

[Exterior angle of a triangle is equal to sum of interior opposite angles]

$$\Rightarrow \angle AEC = \angle ABC + \angle DAE$$

$$\Rightarrow \frac{1}{2} \angle AOC = \angle ABC + \frac{1}{2} \angle DOE$$

$$\Rightarrow \angle ABC = \frac{1}{2} (\angle AOC - \angle DOE).$$

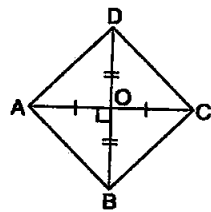


5. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

**Sol.** We know that diagonals of a rhombus intersect each other at right angles.

$\therefore \angle AOB = 90^\circ$ . Also we know that angle in a semicircle is  $90^\circ$ . If a circle is drawn with  $AB$  as diameter, then  $O$  lies on the circle.

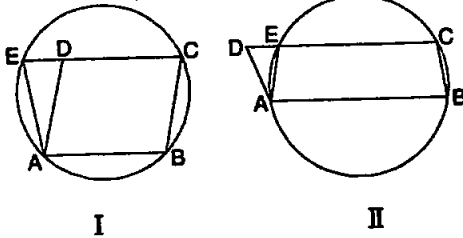
Hence, if a circle is drawn with any side of the rhombus as diameter, then it passes through the point of intersection of diagonals.



6.  $ABCD$  is a parallelogram. The circle through  $A$ ,  $B$  and  $C$  intersects  $CD$  (produced if necessary) at  $E$ . Prove that  $AE = AD$ .

**Sol.**  $ABCD$  is a parallelogram.

$\therefore \angle ABC = \angle CDA$  ... (i) [Opposite angles of a parallelogram]



Also,  $ABCE$  is a cyclic quadrilateral.

$\therefore \angle ABC + \angle AEC = 180^\circ$  ...*(ii)* [Sum of opposite angles of a cyclic quadrilateral is  $180^\circ$ ]

In Fig. I,

Also,  $\angle ADC + \angle ADE = 180^\circ$  [Linear pair]

$\Rightarrow \angle ABC + \angle ADE = 180^\circ$  ...*(iii)* [From *(i)*]

From *(ii)* and *(iii)*, we get

$$\angle AEC = \angle ADE \Rightarrow \angle AED = \angle ADE$$

$\Rightarrow AD = AE$

[Sides opposite to equal angles of a triangle are equal.]

In Fig. II,

$$\angle AEC + \angle AED = 180^\circ \quad \dots\text{(iv)} \text{ [Linear pair]}$$

From *(i)* and *(ii)*, we get

$$\angle CDA + \angle AEC = 180^\circ \Rightarrow \angle EDA + \angle AEC = 180^\circ \quad \dots\text{(v)}$$

From *(iv)* and *(v)*, we get

$$\angle AED = \angle EDA$$

$\Rightarrow AD = AE$ .

[Sides opposite to equal angles of a triangle are equal]

7.  $AC$  and  $BD$  are chords of a circle which bisect each other. Prove that *(i)*  $AC$  and  $BD$  are diameters, *(ii)*  $ABCD$  is a rectangle.

Sol. *(i)* Consider triangles  $AOD$  and  $BOC$ .

$$AO = OC \quad \text{[Given]}$$

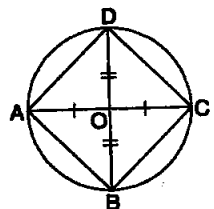
$$BO = OD \quad \text{[Given]}$$

$$\angle AOD = \angle BOC$$

[Vertically opposite angles]

$$\Rightarrow \triangle AOD \cong \triangle COB \quad \text{[SAS]}$$

$$\Rightarrow \angle DAO = \angle BCO$$



...*(i)* [CPCT]

Also,  $\angle ADO = \angle BCO$  ...*(ii)*

[Angles in the same segment of a circle]

From *(i)* and *(ii)*, we have

$$\angle DAO = \angle ADO$$

$$\Rightarrow OA = OD$$

Hence,  $OA = OB = OC = OD$ .

$\Rightarrow O$  is equidistant from  $A, B, C, D$ .

$\Rightarrow O$  is centre of the circle.

Hence,  $AC$  and  $BD$  are diameters.

*(ii)* As  $AC$  and  $BD$  are diameters. Therefore,  $\angle BAD$ ,  $\angle ABC$ ,  $\angle BCD$ ,  $\angle CDA$  are angles in a semicircle.

Therefore, each of them is  $90^\circ$ .

Hence,  $ABCD$  is a rectangle.

8. Bisectors of angles  $A, B$  and  $C$  of a triangle  $ABC$  intersect its circumcircle at  $D, E$  and  $F$  respectively. Prove that the

angles of the triangle  $DEF$  are  $90^\circ - \frac{1}{2}\angle A$ ,  $90^\circ - \frac{1}{2}\angle B$

and  $90^\circ - \frac{1}{2}\angle C$ .

**Sol.**  $\angle ADE = \angle ABE$  ...*(i)*

[Angles in the same segment of a circle]

$$\angle ADF = \angle ACF$$

...*(ii)* [Reason same as above]

Now  $\angle EDF = \angle ADE + \angle ADF$

$$\Rightarrow \angle EDF = \angle ABE + \angle ACF$$

...*(iii)* [From *(i)*, *(ii)*]

As  $BE$  and  $CF$  are bisectors of  $\angle ABC$  and  $\angle ACB$ .

$$\therefore \angle ABE = \frac{1}{2}\angle ABC \text{ and } \angle ACF = \frac{1}{2}\angle ACB \quad \dots\text{(iv)}$$

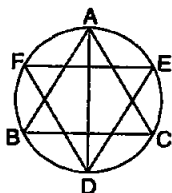
From *(iii)* and *(iv)*, we get

$$\angle EDF = \frac{1}{2}\angle ABC + \frac{1}{2}\angle ACB = \frac{1}{2}(\angle ABC + \angle ACB)$$

...*(v)*

In triangle  $ABC$ ,

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$



$$\Rightarrow \angle ABC + \angle ACB = 180^\circ - \angle BAC.$$

Substituting in (v), we get

$$\angle EDF = \frac{1}{2}(180^\circ - \angle BAC) = 90^\circ - \frac{1}{2}\angle BAC = 90^\circ - \frac{1}{2}\angle A.$$

Similarly, we can show that

$$\angle DEF = 90^\circ - \frac{1}{2}\angle B; \angle DFE = 90^\circ - \frac{1}{2}\angle C.$$

9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

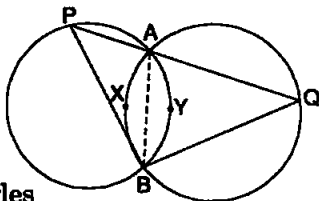
Sol. As AB is common chord.

$$\therefore \text{arc AXB} = \text{arc AYB}$$

$$\Rightarrow \angle AQB = \angle APB$$

[Equal areas of congruent circles subtend equal angles

on the remaining part of the circles.]



Now in triangle BPQ,

$$\angle AQB = \angle APB$$

[Proved above]

$$\Rightarrow \angle PQB = \angle QPB$$

$$\Rightarrow PB = QB.$$

[Sides opposite to equal angles are equal.]

10. In any triangle ABC, if the angle bisector of  $\angle A$  and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

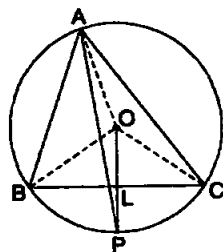
Sol. Let O be the centre of the circumcircle of triangle ABC.

$$\therefore \angle BOC = 2\angle BAC$$

[Angle subtended by an arc at the centre is doubled the angle subtended by it at remaining part of the circle.]

Consider  $\triangle OLB$  and  $\triangle OLC$ .

$$OB = OC$$



[Radii]

OC is common.

$$BL = LC \quad [OL \text{ is perpendicular bisector}]$$

$$\therefore \triangle OLB \cong \triangle OLC \quad [SSS]$$

$$\therefore \angle BOL = \angle COL = \angle CAB \quad \dots(i)$$

Let perpendicular bisector of side BC and angle bisector of  $\angle A$  meet at the point P.

From equation (i), we get

$$\angle COL = \angle CAB$$

$$\Rightarrow \angle COP = 2\angle CAP \quad [\angle CAP = \angle BAP]$$

This proves that points C, A and P are concyclic points as  $\angle COP$  is subtended by arc CP at the centre O and  $\angle CAP$  is subtended by it at the remaining part of the circle.

Again, from equation (i), we get

$$\angle BOL = \angle BAC$$

$$\text{i.e., } \angle BOP = 2\angle BAP$$

This, also proves that points B, A and P are concyclic.

Hence, P lies on the circumcircle of the  $\triangle ABC$ .

□□