

2

Polynomials

EXERCISE 2.1

Choose the correct answer from the given four options in the following questions:

Q1. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3 , then the value of k is

- (a) $4/3$ (b) $-4/3$ (c) $2/3$ (d) $-2/3$

Sol. (a): Main concept: If a is root of a polynomial $f(x)$, then $f(a) = 0$.

Let $f(x) = (k-1)x^2 + kx + 1$

As -3 is a zero of $f(x)$, then

$$\begin{aligned} & f(-3) = 0 \\ \Rightarrow & (k-1)(-3)^2 + k(-3) + 1 = 0 \\ \Rightarrow & 9k - 9 - 3k + 1 = 0 \\ \Rightarrow & 9k - 3k = +9 - 1 \\ \Rightarrow & 6k = 8 \\ \Rightarrow & k = 4/3 \end{aligned}$$

Q2. A quadratic polynomial, whose zeroes are -3 and 4 , is

- (a) $x^2 - x - 12$ (b) $x^2 + x + 12$ (c) $\frac{x^2}{2} - \frac{x}{2} - 6$ (d) $2x^2 + 2x - 24$

Sol. (c): Main concept: Required quadratic polynomial

$$= x^2 - (\alpha + \beta)x + \alpha\beta$$

Here, $\alpha = -3$ and $\beta = 4$

$$\therefore \alpha + \beta = -3 + 4 = 1$$

$$\text{and } \alpha \cdot \beta = -3 \times 4 = -12$$

\therefore The quadratic polynomial is

$$\begin{aligned} & = x^2 - (\alpha + \beta)x + \alpha\beta \\ & = x^2 - 1x - 12 \\ & = \frac{x^2}{2} - \frac{x}{2} - \frac{12}{2} \\ & = \frac{x^2}{2} - \frac{x}{2} - 6 \end{aligned}$$

Q3. If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3 , then

(a) $a = -7, b = -1$

(b) $a = 5, b = -1$

(c) $a = 2, b = -6$

(d) $a = 0, b = -6$

Sol. (d): Main concept: If a is zero of a polynomial $f(x)$, then $f(a) = 0$.

Let $f(x) = x^2 + (a+1)x + b$

As 2, and (-3) are zeroes of polynomial $f(x) = x^2 + (a+1)x + b$, then

$$\begin{array}{l|l} f(2) = 0 & \text{and} & f(-3) = 0 \\ \Rightarrow (2)^2 + (a+1)(2) + b = 0 & \Rightarrow (-3)^2 + (a+1)(-3) + b = 0 \\ \Rightarrow 4 + 2a + 2 + b = 0 & \Rightarrow 9 - 3a - 3 + b = 0 \\ \Rightarrow 2a + b = -6 \dots(i) & \Rightarrow -3a + b = -6 \\ & \Rightarrow 3a - b = 6 \dots(ii) \end{array}$$

[Adding (i) and (ii)]

$$5a = 0$$

$$\Rightarrow a = 0$$

$$\text{But, } 2a + b = -6$$

[From (i)]

$$\Rightarrow 2(0) + b = -6$$

$$\Rightarrow b = -6$$

Hence, $a = 0$ and $b = -6$ verifies option (d).

Q4. The number of polynomials having zeroes as -2 and 5 is

- (a) 1 (b) 2 (c) 3 (d) more than 3

Sol. (d): We know that if we divide or multiply a polynomial by any constant (real number), then the zeroes of polynomial remains same. Here, $\alpha = -2$ and $\beta = +5$

$$\therefore \alpha + \beta = -2 + 5 = 3 \quad \text{and} \quad \alpha \cdot \beta = -2 \times 5 = -10$$

$$\text{So, required polynomial is } x^2 - (\alpha + \beta)x + \alpha\beta \\ = x^2 - 3x - 10$$

If we multiply this polynomial by any real number let 5 and 2, we get

$$5x^2 - 15x - 50$$

and

$$2x^2 - 6x - 20$$

which are different polynomials having same zeroes -2 and 5 .

So, we can obtain so many (infinite polynomials) from two given zeroes.

Q5. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

- (a) $-\frac{c}{a}$ (b) $\frac{c}{a}$ (c) 0 (d) $\frac{-b}{a}$

Sol. (b): Let $f(x) = ax^3 + bx^2 + cx + d$

If α, β, γ are the zeroes of $f(x)$, then

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

One root is zero (Given) so, $\alpha = 0$.

$$\Rightarrow \beta\gamma = \frac{c}{a}$$

Q6. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1 , then the product of other two zeroes is

- (a) $b - a + 1$ (b) $b - a - 1$ (c) $a - b + 1$ (d) $a - b - 1$

Sol. (a): Let $f(x) = x^3 + ax^2 + bx + c$

\therefore Zero of $f(x)$ is -1 so

$$\begin{aligned} & f(-1) = 0 \\ \Rightarrow & (-1)^3 + a(-1)^2 + b(-1) + c = 0 \\ \Rightarrow & -1 + a - b + c = 0 \\ \Rightarrow & a - b + c = 1 \\ \Rightarrow & c = 1 + b - a \end{aligned}$$

Now, $\alpha \cdot \beta \cdot \gamma = \frac{-d}{a}$ [$\because c = b, d = c$]

$$\Rightarrow -1 \beta \gamma = \frac{-c}{1}$$

$$\Rightarrow \beta \gamma = c$$

$$\Rightarrow \beta \gamma = 1 + b - a$$

Q7. The zeroes of quadratic polynomial $x^2 + 99x + 127$ are

- (a) both positive (b) both negative
(c) one positive and one negative (d) both are equal

Sol. (b): Let $f(x) = x^2 + 99x + 127$

Now, $b^2 - 4ac = (99)^2 - 4(1)127$ ($a = 1, b = 99, c = 127$)

$$\Rightarrow b^2 - 4ac = 9801 - 508$$

$$\Rightarrow \sqrt{b^2 - 4ac} = \sqrt{9293}$$

$$\Rightarrow \sqrt{b^2 - 4ac} = 96.4$$

So, zeroes of $f(x)$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = \frac{-99 \pm 96.4}{2 \times 1}$$

\Rightarrow Both roots will be negative as $99 > 96.4$.

Q8. The zeroes of the quadratic polynomial $x^2 + kx + k$ where $k \neq 0$

- (a) cannot both be positive (b) cannot both be negative
(c) are always unequal (d) are always equal

Sol. (a): Let $f(x) = x^2 + kx + k$

For zeroes of $f(x)$, $f(x) = 0$

$$\Rightarrow x^2 + kx + k = 0$$

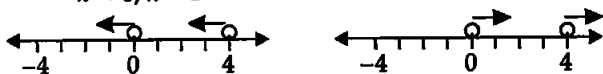
But, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = \frac{-k \pm \sqrt{k^2 - 4 \cdot k}}{2} = \frac{-k \pm \sqrt{k(k-4)}}{2}$$

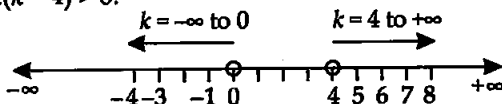
For real roots, $b^2 - 4ac > 0$

$$\Rightarrow k(k-4) > 0 \quad (k \neq 0)$$

⇒ Either $k < 0$ and $k - 4 < 0$ or $k > 0$ and $(k - 4) > 0$
 $k < 0$ and $k < 4$ $k > 0$ and $k > 4$
 $k < 0, k < 4$



So, solution $k(k - 4) > 0$.



Let $k = -4$

(any point on number line)

$$x = \frac{-4 \pm \sqrt{-4(-4-4)}}{2} = \frac{-4 \pm \sqrt{32}}{2}$$

$$= \frac{-4 \pm 4\sqrt{2}}{2} = \frac{4[-1 \pm \sqrt{2}]}{2}$$

$$x = 2[-1 \pm \sqrt{2}]$$

$$x_1 = 2[-1 + \sqrt{2}], \text{ which is positive}$$

$$x_2 = 2[-1 - \sqrt{2}], \text{ which is negative}$$

Let $k = 8$ (any point on number line)

$$x = \frac{-8 \pm \sqrt{8(8-4)}}{2}$$

$$x = \frac{-8 \pm \sqrt{8 \times 4}}{2}$$

$$x = \frac{-8 \pm 4\sqrt{2}}{2}$$

$$x = \frac{+4[-2 \pm \sqrt{2}]}{2}$$

$$x = 2(-2 \pm \sqrt{2})$$

$$x_1 = 2[-2 + \sqrt{2}], \text{ which is negative}$$

$$x_2 = 2[-2 - \sqrt{2}], \text{ which is negative}$$

So, the roots cannot be both positive.

Q9. If the zeroes of the quadratic polynomial

$$ax^2 + bx + c, \text{ where, } c \neq 0 \text{ are equal then}$$

- c and a both have opposite signs
- c and b have opposite signs
- c and a have same sign
- c and b have the same sign

Sol. (c): For equal roots $b^2 - 4ac = 0$

$$\text{or } b^2 = 4ac$$

b^2 is always positive so $4ac$ must be positive or i.e., product of a and c must be positive i.e., a and c must have same sign either positive or negative.

Q10. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other then it

- has no linear term and the constant term is negative
- has no linear term and the constant term is positive
- can have a linear term but the constant term is negative.
- can have a linear term but the constant term is positive.

Sol. (a): Let $f(x) = x^2 + ax + b$ and α, β are the roots of it.

Then, $\beta = -\alpha$ (Given)

$$\alpha + \beta = \frac{-b}{a} \quad \text{and} \quad \alpha \cdot \beta = \frac{c}{a}$$

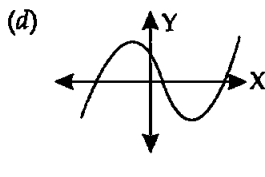
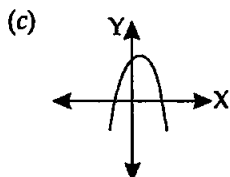
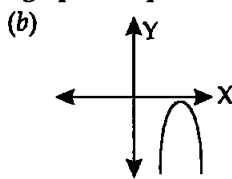
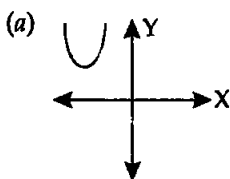
$$\Rightarrow \alpha - \alpha = \frac{-a}{1} \quad \alpha(-\alpha) = \frac{b}{1}$$

$$\Rightarrow -a = 0 \quad -\alpha^2 = b$$

$$\Rightarrow a = 0 \quad \Rightarrow b < 0 \text{ or } b \text{ is negative}$$

So, $f(x) = x^2 + b$ shows that it has no linear term.

Q11. Which of the following is not the graph of a quadratic polynomial?



Sol. (d): Graph 'd' intersect at three points on X-axis so the roots of polynomial of graph is three, so it is cubic polynomial. Other graphs are of quadratic polynomial. Graphs a, b have no real zeroes.

EXERCISE 2.2

Q1. Answer the following and justify.

- (i) Can $x^2 - 1$ be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5?
- (ii) What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + s, p \neq 0$?
- (iii) If on division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero what is the relation between the degrees of $p(x)$ and $g(x)$?
- (iv) If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?
- (v) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?

Sol. (i): Let the divisor of degree 5 is $g(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + 1$

$$\text{Dividend} = p(x) = x^6 + 2x^3 + x - 1,$$

$$q(x) = x^2 - 1 \quad \text{and let remainder be } r(x)$$

So, by Euclid's division algorithm

$$p(x) = g(x)q(x) + r(x)$$

[deg $p(x)$ is 6] = [g(x) of deg 5] [q(x) degree 2] + r(x) of degree less than 5

$$\begin{aligned} \text{degree } p(x) &= \text{degree } g(x) + \text{degree } q(x) + \text{degree } r(x) \\ 6 &= 5 + 2 + \text{any} \end{aligned}$$

So, degree of $q(x)$ can never be 2 it may be only one.

So, $(x^2 - 1)$ can never be the quotient.

$$\begin{aligned} \text{(ii) } p(x) \text{ (dividend)} &= ax^2 + bx + c \\ g(x) \text{ (divisor)} &= px^3 + qx^2 + r(x) + s \end{aligned}$$

As the degree of dividend is always greater than divisor but here degree $p(x) <$ degree $g(x)$.

When we divide $p(x)$ by $g(x)$, quotient will be zero and remainder will be $p(x)$.

$$\begin{aligned} \text{(iii) The dividend} &= p(x), \text{ divisor } g(x) \\ \text{quotient } q(x) &= 0 \\ \text{remainder} &= r(x) \end{aligned}$$

Here, degree of divisor $g(x)$ is more than degree of dividend.

(iv) When $p(x)$ is divided by $g(x)$, the remainder is zero so the $g(x)$ is a factor of $p(x)$ and degree of $g(x)$ will be less than or equal to the degree of $p(x)$ or degree $g(x) \leq$ degree $p(x)$.

$$\text{(v) Let } p(x) = x^2 + kx + k$$

$$\text{For equal zeroes, } b^2 - 4ac = 0$$

$$\Rightarrow (k)^2 - 4(1)(k) = 0$$

$$\Rightarrow k^2 - 4k = 0$$

$$\Rightarrow k(k - 4) = 0$$

$$\Rightarrow k = 0 \quad \text{or} \quad k = 4$$

But $k > 1$ so $k = 4$

The given quadratic polynomial has equal zeroes at $k = 4$.

Q2. Are the following statements true or false? Justify your answers.

- If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a , b and c have the same sign.
- If the graph of polynomial intersects the X-axis at only one point it cannot be a quadratic polynomial.
- If the graph of a polynomial intersects the X-axis at exactly two points, it need not be a quadratic polynomial.
- If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.
- If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of polynomial have the same sign.

- (vi) If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of a , b , and c is non-negative.
- (vii) The only value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeroes is $1/2$.

Sol. (i): False: Let α and β be the roots of the quadratic polynomial. If α

and β are positive then $\alpha + \beta = \frac{-b}{a}$ it shows that $\frac{-b}{a}$ is negative but sum of two positive numbers (α , β) must be +ive i.e. either b or a must be negative. So a , b and c will have different signs.

- (ii) **False:** The given statement is false, because when two zeroes of a quadratic polynomial are equal, then two intersecting points coincide to become one point.
- (iii) **True:** If a polynomial of degree more than two has two real zeroes and other zeroes are not real or are imaginary, then graph of the polynomial will intersect at two points on x-axis.
- (iv) **True:** Let $\beta = 0$, $\gamma = 0$

$$\begin{aligned} f(x) &= (x - \alpha)(x - \beta)(x - \gamma) \\ &= (x - \alpha)x \cdot x \end{aligned}$$

$$\Rightarrow f(x) = x^3 - \alpha x^2$$

which has no linear (coefficient of x) and constant terms.

- (v) **True:** α , β , and γ are all (-)ive for cubic polynomial $ax^3 + bx^2 + cx + d$.

$$\alpha + \beta + \gamma = \frac{-b}{a} \quad \dots(i)$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \quad \dots(ii)$$

$$\alpha\beta\gamma = \frac{-d}{a} \quad \dots(iii)$$

$\therefore \alpha, \beta, \gamma$ are all negative so,

$$\alpha + \beta + \gamma = -x \quad \text{(Any negative number)}$$

$$\Rightarrow \frac{-b}{a} = -x \quad \text{[From (i)]}$$

$$\Rightarrow \frac{b}{a} = x$$

So, a, b , have same sign and product of any two zeroes will be positive.

$$\text{So, } \alpha\beta + \beta\gamma + \gamma\alpha = +y \quad \text{(Any positive number)}$$

$$\Rightarrow \frac{+c}{a} = +y \quad \text{[From (ii)]}$$

$\Rightarrow c$ and a have same sign

$$\alpha\beta\gamma = -z \quad \text{(Any negative number)}$$

$$\Rightarrow \frac{-d}{a} = -z \quad \text{[From (iii)]}$$

$$\Rightarrow \frac{d}{a} = z$$

So, d and a will have same sign.

Hence, signs of b, c, d are same as of a .

So, signs of a, b, c, d will be same either positive or negative.

(vi) **True:** As all zeroes of cubic polynomial are positive

$$\text{Let } f(x) = x^3 + ax^2 - bx + c$$

$$\therefore \alpha + \beta + \gamma = + \text{ive say } +x$$

$$\Rightarrow \frac{-b}{a} = x$$

$\Rightarrow a$ and b has opposite signs ...(i)

$$\alpha\beta + \beta\gamma + \gamma\alpha = +y$$

$$\Rightarrow \frac{c}{a} = y$$

So, signs of a and c are same. ...(ii)

$$\text{Now, } \alpha\beta\gamma = + \text{ive} = +z$$

$$\Rightarrow \frac{-d}{a} = z$$

$\Rightarrow a$ and d have opposite signs. [From (i)]

From (i), if a is positive, then b is negative.

From (ii) if a is positive, then c is also positive.

From (iii) if a is positive, then d is negative.

Hence, if zeroes α, β, γ of cubic polynomial are positive then out of a, b, c at least one is positive.

(vii) **False:** $f(x) = kx^2 + x + k$ ($a = k, b = 1, c = k$)

For equal roots

$$b^2 - 4ac = 0$$

$$\Rightarrow (1)^2 - 4(k)(k) = 0$$

$$\Rightarrow 4k^2 = 1$$

$$\Rightarrow k^2 = 1/4$$

$$\Rightarrow k = \pm \frac{1}{2}$$

So, there are $\frac{1}{2}$ and $-\frac{1}{2}$ values of k so that the given equation has equal roots.

EXERCISE 2.3

Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and coefficients of the polynomials.

Q1. $4x^2 - 3x - 1$

Sol. Let $f(x) = 4x^2 - 3x - 1$

Splitting the middle term, we get

$$\begin{aligned} &= 4x^2 - 4x + 1x - 1 \\ &= 4x(x-1) + 1(x-1) \\ &= (x-1)(4x+1) \end{aligned}$$

For $f(x) = 0$, we have

$$4x^2 - 3x - 1 = 0$$

or $(x-1)(4x+1) = 0$

Either $x-1 = 0 \Rightarrow x = 1$

or $4x+1 = 0 \Rightarrow 4x = -1 \Rightarrow x = \frac{-1}{4}$

\therefore The zeroes of $f(x)$ are 1 and $\frac{-1}{4}$.

Verification: $\alpha = 1, \beta = \frac{-1}{4}$

$a = 4, b = -3$ and $c = -1$

$$\therefore \alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow 1 - \frac{1}{4} = \frac{-(-3)}{4}$$

$$\Rightarrow \frac{3}{4} = \frac{3}{4}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow 1 \times \left(\frac{-1}{4}\right) = \frac{-1}{4}$$

$$\Rightarrow \frac{-1}{4} = \frac{-1}{4}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified

Q2. $3x^2 + 4x - 4$

Sol. Let $f(x) = 3x^2 + 4x - 4$

For zeroes of $f(x)$, $f(x) = 0$

$$\therefore 3x^2 + 4x - 4 = 0$$

Splitting the middle term, we get

$$3x^2 + 6x - 2x - 4 = 0$$

$$\Rightarrow 3x(x+2) - 2(x+2) = 0$$

$$\Rightarrow (x+2)(3x-2) = 0$$

$$\Rightarrow x+2 = 0 \quad \text{or} \quad 3x-2 = 0$$

$$\Rightarrow x = -2 \quad \text{or} \quad 3x = +2 \Rightarrow x = \frac{2}{3}$$

So, zeroes of $f(x)$ are -2 , and $2/3$.

$$\text{Sum of roots} = \frac{-b}{a}$$

$$\Rightarrow -2 + \frac{2}{3} = \frac{-4}{3}$$

$$\Rightarrow \frac{-6+2}{3} = \frac{-4}{3}$$

$$(a=3, b=4, c=-4)$$

$$\Rightarrow \frac{-4}{3} = \frac{-4}{3}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

$$\text{Product of roots} = \frac{c}{a}$$

$$\Rightarrow -2 \times \frac{2}{3} = \frac{-4}{3}$$

$$\Rightarrow \frac{-4}{3} = \frac{-4}{3}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

Q3. $5t^2 + 12t + 7$

Sol. Let $f(t) = 5t^2 + 12t + 7$

For zeroes of $f(t)$, $f(t) = 0$

$$\Rightarrow 5t^2 + 12t + 7 = 0$$

$$\Rightarrow 5t^2 + 7t + 5t + 7 = 0$$

$$\Rightarrow t(5t + 7) + 1(5t + 7) = 0$$

$$\Rightarrow (5t + 7)(t + 1) = 0$$

$$\Rightarrow 5t + 7 = 0 \quad \text{or} \quad (t + 1) = 0$$

$$\Rightarrow t = \frac{-7}{5} \quad \text{or} \quad t = -1$$

$$\Rightarrow \alpha = \frac{-7}{5} \quad \beta = -1$$

Verification:

$$a = 5, \quad b = 12, \quad c = +7$$

$$\Rightarrow \alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \frac{-7}{5} - 1 = \frac{-(+12)}{5}$$

$$\Rightarrow \frac{-7 - 5}{5} = \frac{-12}{5}$$

$$\Rightarrow \frac{-12}{5} = \frac{-12}{5}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

Q4. $t^3 - 2t^2 - 15t$

Sol. Let $f(t) = t^3 - 2t^2 - 15t$

For zeroes of $f(t)$, $f(t) = 0$

$$\Rightarrow t^3 - 2t^2 - 15t = 0$$

$$\Rightarrow \alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \left(\frac{-7}{5}\right)(-1) = \frac{7}{5}$$

$$\Rightarrow +\frac{7}{5} = \frac{7}{5}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

$$\Rightarrow t[t^2 - 2 \cdot t - 15] = 0$$

$$\Rightarrow t[t^2 - 5t + 3t - 15] = 0$$

$$\Rightarrow t[t(t-5) + 3(t-5)] = 0$$

$$\Rightarrow t(t-5)(t+3) = 0$$

$$\Rightarrow t=0 \text{ or } t-5=0 \quad \text{or} \quad t+3=0$$

$$\Rightarrow t=0 \text{ or } t=5 \quad \text{or} \quad t=-3$$

So, zeroes of cubic polynomial are $\alpha=0$, $\beta=5$, $\gamma=-3$

Verification: $\alpha=0$, $\beta=5$, $\gamma=-3$

Cubic polynomial,

$$f(t) = t^3 - 2t^2 - 15t, \text{ which is of the form } at^3 + bt^2 + ct + d$$

where $a=1$, $b=-2$, $c=-15$ and $d=0$

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\Rightarrow 0 + 5 - 3 = \frac{-(-2)}{1}$$

$$\Rightarrow 2 = 2$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\Rightarrow (0)(5) + (5)(-3) + (-3)(0) = \frac{-15}{1}$$

$$\Rightarrow 0 - 15 + 0 = -15$$

$$\Rightarrow -15 = -15$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

$$\alpha\beta\gamma = \frac{-d}{a}$$

$$\Rightarrow (0)(5)(-3) = \frac{-0}{1}$$

$$\Rightarrow 0 = 0$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

$$\text{Q5. } 2x^2 + \frac{7}{2}x + \frac{3}{4}$$

$$\text{Sol. Let } f(x) = 2x^2 + \frac{7}{2}x + \frac{3}{4}$$

For zeroes of $f(x)$, $f(x) = 0$

$$\Rightarrow 2x^2 + \frac{7}{2}x + \frac{3}{4} = 0$$

$$\Rightarrow 8x^2 + 14x + 3 = 0$$

[As c is positive (+3) so sum of (8×3) factors should be equal to 14]

$$\Rightarrow 8x^2 + 12x + 2x + 3 = 0$$

$$\Rightarrow 4x(2x+3) + 1(2x+3) = 0$$

$$\Rightarrow (2x+3)(4x+1) = 0$$

$$\Rightarrow 2x+3=0 \quad \text{or} \quad 4x+1=0$$

$$\Rightarrow 2x=-3 \quad \text{or} \quad 4x=-1$$

$$\Rightarrow x = \frac{-3}{2} \quad \text{or} \quad x = \frac{-1}{4}$$

Verification: $\alpha = \frac{-3}{2}$ and $\beta = \frac{-1}{4}$

Quadratic polynomial $f(x) = 2x^2 + \frac{7}{2}x + \frac{3}{4}$, which is of the form $ax^2 + bx + c$.

$$\therefore a = 2, \quad b = \frac{7}{2} \quad \text{and} \quad c = \frac{3}{4}$$

$\alpha + \beta = \frac{-b}{a}$	$\alpha \cdot \beta = \frac{c}{a}$
$\Rightarrow \frac{-3}{2} - \frac{1}{4} = \frac{-7}{2}$	$\Rightarrow \left(\frac{-3}{2}\right)\left(\frac{-1}{4}\right) = \frac{3}{8}$
$\Rightarrow \frac{-7}{4} = \frac{-7}{2} \times \frac{1}{2}$	$\Rightarrow \frac{+3}{8} = \frac{3}{4} \times \frac{1}{2}$
$\Rightarrow \frac{-7}{4} = \frac{-7}{4}$	$\Rightarrow \frac{3}{8} = \frac{3}{8}$
$\Rightarrow \text{LHS} = \text{RHS}$	$\Rightarrow \text{LHS} = \text{RHS}$

Hence, verified.

Hence, verified.

Q6. $4x^2 + 5\sqrt{2}x - 3$

Sol. Let $f(x) = 4x^2 + 5\sqrt{2}x - 3$

For zeroes of $f(x)$, $f(x) = 0$

$$\begin{aligned} \Rightarrow 4x^2 + 5\sqrt{2}x - 3 &= 0 \\ \Rightarrow 4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3 &= 0 \\ \Rightarrow 2x[2x + 3\sqrt{2}] - 1[\sqrt{2}x + 3] &= 0 \quad \left[\begin{array}{l} \because 4 \times 3 = 2 \times 2 \times 3 \\ = \sqrt{2} \times \sqrt{2} \times 2 \times 3 \\ = 6\sqrt{2} \times \sqrt{2} \end{array} \right] \\ \Rightarrow 2\sqrt{2}x[\sqrt{2}x + 3] - 1[\sqrt{2}x + 3] &= 0 \\ \Rightarrow (\sqrt{2}x + 3)(2\sqrt{2}x - 1) &= 0 \\ \Rightarrow \sqrt{2}x + 3 = 0 &\quad \text{or} \quad 2\sqrt{2}x - 1 = 0 \\ \Rightarrow x = \frac{-3}{\sqrt{2}} &\quad \text{or} \quad x = \frac{1}{2\sqrt{2}} \end{aligned}$$

Verification:

$$\alpha = \frac{-3}{\sqrt{2}}, \beta = \frac{1}{2\sqrt{2}}, a = 4, b = 5\sqrt{2}, c = -3$$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \frac{-3}{\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{-5\sqrt{2}}{4}$$

$$\Rightarrow \frac{(-6+1)}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-5\sqrt{2}}{4}$$

$$\Rightarrow \frac{-5\sqrt{2}}{4} = \frac{-5\sqrt{2}}{4}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

Q7. $2s^2 - (1+2\sqrt{2})s + \sqrt{2}$

Sol. Let $f(s) = 2s^2 - (1+2\sqrt{2})s + \sqrt{2}$

For zeroes of $f(s)$, $f(s) = 0$

$$\Rightarrow 2s^2 - (1+2\sqrt{2})s + \sqrt{2} = 0$$

$$\Rightarrow 2s^2 - 1s - 2\sqrt{2}s + \sqrt{2} = 0$$

$$\Rightarrow s(2s-1) - \sqrt{2}(2s-1) = 0$$

$$\Rightarrow (2s-1)(s-\sqrt{2}) = 0$$

$$\Rightarrow 2s-1 = 0 \quad \text{or} \quad s-\sqrt{2} = 0$$

$$\Rightarrow s = \frac{1}{2} \quad \text{or} \quad s = \sqrt{2}$$

Verification of the relation between α , β , a , b and c

$$\alpha = \frac{1}{2}, \quad \beta = \sqrt{2}, \quad a = 2, \quad b = -(1+2\sqrt{2}), \quad c = \sqrt{2}$$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \frac{1}{2} + \sqrt{2} = \frac{+(1+2\sqrt{2})}{2}$$

$$\Rightarrow \frac{1}{2} + \sqrt{2} = \frac{1}{2} + \frac{2\sqrt{2}}{2}$$

$$\Rightarrow \frac{1}{2} + \sqrt{2} = \frac{1}{2} + \sqrt{2}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \left(-\frac{3}{\sqrt{2}}\right)\left(\frac{1}{2\sqrt{2}}\right) = \frac{-3}{4}$$

$$\Rightarrow \frac{-3}{4} = \frac{-3}{4}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

$$[\because a \times c = (2\sqrt{2})]$$

[Open the brackets]

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \left(\frac{1}{2}\right)(\sqrt{2}) = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

Q8. $v^2 + 4\sqrt{3}v - 15$

Sol. Let $f(v) = v^2 + 4\sqrt{3}v - 15$

For zeroes of $f(v)$, $f(v) = 0$

$$\Rightarrow v^2 + 4\sqrt{3}v - 15 = 0$$

$$\Rightarrow v^2 + 5\sqrt{3}v - 1\sqrt{3}v - 15 = 0$$

$$\Rightarrow v(v + 5\sqrt{3}) - \sqrt{3}(v + 5\sqrt{3}) = 0$$

$$\Rightarrow (v + 5\sqrt{3})(v - \sqrt{3}) = 0$$

$$\Rightarrow (v + 5\sqrt{3}) = 0 \quad \text{or} \quad (v - \sqrt{3}) = 0$$

$$\Rightarrow v = -5\sqrt{3} \quad \text{or} \quad v = \sqrt{3}$$

Verification of relations between α , β , a , b , c

$$\alpha = -5\sqrt{3}, \quad \beta = \sqrt{3}, \quad a = 1, \quad b = 4\sqrt{3} \quad \text{and} \quad c = -15$$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow -5\sqrt{3} + \sqrt{3} = \frac{-4\sqrt{3}}{1}$$

$$\Rightarrow -4\sqrt{3} = -4\sqrt{3}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow (-5\sqrt{3})(\sqrt{3}) = \frac{-15}{1}$$

$$\Rightarrow -5 \times 3 = -15$$

$$\Rightarrow -15 = -15$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, verified.

$$\text{Q9. } y^2 + \frac{3}{2}\sqrt{5}y - 5$$

$$\text{Sol. Let } f(y) = y^2 + \frac{3}{2}\sqrt{5}y - 5$$

For zeroes of $f(y)$, $f(y) = 0$

$$\Rightarrow y^2 + \frac{3}{2}\sqrt{5}y - 5 = 0$$

$$\Rightarrow 2y^2 + 3 \cdot \sqrt{5}y - 10 = 0$$

$$\Rightarrow 2y^2 + 4\sqrt{5}y - 1\sqrt{5}y - 10 = 0$$

$$\Rightarrow 2y(y + 2\sqrt{5}) - \sqrt{5}[y + 2\sqrt{5}] = 0$$

$$\Rightarrow (y + 2\sqrt{5})(2y - \sqrt{5}) = 0$$

$$\Rightarrow y + 2\sqrt{5} = 0 \quad \text{or} \quad 2y - \sqrt{5} = 0$$

$$\Rightarrow y = -2\sqrt{5} \quad \text{or} \quad y = \frac{\sqrt{5}}{2}$$

Verification of the relations between α , β , and a , b , c

$$\alpha = -2\sqrt{5}, \quad \beta = \frac{\sqrt{5}}{2}, \quad a = 1, \quad b = \frac{3}{2}\sqrt{5} \quad \text{and} \quad c = -5$$

$$\begin{aligned}\alpha + \beta &= \frac{-b}{a} \\ \Rightarrow -2\sqrt{5} + \frac{\sqrt{5}}{2} &= \frac{-3\sqrt{5}}{2} \\ \Rightarrow \frac{-4\sqrt{5} + \sqrt{5}}{2} &= \frac{-3\sqrt{5}}{2} \\ \Rightarrow \frac{-3\sqrt{5}}{2} &= \frac{-3\sqrt{5}}{2} \\ \Rightarrow \text{LHS} &= \text{RHS} \\ \text{Hence, verified.}\end{aligned}$$

$$\begin{aligned}\alpha \cdot \beta &= \frac{c}{a} \\ \Rightarrow (-2\sqrt{5})\left(\frac{\sqrt{5}}{2}\right) &= \frac{-5}{1} \\ \Rightarrow -5 &= -5 \\ \Rightarrow \text{LHS} &= \text{RHS} \\ \text{Hence, verified.}\end{aligned}$$

Q10. $7y^2 - \frac{11}{3}y - \frac{2}{3}$

Sol. Let $f(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

For zeroes of $f(y)$, $f(y) = 0$

$$\begin{aligned}\Rightarrow 7y^2 - \frac{11}{3}y - \frac{2}{3} &= 0 \\ \Rightarrow 21y^2 - 11y - 2 &= 0 \\ \Rightarrow 21y^2 - 14y + 3y - 2 &= 0 \\ \Rightarrow 7y(3y - 2) + 1(3y - 2) &= 0 \\ \Rightarrow (3y - 2)(7y + 1) &= 0 \\ \Rightarrow 3y - 2 = 0 &\quad \text{or} \quad 7y + 1 = 0 \\ \Rightarrow y = \frac{2}{3} &\quad \text{or} \quad y = \frac{-1}{7}\end{aligned}$$

Verification of the relations between α , β , a , b and c

$$\alpha = \frac{2}{3}, \quad \beta = \frac{-1}{7}, \quad a = 7, \quad b = -\frac{11}{3}, \quad c = \frac{-2}{3}$$

$$\begin{aligned}\Rightarrow \alpha + \beta &= \frac{-b}{a} \\ \Rightarrow \left(\frac{2}{3}\right) - \frac{1}{7} &= \frac{+\frac{11}{3}}{7} \\ \Rightarrow \frac{14 - 3}{21} &= \frac{11}{3} \times \frac{1}{7} \\ \Rightarrow \frac{11}{21} &= \frac{11}{21} \\ \Rightarrow \text{LHS} &= \text{RHS}\end{aligned}$$

Hence, verified.

$$\begin{aligned}\alpha \cdot \beta &= \frac{c}{a} \\ \Rightarrow \left(\frac{2}{3}\right) \times \left(\frac{-1}{7}\right) &= \frac{\frac{-2}{3}}{7} \\ \Rightarrow \frac{-2}{21} &= \frac{-2}{3} \times \frac{1}{7} \\ \Rightarrow \frac{-2}{21} &= \frac{-2}{21} \\ \Rightarrow \text{LHS} &= \text{RHS}\end{aligned}$$

Hence, verified.

EXERCISE 2.4

Q1. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomial by factorisation.

(i) $\frac{-8}{3}, \frac{4}{3}$ (ii) $\frac{21}{8}, \frac{5}{16}$ (iii) $-2\sqrt{3}, -9$ (iv) $\frac{-3}{2\sqrt{5}}, \frac{-1}{2}$

Sol. **Main concept:** (a) If α, β are the zeroes of $f(x)$, then

$$f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

(b) The zeroes of $f(x)$ are given by $f(x) = 0$.

(i) $\alpha + \beta = \frac{-8}{3}$ and $\alpha \cdot \beta = \frac{4}{3}$ [Given]

\therefore $f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$ [Formula]

$$= x^2 - \left(\frac{-8}{3}\right)x + \frac{4}{3}$$

Multiplying or dividing $f(x)$ by any real number does not affect the zeroes of polynomial.

So, $f(x) = 3x^2 + 8x + 4$ [Multiplying by LCM 3]

For zeroes of $f(x)$, $f(x) = 0$

$$\Rightarrow 3x^2 + 8x + 4 = 0$$

$$\Rightarrow 3x^2 + 6x + 2x + 4 = 0$$

$$\Rightarrow 3x(x + 2) + 2(x + 2) = 0$$

$$\Rightarrow (x + 2)(3x + 2) = 0$$

$$\Rightarrow x + 2 = 0 \quad \text{or} \quad 3x + 2 = 0$$

$$\Rightarrow x = -2 \quad \text{or} \quad x = \frac{-2}{3}$$

$$\therefore \alpha = -2 \text{ and } \beta = \frac{-2}{3}$$

(ii) $\alpha + \beta = \frac{21}{8}$ and $\alpha \cdot \beta = \frac{5}{16}$ [Given]

$$f(x) = x^2 - (\alpha + \beta)x + \alpha \cdot \beta$$
 [Formula]

$$\Rightarrow f(x) = x^2 - \left(\frac{21}{8}\right)x + \left(\frac{5}{16}\right)$$

Multiplying (or dividing) $f(x)$ by any real number does not affect the zeroes of $f(x)$ so, multiplying $f(x)$ by 16 (LCM), we get

$$f(x) = 16x^2 - 42x + 5$$

For zeroes of polynomial $f(x)$, $f(x) = 0$

$$\Rightarrow 16x^2 - 42x + 5 = 0$$

$$\Rightarrow 16x^2 - 40x - 2x + 5 = 0$$

$$\Rightarrow 8x(2x - 5) - 1(2x - 5) = 0$$

$$\begin{aligned} \Rightarrow & (2x-5)(8x-1) = 0 \\ \Rightarrow & 2x-5 = 0 \quad \text{or} \quad 8x-1 = 0 \\ \Rightarrow & x = \frac{5}{2} \quad \text{or} \quad x = \frac{1}{8} \end{aligned}$$

$$\therefore \alpha = \frac{5}{2} \text{ and } \beta = \frac{1}{8}$$

$$(iii) \alpha + \beta = -2\sqrt{3} \quad \text{and} \quad \alpha\beta = -9 \quad \text{[Given]}$$

$$f(x) = x^2 - (\alpha + \beta)x + \alpha\beta \quad \text{[Formula]}$$

$$= x^2 - (-2\sqrt{3})x + (-9)$$

$$\Rightarrow f(x) = x^2 + 2\sqrt{3}x - 9$$

For zeroes of polynomial $f(x)$, $f(x) = 0$

$$\Rightarrow x^2 + 2\sqrt{3}x - 9 = 0$$

$$\Rightarrow x^2 + 3\sqrt{3}x - 1\sqrt{3}x - 9 = 0$$

$$\Rightarrow x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3}) = 0$$

$$\Rightarrow (x + 3\sqrt{3})(x - \sqrt{3}) = 0$$

$$\Rightarrow x + 3\sqrt{3} = 0 \quad \text{or} \quad (x - \sqrt{3}) = 0$$

$$\Rightarrow x = -3\sqrt{3} \quad \text{or} \quad x = \sqrt{3}$$

$$\therefore \alpha = -3\sqrt{3} \quad \text{and} \quad \beta = \sqrt{3}$$

$$(iv) \alpha + \beta = \frac{-3}{2\sqrt{5}} \quad \text{and} \quad \alpha \cdot \beta = -\frac{1}{2} \quad \text{[Given]}$$

$$f(x) = x^2 - (\alpha + \beta)x + \alpha\beta \quad \text{[Formula]}$$

$$= x^2 - \left(\frac{-3}{2\sqrt{5}}\right)x + \left(-\frac{1}{2}\right)$$

$$\Rightarrow f(x) = x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}$$

Multiplying or dividing $f(x)$ by any real number does not affect the zeroes of $f(x)$. On multiplying $f(x)$ by $2\sqrt{5}$ (LCM), we get

$$f(x) = 2\sqrt{5}x^2 + 3x - \sqrt{5}$$

For zeroes of polynomial $f(x)$, $f(x) = 0$

$$\Rightarrow 2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$$

$$\Rightarrow 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5} = 0$$

$$\Rightarrow \sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = 0$$

$$\Rightarrow (2x + \sqrt{5})(\sqrt{5}x - 1) = 0$$

$$\Rightarrow (2x + \sqrt{5}) = 0 \quad \text{or} \quad \sqrt{5}x - 1 = 0$$

$$\Rightarrow x = \frac{-\sqrt{5}}{2} \quad \text{or} \quad x = \frac{1}{\sqrt{5}}$$

$$\therefore \alpha = \frac{-\sqrt{5}}{2} \quad \text{and} \quad \beta = \frac{1}{\sqrt{5}}$$

Q2. Given that the zeroes of cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form $a, (a + b), (a + 2b)$ for some real numbers a and b , find the values of a and b as well as the zeroes of the given polynomial.

Sol. Main concept: $\alpha + \beta + \gamma = \frac{-b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{+c}{a}$ and $\alpha\beta\gamma = \frac{-d}{a}$
 Let $f(x) = x^3 - 6x^2 + 3x + 10$ [given] ... (i)

$$\alpha = a, \quad \beta = a + b \quad \text{and} \quad \gamma = a + 2b \quad \text{[Given]}$$

But, $f(x) = ax^3 + bx^2 + cx + d$... (ii)

$\therefore a = 1, \quad b = -6, \quad c = 3$ and $d = +10$ [Comparing (i) and (ii)]

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\Rightarrow a + a + b + a + 2b = \frac{+6}{1} \Rightarrow 3a + 3b = 6$$

$$\Rightarrow a + b = 2$$

$$\Rightarrow b = 2 - a \quad \dots (iii)$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\Rightarrow a(a + b) + (a + b)(a + 2b) + (a + 2b)(a) = \frac{3}{1}$$

$$\Rightarrow a^2 + ab + a^2 + 2ab + ab + 2b^2 + a^2 + 2ab = 3$$

$$\Rightarrow 3a^2 + 6ab + 2b^2 = 3$$

$$\Rightarrow 3a^2 + 6a(2 - a) + 2(2 - a)^2 = 3 \quad \text{[Using (iii)]}$$

$$\Rightarrow 3a^2 + 12a - 6a^2 + 2(4 + a^2 - 4a) = 3$$

$$\Rightarrow -3a^2 + 12a + 8 + 2a^2 - 8a - 3 = 0$$

$$\Rightarrow -a^2 + 4a + 5 = 0$$

$$\Rightarrow a^2 - 4a - 5 = 0$$

$$\Rightarrow a^2 - 5a + a - 5 = 0$$

$$\Rightarrow a(a - 5) + 1(a - 5) = 0$$

$$\Rightarrow (a + 1)(a - 5) = 0$$

$$\Rightarrow (a + 1) = 0 \quad \text{or} \quad (a - 5) = 0$$

$$\Rightarrow a = -1 \quad \text{or} \quad a = 5$$

Now, $b = 2 - a$

[From (iii)]

When $a = 5, b = 2 - 5 = -3$

When $a = -1$, $b = 2 - (-1) = 3$

If $a = -1$ and $b = 3$, then zeroes are, a , $(a + b)$, $(a + 2b)$
 $= -1, (-1 + 3), [-1 + 2(3)]$
 $= -1, 2, 5$

If $a = 5$, and $b = -3$, then zeroes are 5 , $[5 + (-3)]$, $[5 + 2(-3)] = 5, 2, -1$

So, zeroes in both cases are $\beta = 2$, $\gamma = -1$ and $\alpha = 5$.

Q3. Given that $\sqrt{2}$ is a zero of a cubic polynomial

$6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.

Sol. Main concept: Using Euclid's division algorithm here, remainder is zero. Then quotient will be quadratic whose zeroes can be find out by factorisation.

Let $f(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

If $\sqrt{2}$ is the zero of $f(x)$, then $(x - \sqrt{2})$ will be a factor of $f(x)$. So, by remainder theorem when $f(x)$ is divided by $(x - \sqrt{2})$, the quotient comes out to be quadratic.

$$\begin{array}{r} 6x^2 + 7\sqrt{2}x + 4 \\ x - \sqrt{2} \overline{) 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}} \\ \underline{- 6x^3 + 6\sqrt{2}x^2} \phantom{- 10x - 4\sqrt{2}} \\ 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\ \underline{- 7\sqrt{2}x^2 + 14x} \phantom{- 4\sqrt{2}} \\ 4x - 4\sqrt{2} \\ \underline{- 4x + 4\sqrt{2}} \\ 0 \end{array}$$

$$\begin{aligned} \therefore f(x) &= (x - \sqrt{2})(6x^2 + 7\sqrt{2}x + 4) \text{ (By Euclid's division algorithm)} \\ &= (x - \sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4) \end{aligned}$$

For zeroes of $f(x)$, $f(x) = 0$

$$\therefore (x - \sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4) = 0$$

$$\Rightarrow (x - \sqrt{2})[2x(3x + 2\sqrt{2}) + \sqrt{2}(3x + 2\sqrt{2})] = 0$$

$$\Rightarrow (x - \sqrt{2})(3x + 2\sqrt{2})(2x + \sqrt{2}) = 0$$

$$\Rightarrow x - \sqrt{2} = 0 \quad \text{or} \quad 3x + 2\sqrt{2} = 0 \quad \text{or} \quad 2x + \sqrt{2} = 0$$

$$\Rightarrow x = \sqrt{2} \quad \text{or} \quad x = \frac{-2\sqrt{2}}{3} \quad \text{or} \quad x = \frac{-\sqrt{2}}{2}$$

So, other two roots are $= \frac{-2\sqrt{2}}{3}$ and $\frac{-\sqrt{2}}{2}$.

Q4. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of two polynomials.

Sol. Main concept: Factor theorem and Euclid's division algorithm.

By factor theorem and Euclid's division algorithm, we get

$$f(x) = g(x) \times q(x) + r(x)$$

$$\text{Let } f(x) = 2x^4 + x^3 - 14x^2 + 5x + 6 \quad \dots(i)$$

$$\text{and } g(x) = x^2 + 2x + k$$

$$\begin{array}{r}
 \therefore \quad x^2 + 2x + k \overline{) 2x^4 + x^3 - 14x^2 + 5x + 6} \\
 \underline{2x^4 + 4x^3 + 2kx^2} \\
 -3x^3 - 14x^2 - 2kx^2 + 5x + 6 \\
 \underline{-3x^3 - 6x^2} \\
 -8x^2 - 2kx^2 + 5x + 3kx + 6 \\
 \underline{-8x^2} \\
 -2kx^2 + 21x + 3kx + 8k + 6 \\
 \underline{-2kx^2} \\
 21x + 7kx + 2k^2 + 8k + 6
 \end{array}$$

$$\text{But, } r(x) = 0$$

$$\therefore (21 + 7k)x + 2k^2 + 8k + 6 = 0x + 0$$

$$\Rightarrow \quad 21 + 7k = 0 \quad \text{and} \quad \begin{array}{l} 2k^2 + 8k + 6 = 0 \\ 2k^2 + 6k + 2k + 6 = 0 \end{array}$$

$$\begin{array}{l} \Rightarrow \quad k = \frac{-21}{7} \\ \Rightarrow \quad k = -3 \end{array} \quad \left| \begin{array}{l} \Rightarrow 2k(k+3) + 2(k+3) = 0 \\ \Rightarrow (k+3)(2k+2) = 0 \\ \Rightarrow k+3 = 0 \text{ or } 2k+2 = 0 \\ \Rightarrow k = -3 \text{ or } k = -1 \end{array} \right.$$

\therefore Common solution is $k = -3$

$$\text{So, } \quad q(x) = 2x^2 - 3x - 8 - 2(-3)$$

$$\quad \quad \quad = 2x^2 - 3x - 8 + 6$$

$$\Rightarrow \quad q(x) = 2x^2 - 3x - 2$$

$$\therefore \quad f(x) = g(x)q(x) + 0$$

$$\quad \quad \quad = (x^2 + 2x - 3)(2x^2 - 3x - 2)$$

$$\quad \quad \quad = (2x^2 - 4x + 1x - 2)(x^2 + 3x - 1x - 3)$$

$$= [2x(x-2) + 1(x-2)] [x(x+3) - 1(x+3)]$$

$$\Rightarrow f(x) = (x-2)(2x+1)(x+3)(x-1)$$

For zeroes of $f(x)$, $f(x) = 0$

$$\therefore (x-1)(x-2)(x+3)(2x+1) = 0$$

$$\Rightarrow (x-1) = 0, (x-2) = 0, (x+3) = 0 \text{ and } 2x+1 = 0$$

$$\Rightarrow x = 1, x = 2, x = -3 \text{ and } x = -\frac{1}{2}$$

So, zeroes of $f(x)$ are 1, 2, -3, and $-\frac{1}{2}$.

Q5. Given that $(x - \sqrt{5})$ is a factor of cubic polynomial

$x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, find all the zeroes of the polynomial.

Sol. Main concept: Factor theorem, Euclid's division algorithm.

Let $f(x) = x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$

and $g(x) = (x - \sqrt{5})$

$\therefore g(x)$ is a factor of $f(x)$ so $f(x) = q(x)(x - \sqrt{5})$

$$\begin{array}{r} x^2 - 2\sqrt{5}x + 3 \\ x - \sqrt{5} \overline{) x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}} \\ \underline{-x^3 + \sqrt{5}x^2} \phantom{+ 13x - 3\sqrt{5}} \\ -2\sqrt{5}x^2 + 13x - 2\sqrt{5} \\ \underline{-2\sqrt{5}x^2 + 10x} \phantom{- 2\sqrt{5}} \\ +3x - 3\sqrt{5} \\ \underline{+3x - 3\sqrt{5}} \\ 0 \end{array}$$

But, $f(x) = q(x)g(x)$

$$\therefore f(x) = (x^2 - 2\sqrt{5}x + 3)(x - \sqrt{5})$$

$$\begin{aligned} \Rightarrow f(x) &= [x^2 - \{(\sqrt{5} + \sqrt{2}) + (\sqrt{5} - \sqrt{2})\}x + (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})][x - \sqrt{5}] \\ &= [x^2 - (\sqrt{5} + \sqrt{2})x - (\sqrt{5} - \sqrt{2})x + (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})][x - \sqrt{5}] \\ &= x[x - (\sqrt{5} + \sqrt{2})] - (\sqrt{5} - \sqrt{2})[x - (\sqrt{5} + \sqrt{2})][x - \sqrt{5}] \end{aligned}$$

$$\Rightarrow f(x) = (x - \sqrt{5} - \sqrt{2})(x - \sqrt{5} + \sqrt{2})(x - \sqrt{5})$$

For zeroes of $f(x)$, $f(x) = 0$

$$\Rightarrow (x - \sqrt{5} - \sqrt{2})(x - \sqrt{5} + \sqrt{2})(x - \sqrt{5}) = 0$$

$$\Rightarrow (x - \sqrt{5} - \sqrt{2}) = 0 \text{ or } (x - \sqrt{5} + \sqrt{2}) = 0 \text{ or } (x - \sqrt{5}) = 0$$

$$\Rightarrow x = \sqrt{5} + \sqrt{2} \text{ or } x = \sqrt{5} - \sqrt{2} \text{ or } x = +\sqrt{5}$$

\therefore Zeroes are $(\sqrt{5} + \sqrt{2})$, $(\sqrt{5} - \sqrt{2})$ and $\sqrt{5}$.

Q6. For which values of a and b are the zeroes of $q(x) = x^3 + 2x^2 + a$ also the zeroes of polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? Which zeroes of $p(x)$ are not the zeroes of $q(x)$?

Sol. **Main concept:** Factor theorem and Euclid's division algorithm.

By factor theorem if $q(x)$ is a factor of $p(x)$, then $r(x)$ must be zero.

$$\begin{array}{r}
 p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b \\
 q(x) = x^3 + 2x^2 + a \\
 \begin{array}{r}
 x^2 - 3x + 2 \\
 \hline
 x^3 + 2x^2 + a \) \ x^5 - x^4 - 4x^3 + 3x^2 + 3x + b \\
 \underline{-x^5 + 2x^4} \qquad \qquad \underline{+ax^2} \\
 \hline
 -3x^4 - 4x^3 - ax^2 + 3x^2 + 3x + b \\
 \underline{-3x^4 - 6x^3} \qquad \qquad \underline{-3ax} \\
 \hline
 \qquad \qquad \underline{+3x^2 - ax^2 + 3x^2 + 3ax + 3x + b} \\
 \qquad \underline{-2x^3} \qquad \qquad \underline{+4x^2} \qquad \qquad \underline{+2a} \\
 \hline
 \qquad \qquad \qquad \qquad \underline{-ax^2 - x^2 + 3ax + 3x - 2a + b}
 \end{array}
 \end{array}$$

So, by factor theorem remainder must be zero i.e.,

$$\begin{aligned} r(x) &= 0 \\ \Rightarrow -(a+1)x^2 + (3a+3)x + (b-2a) &= 0x^2 + 0x + 0 \end{aligned}$$

Comparing the coefficients of x^2 , x and constt. on both sides, we get

$$\begin{aligned}
 -(a+1) &= 0 & \text{and} & & 3a+3 &= 0 & \text{and} & & b-2a &= 0 \\
 \Rightarrow a &= -1 & \text{and} & & a &= -1 & \text{and} & & b-2(-1) &= 0 \\
 & & & & & & \Rightarrow & & b &= -2
 \end{aligned}$$

For $a = -1$ and $b = -2$, zeroes of $q(x)$ will be zeroes of $p(x)$.

For zeroes of $p(x)$, $p(x) = 0$

$$\Rightarrow (x^3 + 2x^2 + a)(x^2 - 3x + 2) = 0 \quad [\because a = -1]$$

$$\Rightarrow [x^3 + 2x^2 - 1][x^2 - 2x - 1x + 2] = 0$$

$$\Rightarrow (x^3 + 2x^2 - 1)[x(x-2) - 1(x-2)] = 0$$

$$\Rightarrow (x^3 + 2x^2 - 1)(x-2)(x-1) = 0$$

Hence, $x = 2$ and 1 are not the zeroes of $q(x)$.

□□□