5 Arithmetic Progressions

EXERCISE 5.1

Choose the correct answer from the given four options in the following questions:

Q1. In an A.P., if $d = -4$, $n = 7$, $a_n = 4$, then a is

(*a*) 6 (*b*) 7 (*c*) 20 (*d*) 28

Sol. (*d*): **Main concept used:** $\boxed{a_n = a + (n-1)d}$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad 4 = a + (7-1)(-4)$ (By the given condition)

$\Rightarrow \quad -a = -4 - 24$

$\Rightarrow \quad a = 28$

Q2. In an A.P., if $a = 3.5$, $d = 0$, $n = 101$, then a_n will be

(*a*) 0 (*b*) 3.5 (*c*) 103.5 (*d*) 104.5

Sol. (*b*): $a_n = a + (n-1)d$

$\Rightarrow \quad a_n = 3.5 + (101-1) \times 0$ (By the given condition)

$\Rightarrow \quad a_n = 3.5 + 100 \times 0$

$\Rightarrow \quad a_n = 3.5$

OR

As $d = 0$ so all terms are same.

Q3. The list of numbers $-10, -6, -2, 2, \ldots$ is

(*a*) an A.P. with $d = -16$ (*b*) an A.P. with $d = 4$

(*c*) an A.P. with $d = -4$ (*d*) not an A.P.

Sol. (*b*): **Main concept used:** A series of numbers will be an A.P. if $d_1 = d_2 = d_3 = \ldots$

where $d_1 = a_2 - a_1, d_2 = a_3 - a_2, d_3 = a_4 - a_3$

$d_1 = a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$d_2 = a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$

$d_3 = a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

As $d_1 = d_2 = d_3 = 4$

So, the given series is an A.P. with $d = 4$.

Q4. The 11th term of an A.P. $-5, \frac{-5}{2}, 0, \frac{5}{2} \ldots$ is

(*a*) -20 (*b*) 20 (*c*) -30 (*d*) 30

Sol. (*b*): Here, $n = 11$, $a = -5$, $d = \frac{5}{2} - 0 = \frac{5}{2}$

$a_n = a + (n-1)d$

$\therefore \quad a_{11} = -5 + (11-1)\left(\frac{5}{2}\right)$

$= -5 + 10 \times \frac{5}{2} = -5 + 25 = 20$

Q5. The first four terms of an A.P. whose first term is -2 and the common difference is (-2), are

(*a*) $-2, 0, 2, 4$ (*b*) $-2, 4, -8, 16$
(*c*) $-2, -4, -6, -8$ (*d*) $-2, -4, -8, -16$

Sol. (*c*): **Main concept used:** $a_n = a + (n-1)d$

$a_1 = -2, d = -2$

$\therefore \quad a_2 = a_1 + d$

$\Rightarrow \quad a_2 = -2 - 2 = -4$

and $\quad a_3 = a_2 + d = -4 + (-2) = -6$

and $\quad a_4 = a_3 + d = -6 + (-2) = -8$

So, the first four terms are $-2, -4, -6, -8$.

Q6. The 21st term of an A.P. whose first two terms are -3 and 4 is

(*a*) 17 (*b*) 137 (*c*) 143 (*d*) -143

Sol. (*b*): **Main concept used:** $a_n = a + (n-1)d$

Here, $\quad a = a_1 = -3, a_2 = 4$

$\therefore \quad d = a_2 - a_1 = 4 - (-3) = 4 + 3 = 7$

Hence, $d = 7$

Now, $\quad a_n = a + (n-1)d$

$\Rightarrow \quad a_{21} = -3 + (21-1) \times 7 = -3 + 20 \times 7 = -3 + 140$

$\Rightarrow \quad a_{21} = 137$

Hence, (*b*) is the correct answer.

Q7. If the 2nd term of an A.P. is 13 and 5th term is 25, what is its 7th term?

(*a*) 30 (*b*) 33 (*c*) 37 (*d*) 38

Sol. (*b*): Here, $a_2 = 13$ and $a_5 = 25$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad a_2 = a + (2-1)d$

$\Rightarrow \quad 13 = a + d$

$\Rightarrow \quad a + d = 13 \quad ...(i)$

and $\quad a_5 = a + (5-1)d$

$\Rightarrow \quad 25 = a + 4d$

$\Rightarrow \quad a + 4d = 25 \quad ...(ii)$

Now, subtracting (*i*) from (*ii*), we get

$$\begin{array}{rcl} a + 4d &=& 25 \quad ...(ii) \\ a + d &=& 13 \quad ...(i) \\ - \quad - & & - \\ \hline 3d &=& 12 \end{array}$$

$\Rightarrow \quad d = \frac{12}{3}$

$\Rightarrow \quad d = 4$

Now, $\quad a + d = 13$ [From (*i*)]

$\Rightarrow \quad a + 4 = 13$

$\Rightarrow \quad a = 13 - 4 = 9$

Now, $\quad a_7 = a + 6d = 9 + 6(4) = 9 + 24$

$\Rightarrow \quad a_7 = 33$

Hence, (*b*) is the correct answer.

Q8. Which term of an A.P.: 21, 42, 63, 84, ... is 210?

(*a*) 9th (*b*) 10th (*c*) 11th (*d*) 12th

Sol. (*b*): Given A.P. is 21, 42, 63, 84, ...

So, $a = 21$, $d = 42 - 21 = 21$, $a_n = 210$

We know that $\quad a_n = a + (n-1)d$

$\Rightarrow \quad 210 = 21 + (n-1)21$

$\Rightarrow \quad 210 - 21 = (n-1)21$

$\Rightarrow \quad \frac{189}{21} = (n-1)$

$\Rightarrow \quad n - 1 = 9$

$\Rightarrow \quad n = 10$

Hence, (*b*) is the correct answer.

Q9. If the common difference of an A.P. is 5, then what is $a_{18} - a_{13}$?

(*a*) 5 (*b*) 20 (*c*) 25 (*d*) 30

Sol. (*c*): Here, $d = 5$.

$\because \quad a_n = a + (n-1)d.$

$\therefore \quad a_{18} - a_{13} = [a + (18-1)d] - [a + (13-1)d]$

$= a + 17d - a - 12d$

$= 5d = 5 \times 5 = 25$

Hence, (*c*) is the correct answer.

Q10. What is the common difference of an A.P. in which $a_{18} - a_{14} = 32$?

(*a*) 8 (*b*) -8 (*c*) -4 (*d*) 4

Sol. (*a*): Here, $a_{18} - a_{14} = 32$

$\Rightarrow \quad [a + (18-1)d] - [a + (14-1)d] = 32$ $[\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + 17d - a - 13d = 32$

$\Rightarrow \quad 4d = 32$

$\Rightarrow \quad d = \frac{32}{4} = 8$

Hence, (*a*) is the correct answer.

Q11. Two A.P.s have the same common difference. The Ist term of one of these is -1, and that of other is -8. The difference between their 4th terms is

(*a*) -1 (*b*) -8 (*c*) 7 (*d*) -9

Sol. (*c*): Given: $a_1 = -1$ and $a'_1 = -8$
Let d be the same common difference of two A.Ps.
So, $d_1 = d$, $d'_1 = d$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad a_4 - a'_4 = [a_1 + (4-1)\,d_1] - [\,a'_1 + (4-1)\,d'_1]$

$\Rightarrow \quad a_4 - a'_4 = (-1 + 3d) - [-8 + 3d]$

$= -1 + 3d + 8 - 3d = 7$

Hence, the required answer is (*c*).

Q12. If 7 times the 7th term of an A.P. is equal to 11 times its 11th term, then its 18th term will be

(*a*) 7 (*b*) 11 (*c*) 18 (*d*) 0

Sol. (*d*): $a_{18} = a + (18-1)d = a + 17d$

Now, $7a_7 = 11a_{11}$ [Given]

$\Rightarrow 7[a + (7-1)d] = 11[a + (11-1)d]$

$\Rightarrow \quad 7[a + 6d] = 11[a + 10d]$

$\Rightarrow \quad 7a + 42d = 11a + 110d$

$\Rightarrow \quad 0 = 11a - 7a + 110d - 42d$

$\Rightarrow \quad 0 = 4a + 68d$

$\Rightarrow \quad 0 = a + 17d$

$\Rightarrow \quad a_{18} = 0$

Hence, (*d*) is the correct answer.

Q13. The 4th term from the end of an A.P. $-11, -8, -5, \ldots, 49$ is

(*a*) 37 (*b*) 40 (*c*) 43 (*d*) 58

Sol. (*b*): Reversing the A.P., we get

$49, \ldots, -5, -8, -11$

$\therefore \quad d = -8 - (-5) = -8 + 5 = -3$

$a = 49$ and $n = 4$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad a_4 = 49 + (4-1)(-3)$

$= 49 + 3(-3) = 49 - 9$

$\Rightarrow \quad a_4 = 40$

Hence, the required value of a_4 is 40 and answer is (*b*).

Q14. The famous mathematician associated with finding the sum of the first 100 natural numbers is

(*a*) Pythagoras (*b*) Newton (*c*) Gauss (*d*) Euclid

Sol. (*c*): Gauss is the famous mathematician associated with finding the sum of first 100 natural numbers, i.e., $1 + 2 + 3 + 4 + 5 + \ldots + 100$

Here, $a = 1, \quad d = 1, \quad n = 100$

As $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{100} = \frac{100}{2}[2(1) + (100-1)1]$

$= \frac{100}{2}[2+99] = \frac{100 \times 101}{2} = 50 \times 101$

$= 5050$

Q15. If the first term of an A.P. is -5 and the common difference is 2, then the sum of first 6 terms is

(*a*) 0 (*b*) 5 (*c*) 6 (*d*) 15

Sol. (*a*) Here, $a = -5, \quad d = 2, \quad n = 6$

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_6 = \frac{6}{2}[2(-5) + (6-1)2]$

$= 3[-10 + 5 \times 2]$

$= 3[-10 + 10]$

$= 3[0]$

$\Rightarrow \quad S_6 = 0$

Hence, (*a*) is the correct answer.

Q16. The sum of first 16 terms of an A.P. 10, 6, 2, ... is

(*a*) -320 (*b*) 320 (*c*) -352 (*d*) -400

Sol. (*a*): Here, $a = 10, \quad n = 16, \quad d = 6 - 10 = -4$

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{16} = \frac{16}{2}[2 \times 10 + (16-1)(-4)]$

$= 8[20 + 15(-4)] = 8[20 - 60] = -8 \times 40$

$\Rightarrow \quad S_{16} = -320$

So, the required answer is (*a*).

Q17. In an A.P., if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is

(*a*) 19 (*b*) 21 (*c*) 38 (*d*) 42

Sol. (*c*): We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_n = \frac{n}{2}[a + a + (n-1)d]$

$\Rightarrow \quad 399 = \frac{n}{2}[a + a_n]$ $\quad [a_n = \text{last term}]$

$\Rightarrow \quad 399 = \frac{n}{2}[1 + 20] \quad \Rightarrow \quad n = \frac{399 \times 2}{21} = 38$

Hence, (*c*) is the correct answer.

Q18. The sum of first five multiples of 3 is

(*a*) 45 (*b*) 55 (*c*) 65 (*d*) 75

Sol. (*a*): Ist five multiples of 3 are 3, 6, 9, 12, 15, ...

Here, $a = 3$, $n = 5$, $d = 6 - 3 = 3$

$$\therefore \quad S_5 = \frac{5}{2}[2 \times 3 + (5-1)3] \qquad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$\Rightarrow \quad S_5 = \frac{5}{2}[6 + 12] = \frac{5}{2} \times 18 = 45$$

Hence, (*a*) is the correct answer.

EXERCISE 5.2

Q1. Which of the following form an A.P.? Justify your answer.

(*i*) $-1, -1, -1, -1, \ldots$ (*ii*) $0, 2, 0, 2, \ldots$

(*iii*) $1, 1, 2, 2, 3, 3, \ldots$ (*iv*) $11, 22, 33, \ldots$

(*v*) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ (*vi*) $2, 2^2, 2^3, 2^4, \ldots$

(*vii*) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

Sol. (*i*) $-1, -1, -1, -1, \ldots$

A series of numbers will be in A.P. if $d_1 = d_2 = d_3 \ldots$

So, $d_1 = -1 - (-1) = 0$

$d_2 = -1 - (-1) = 0$

$d_3 = -1 - (-1) = 0$

$\therefore \quad d_1 = d_2 = d_3 \ldots$

So, the given series form an A.P.

(*ii*) $0, 2, 0, 2, \ldots$

Given form of numbers will be in A.P. if $d_1 = d_2 = d_3 \ldots$

So, $d_1 = 2 - 0 = 2$

$d_2 = 0 - 2 = -2$

$\therefore \quad d_1 \neq d_2$

So, the given form of numbers is not an A.P.

(*iii*) $1, 1, 2, 2, 3, 3, \ldots$

Given form of numbers will form an A.P. if $d_1 = d_2 = d_3 \ldots$

So, $d_1 = 1 - 1 = 0$

$d_2 = 2 - 1 = 1$

$\therefore \quad d_1 \neq d_2$

Hence, the given form of numbers will not form an A.P.

(*iv*) $11, 22, 33, \ldots$

Given form of numbers will form an A.P. if $d_1 = d_2 = d_3 = \ldots$

So, $d_1 = 22 - 11 = 11$

$d_2 = 33 - 22 = 11$

$\therefore \quad d_1 = d_2 = 11$

Hence, the given form of numbers will form an A.P.

(*v*) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

Given form of numbers will form an A.P. if $d_1 = d_2 = d_3 \ldots$

So, $$d_1 = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \frac{-1}{6}$$

$$d_2 = \frac{1}{4} - \frac{1}{3} = \frac{3-4}{12} = \frac{-1}{12}$$

$\therefore$ $d_1 \neq d_2$

Hence, the given form of numbers will not form an A.P.

(*vi*) $2, 2^2, 2^3, 2^4, \ldots$

Given form of numbers will form an A.P. if $d_1 = d_2 = d_3 \ldots$

So, $$d_1 = 2^2 - 2 = 4 - 2 = 2$$

$$d_2 = 2^3 - 2^2 = 8 - 4 = 4$$

$$d_3 = 2^4 - 2^3 = 16 - 8 = 8$$

$\therefore$ $d_1 \neq d_2 \neq d_3$

Hence, the given form of numbers will not form an A.P.

(*vii*) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

Given form of numbers will form an A.P. if $d_1 = d_2 = d_3 \ldots$

So, $$d_1 = \sqrt{12} - \sqrt{3} = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$$

$$d_2 = \sqrt{27} - \sqrt{12} = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$$

$$d_3 = \sqrt{48} - \sqrt{27} = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$$

$\therefore$ $d_1 = d_2 = d_3 = \sqrt{3}$

Hence, the given form of numbers will form an A.P.

Q2. Justify whether it is true to say that $-1, \frac{-3}{2}, -2, \frac{5}{2} \ldots$ forms an A.P. as $a_2 - a_1 = a_3 - a_2$.

Sol. Main concept used: A form of numbers will form an A.P. if $d_1 = d_2 = d_3 = \ldots d_n = d$.

Given form of numbers will form an A.P. if $d_1 = d_2 = d_3 = d$ otherwise not.

So, $$d_1 = a_2 - a_1 = \frac{-3}{2} - (-1) = \frac{-3}{2} + 1 = \frac{-3+2}{2} = \frac{-1}{2}$$

$$d_2 = a_3 - a_2 = -2 - \left(\frac{-3}{2}\right) = -2 + \frac{3}{2} = \frac{-4+3}{2} = \frac{-1}{2}$$

$$d_3 = a_4 - a_3 = \frac{5}{2} - (-2) = \frac{5}{2} + 2 = \frac{5+4}{2} = \frac{9}{2}$$

$\therefore$ $d_1 = d_2 \neq d_3$

Although $a_2 - a_1 = a_3 - a_2 = \frac{-1}{2}$ but $a_4 - a_3 \neq \frac{-1}{2}$

So, the given form of numbers will not form an A.P. Hence, the given statement is false.

Q3. For the A.P. –3, –7, –11, ..., can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20}? Give reasons for your answer.

Sol. Here, $a = -3$,

$$d_1 = -7 - (-3) = -7 + 3 = -4$$
$$d_2 = -11 - (-7) = -11 + 7 = -4$$
$$\therefore \quad d = d_1 = d_2 = -4$$

Now, $a_{30} = a + (30 - 1)d = a + 29d$

and $a_{20} = a + (20 - 1)d = a + 19d$

So, $a_{30} - a_{20} = (a + 29d) - (a + 19d) = a + 29d - a - 19d$

$$\Rightarrow \quad a_{30} - a_{20} = 10d$$
$$= 10 \times (-4) = -40$$

So, we can find $a_{30} - a_{20}$ without finding a_{30} and a_{20}.

Hence, $a_{30} - a_{20} = -40$.

Q4. Two A.P.s have the same common difference. The first term of one A.P. is 2, and that of the other is 7. The difference between their 10th terms is same as the difference between their 21th terms, which is the same as the difference between any two corresponding terms. Why?

Sol. Given: $a_1 = 2$ and $a'_1 = 7$

Let d be the same common difference of two A.P.s.

So, $d_1 = d$ and $d'_1 = d$

Now, $a_{10} - a'_{10} = a_1 + (10 - 1)d_1 - [a'_1 + (10 - 1)\, d'_1]$

$$= 2 + 9d - [7 + 9d] = 2 + 9d - 7 - 9d$$
$$\Rightarrow \quad a_{10} - a'_{10} = -5$$

Also, $a_{21} - a'_{21} = a_1 + (21 - 1)\, d_1 - [a'_1 + (21 - 1)\, d'_1]$

$$= 2 + 20d - [7 + 20d] = 2 + 20d - 7 - 20d$$
$$\Rightarrow \quad a_{21} - a'_{21} = -5$$
$$\Rightarrow \quad a_{21} - a'_{21} = a_{10} - a'_{10} = -5$$

Now, $a_n - a'_n = a_1 + (n - 1)d_1 - [\, a'_1 + (n - 1)\, d'_1]$

$$= 2 + (n - 1)d - [7 + (n - 1)d]$$
$$= 2 + nd - d - [7 + nd - d]$$
$$= 2 + nd - d - 7 - nd + d$$
$$= 2 - 7$$
$$\Rightarrow \quad a_n - a'_n = -5$$

Hence, the difference between any two corresponding terms of such A.P.'s is same (–5) as the difference between their 10th terms and 21st terms.

Q5. Is 0 a term of the A.P. 31, 28, 25, ...? Justify your answer.

Sol. Main concept used: $a_n = a + (n-1)d$

If we substitute the values of a_n, a, and d in the above equation and if n comes out to be a natural number then, the given a_n will be the term of the given series.

Here, $a_n = 0, \quad a = 31$

$d_1 = 28 - 31 = -3, \ d_2 = 25 - 28 = -3$

So, $d_1 = d_2 = -3$

$\because \quad a_n = a + (n-1)d \Rightarrow 0 = 31 + (n-1) \times (-3)$

$\Rightarrow \quad -31 = -(n-1) \times 3 \Rightarrow (n-1) = \dfrac{31}{3}$

$\Rightarrow \quad n = \dfrac{31}{3} + 1 \Rightarrow n = \dfrac{31+3}{3} = \dfrac{34}{3} = 11\dfrac{1}{3} \neq$ natural number.

Since n is in fraction and is not natural number so 0 (a_n) is not any term of the given A.P..

Q6. The taxi fare after each km, when the fare is ₹ 15 for the first km and ₹ 8 for each additional km, does not form an A.P., as the total fare (in ₹) after each km is 15, 8, 8, 8, ... Is the statement true? Give reasons.

Sol. 15, 8, 8, 8, ... are not the total fare for 1, 2, 3, 4, km respectively.

Total fare for Ist km = ₹ 15.

Total fare for 2 km = ₹ 15 + ₹ 8 = ₹ 23

Total fare for 3 km = ₹ 23 + ₹ 8 = ₹ 31

Total fare for 4 km = ₹ 31 + ₹ 8 = ₹ 39

$\therefore$ Total fare for 1 km, 2 km, 3 km, 4 km, ... are 15, 23, 31, 39, ... respectively.

Now, $d_1 = 23 - 15 = 8$

$d_2 = 31 - 23 = 8$

$d_3 = 39 - 31 = 8$

Hence, the total fare form an A.P. as 15, 23, 31, 39, ...

But, fare for each km does not form A.P. as 15, 8, 8, 8 ...

Q7. In which of the following situations do the lists of numbers involved form an A.P.? Give reasons for your answers.

(*i*) The fee charged from a student every month by a school for the whole session, when the monthly fee is ₹ 400.

(*ii*) The fee charged every month by a school from classes I to XII, when the monthly fee for class I is ₹ 250 and it increases by ₹ 50 for the next higher class.

(*iii*) The amount of money in the account of Varun at the end of every year when ₹ 1000 is deposited at simple interest of 10% per annum.

(*iv*) The number of bacteria in a certain food item after each second, when they double in every second.

Sol. (*i*) The fee charged from a student every month by a school is ₹ 400. So, the fee charged from a student the whole session is 400, 400, 400, 400, ... As $d_1 = d_2 = d_3 = - d_{12} = 0$ so, the series of numbers is an A.P.

(*ii*) Fee for Ist class = ₹ 250

Fee for IInd class = ₹ (250 + 50) = ₹ 300

Fee for IIIrd class = ₹ (300 + 50) = ₹ 350

Fee for IV class = ₹ (350 + 50) = ₹ 400

$\therefore$ 250, 300, 350, 400, ... is a series consisting of 12 terms.

So, $d_1 = 300 - 250 =$ ₹ 50, $d_2 = 350 - 300 =$ ₹ 50, $d_3 = 400 - 350 =$ ₹ 50

$\therefore$ $d_1 = d_2 = d_3 =$ ₹ 50

So, the list of numbers 250, 300, 350, 400, ... is in A.P.

(*iii*) $SI = \frac{PRT}{100} = \frac{1000 \times 10 \times 1}{100} =$ ₹ 100

So, ₹ 100 is credited at the end of each year in the account of Varun.

Money in the beginning of Ist year (deposited) = ₹ 1000

Money at the end of Ist year when interest credited

= 1000 + 100 = ₹ 1100

Money at the end of IInd year = 1100 + 100 = ₹ 1200

Money at the end of IIIrd your = 1200 + 100 = ₹ 1300

Money at the end of IV year = 1300 + 100 = ₹ 1400

$\therefore$ Amount of money at the end of each year starting initially from Ist year is given by 1000, 1100, 1200, 1300, 1400 ...

$\therefore$ $d_1 = d_2 = d_3 = d_4 = 100$

So, the list of numbers is an A.P.

(*iv*) Let the number of bacteria present initially = x

Then, the number of bacteria present after 1 sec = $2(x) = 2x$

Number of bacteria present after 2 sec = $2(2x) = 4x$

Number of bacteria present after 3 sec = $2(4x) = 8x$

Number of bacteria present after 4 second = $2(8x) = 16x$

So, the number of bacteria from starting to end of each second are given by $x, 2x, 4x, 8x, 16x, \ldots$

Now, $d_1 = 2x - x = x$, $d_2 = 4x - 2x = 2x$

As $d_1 \neq d_2$, so the list of numbers does not form an A.P.

Q8. Justify whether it is true to say that the following are the nth terms of an A.P.

(*i*) $2n - 3$ (*ii*) $3n^2 + 5$ (*iii*) $1 + n + n^2$

Sol. (*i*) $a_n = 2n - 3$

$\therefore$ $a_1 = 2(1) - 3 = 2 - 3 = -1$, $a_2 = 2(2) - 3 = 4 - 3 = 1$

$a_3 = 2(3) - 3 = 6 - 3 = 3$, $a_4 = 2(4) - 3 = 8 - 3 = 5$

So, $d_1 = 1-(-1) = 1+1 = 2$, $d_2 = 3-1 = 2$, $d_3 = 5-3 = 2$

As $d_1 = d_2 = d_3 = 2$, hence, $a_n = 2n-3$ form nth term of an A.P.

(*ii*) $a_n = 3n^2 + 5$

$\therefore$ $a_1 = 3(1)^2 + 5 = 3\times 1 + 5 = 3 + 5 = 8$

$a_2 = 3(2)^2 + 5 = 3\times 4 + 5 = 12 + 5 = 17$

$a_3 = 3(3)^2 + 5 = 3\times 9 + 5 = 27 + 5 = 32$

$a_4 = 3(4)^2 + 5 = 3\times 16 + 5 = 48 + 5 = 53$

$a_5 = 3(5)^2 + 5 = 3\times 25 + 5 = 75 + 5 = 80$

$\therefore$ $d_1 = a_2 - a_1 = 17 - 8 = 9$, $d_2 = a_3 - a_2 = 32 - 17 = 15$

$d_3 = a_4 - a_3 = 53 - 32 = 21$, $d_4 = a_5 - a_4 = 80 - 53 = 27$

Since $d_1 \neq d_2$, so the list of numbers 8, 17, 32, 53, ... is not in A.P.

(*iii*) $a_n = 1 + n + n^2$

$\therefore$ $a_1 = 1 + (1) + (1)^2 = 1 + 1 + 1 = 3$

$a_2 = 1 + (2) + (2)^2 = 1 + 2 + 4 = 7$

$a_3 = 1 + (3) + (3)^2 = 1 + 3 + 9 = 13$

$a_4 = 1 + (4) + (4)^2 = 1 + 4 + 16 = 21$

$a_5 = 1 + (5) + (5)^2 = 1 + 5 + 25 = 31$

So, $d_1 = a_2 - a_1 = 7 - 3 = 4$

$d_2 = a_3 - a_2 = 13 - 7 = 6$

$d_3 = a_4 - a_3 = 21 - 13 = 8$

$d_4 = a_5 - a_4 = 31 - 21 = 10$

As $d_1 \neq d_2$, so the list of numbers 3, 7, 13, 21, 31, ... is not in A.P.

EXERCISE 5.3

Q1. Match the A.P.s given in column A with suitable common differences given in column B.

Column A	Column B
(A_1) 2, −2, −6, −10, ...	(B_1) 2/3
(A_2) $a = -18$, $n = 10$, $a_n = 0$	(B_2) −5
(A_3) $a = 0$, $a_{10} = 6$	(B_3) 4
(A_4) $a_2 = 13$, $a_4 = 3$	(B_4) −4
	(B_5) 2
	(B_6) 1/2
	(B_7) 5

Sol. (*i*) Here, $a_1 = 2$

$\therefore$ $d_1 = -2 - 2 = -4$

and $d_2 = -6 - (-2) = -6 + 2 = -4$

Hence, A_1 matches to B_4.

(*ii*) Given: $a_n = 0, a = -18, n = 10$

Now, $a_n = a + (n-1)d$

$\Rightarrow \quad 0 = -18 + (10-1)d$

$\Rightarrow \quad -9d = -18$

$\Rightarrow \quad d = 2$

Hence, A_2 matches to B_5.

(*iii*) Given: $a = 0, a_{10} = 6$

Now, $a_n = a + (n-1)d$

$\Rightarrow \quad 6 = 0 + (10-1)d$

$\Rightarrow \quad 9d = 6$

$\Rightarrow \quad d = \frac{6}{9} \Rightarrow d = \frac{2}{3}$

Hence, A_3 matches to B_1.

(*iv*) $a_2 = 13$ [Given]

$\therefore \quad a + (2-1)d = 13$ $[\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + d = 13$

$\Rightarrow \quad a = 13 - d$ ...(*i*)

Also, $a_4 = 3$ [Given]

$\therefore \quad a + (4-1)d = 3$ $[\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + 3d = 3$

$\Rightarrow \quad 13 - d + 3d = 3$ [Using (*i*)]

$\Rightarrow \quad 2d = 3 - 13$

$\Rightarrow \quad 2d = -10$

$\Rightarrow \quad d = -5$

Hence, A_4 matches to B_2.

Q2. Verify that each of the following is an A.P. and then write its next three terms.

(*i*) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \cdots$ (*ii*) $5, \frac{14}{3}, \frac{13}{3}, 4, \cdots$

(*iii*) $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$ (*iv*) $a+b, (a+1)+b, (a+1)+(b+1), \ldots$

(*v*) $a, 2a+1, 3a+2, 4a+3, \ldots$

Sol. Main concept used: (*a*) List of numbers will form an A.P. if $d_1 = d_2 = d_3 \ldots, = d$ (*b*) $a_{n+1} = a_n + d$

(*i*) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \cdots$

$$d_1 = \frac{1}{4} - 0 = \frac{1}{4}, d_2 = \frac{1}{2} - \frac{1}{4} = \frac{2-1}{4} = \frac{1}{4}, d_3 = \frac{3}{4} - \frac{1}{2} = \frac{3-2}{4} = \frac{1}{4}$$

$\therefore \quad d_1 = d_2 = d_3 = \frac{1}{4}$

So, the given list of numbers form an A.P.

Now, $a_5 = a_4 + d = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

$a_6 = a_5 + d = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$

$a_7 = a_6 + d = \frac{5}{4} + \frac{1}{4} = \frac{6}{4} = \frac{3}{2}$

So, the next three terms are $1, \frac{5}{4}$ and $\frac{3}{2}$.

(*ii*) $5, \frac{14}{3}, \frac{13}{3}, 4, \cdots$

$d_1 = \frac{14}{3} - 5 = \frac{14-15}{3} = \frac{-1}{3}$

$d_2 = \frac{13}{3} - \frac{14}{3} = \frac{13-14}{3} = \frac{-1}{3}$

$d_3 = 4 - \frac{13}{3} = \frac{12-13}{3} = -\frac{1}{3}$

Since, $d_1 = d_2 = d_3 = -\frac{1}{3}$ so, the given list of numbers is in A.P.

For next 3 terms, we have

$a_5 = a_4 + d = 4 + \left(\frac{-1}{3}\right) = \frac{12-1}{3} = \frac{11}{3}$

$a_6 = a_5 + d = \frac{11}{3} + \left(\frac{-1}{3}\right) = \frac{11-1}{3} = \frac{10}{3}$

$a_7 = a_6 + d = \frac{10}{3} + \left(\frac{-1}{3}\right) = \frac{10-1}{3} = \frac{9}{3}$

Hence, the next three terms are $\frac{11}{3}, \frac{10}{3}$ and $\frac{9}{3}$.

(*iii*) $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$

$d_1 = a_2 - a_1 = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

$d_2 = a_3 - a_2 = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$

$\therefore\ d_1 = d_2 = \sqrt{3}$ verifies that the given list of numbers form an A.P.

For next three terms, we have

$a_4 = a_3 + d = 3\sqrt{3} + \sqrt{3} = 4\sqrt{3}$

$a_5 = a_4 + d = 4\sqrt{3} + \sqrt{3} = 5\sqrt{3}$

$a_6 = a_5 + d = 5\sqrt{3} + \sqrt{3} = 6\sqrt{3}$

Hence, the next three terms are $4\sqrt{3}, 5\sqrt{3}$ and $6\sqrt{3}$.

(*iv*) $a+b, (a+1)+b, (a+1)+(b+1), \ldots$

$d_1 = a+1+b-(a+b) = a+1+b-a-b = 1$

$d_2 = (a+1)+(b+1)-[(a+1)+b] = a+1+b+1-a-1-b = 1$

$\therefore d_1 = d_2 = 1$ verifies that the given list of numbers form an A.P.

For next three terms, we have

$a_4 = a_3 + d = (a+1)+(b+1)+1 = (a+2)+(b+1)$

$a_5 = a_4 + d = (a+2)+(b+1)+1 = (a+2)+(b+2)$

$a_6 = a_5 + d = (a+2)+(b+2)+1 = (a+3)+(b+2)$

(*v*) $a, 2a+1, 3a+2, 4a+3, \ldots$

$d_1 = a_2 - a_1 = 2a+1-a = a+1$

$d_2 = 3a+2-(2a+1) = a+2-1 = a+1$

$d_3 = 4a+3-(3a+2) = 4a+3-3a-2 = a+1$

$\Rightarrow \quad d_1 = d_2 = d_3 = a+1$ verifies that the given list of numbers form an A.P.

For next three terms, we have

$a_5 = a_4 + d = 4a+3+a+1 = 5a+4$

$a_6 = a \;\; + d = 5a+4+a+1 = 6a+5$

$a_7 = a_6 + d = 6a+5+a+1 = 7a+6$

Hence, the next three terms are $(5a+4)$, $(6a+5)$ and $(7a+6)$.

Q3. Write the first three terms of the A.P.s when a and d are as given below.

(*i*) $a = \frac{1}{2}, d = \frac{-1}{6}$ (*ii*) $a = -5, d = -3$ (*iii*) $a = \sqrt{2}, d = \frac{1}{\sqrt{2}}$

Sol. Main concept used: $a_n = a + (n-1)d$

(*i*) Here, $a = \frac{1}{2}, d = \frac{-1}{6}$

We know that

$$a_n = a + (n-1)\,d$$

$$\Rightarrow a_n = \frac{1}{2} + (n-1)\left(\frac{-1}{6}\right)$$

$$\Rightarrow a_n = \frac{1}{2} - \frac{n}{6} + \frac{1}{6} = \frac{1}{2} + \frac{1}{6} - \frac{n}{6} = \frac{3+1-n}{6} \quad \Rightarrow \quad a_n = \frac{4-n}{6}$$

$$\therefore \; a_1 = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}, \quad a_2 = \frac{4-2}{6} = \frac{2}{6} = \frac{1}{3}, \quad a_3 = \frac{4-3}{6} = \frac{1}{6}$$

Hence, the required first three terms are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{6}$.

(*ii*) Here, $a = -5, d = -3$

We know that

$$a_n = a + (n-1)\,d$$

$\Rightarrow \quad a_n = -5 + (n-1)(-3) = -5 - 3n + 3 = -2 - 3n$

$\Rightarrow \quad a_n = -(2 + 3n)$

$\therefore \quad a_1 = -[2 + 3(1)] = -(2+3) = -5$

$a_2 = -[2 + 3 \times 2] = -[2+6] = -8$

$a_3 = -[2 + 3 \times 3] = -[2+9] = -11$

Hence, the first three terms are $-5, -8$ and -11.

(*iii*) Here, $a = \sqrt{2}, d = \dfrac{1}{\sqrt{2}}$

We know that $a_n = a + (n-1)d$

$$\Rightarrow \quad a_n = \sqrt{2} + (n-1)\frac{1}{\sqrt{2}} = \sqrt{2} + \frac{n}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \sqrt{2} - \frac{1}{\sqrt{2}} + \frac{n}{\sqrt{2}}$$

$$\Rightarrow \quad a_n = \frac{2 - 1 + n}{\sqrt{2}} \quad \Rightarrow \quad a_n = \frac{1+n}{\sqrt{2}}$$

$$\therefore \quad a_1 = \frac{1+1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2},$$

$$a_2 = \frac{1+2}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

and $$a_3 = \frac{1+3}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

Hence, the first three terms are $\sqrt{2}, \dfrac{3\sqrt{2}}{2}$ and $2\sqrt{2}$.

Q4. Find a, b, c such that the following numbers are in A.P.: $a, 7, b, 23, c$.

Sol. We have

$d_1 = a_2 - a_1 = 7 - a$

$d_2 = a_3 - a_2 = b - 7$

$d_3 = a_4 - a_3 = 23 - b$

$d_4 = a_5 - a_4 = c - 23$

As list of numbers is in A.P.,

so $\quad d_1 = d_2 = d_3 = d_4$

Now, $\quad d_2 = d_3$

$\Rightarrow \quad b - 7 = 23 - b$

$\Rightarrow \quad b + b = 30 \quad \Rightarrow \quad 2b = 30 \quad \Rightarrow \quad b = 15$

Now, $\quad d_2 = d_1$

$\Rightarrow \quad b - 7 = 7 - a$

$\Rightarrow \quad 15 - 7 = 7 - a \quad \Rightarrow \quad 8 = 7 - a$

$a = 7 - 8 = -1$

Now, $\quad d_4 = d_2$

$\Rightarrow \quad c - 23 = b - 7$

$\Rightarrow \quad c = 23 + 15 - 7 = 38 - 7$

$\Rightarrow \quad c = 31$

Hence, $a = -1$, $b = 15$, and $c = 31$.

Q5. Determine the A.P. whose 5th term is 19 and the difference of 8th term from 13th term is 20.

Sol. Main concept used: (*i*) $a_n = a + (n-1)d$ (*ii*) Solution of linear eqn.

Given: $a_5 = 19$, $a_{13} - a_8 = 20$

Let us consider an A.P. whose Ist term and common difference are a and d respectively.

$a_5 = 19$ [Given]

$\Rightarrow \quad a + (5-1)d = 19$

$\Rightarrow \quad a + 4d = 19$ ...(*i*)

Also, $a_{13} - a_8 = 20$ [Given]

$\Rightarrow \quad a + (13-1)d - [a + (8-1)d] = 20$

$\Rightarrow \quad a + 12d - [a + 7d] = 20$

$\Rightarrow \quad a + 12d - a - 7d = 20$

$\Rightarrow \quad 5d = 20$

$\Rightarrow \quad d = \frac{20}{5} \Rightarrow d = 4$

Now, $a + 4d = 19$ [From (*i*)]

$\Rightarrow \quad a + 4 \times 4 = 19$

$\Rightarrow \quad a = 19 - 16 = 3$

A.P. is given by $a, a+d, a+2d, a+3d, \ldots$

Hence, the required A.P. is 3, 7, 11, 15, ...

Q6. The 26th, 11th and the last term of an A.P. are 0, 3, and $-\frac{1}{5}$ respectively. Find the common difference and the number of terms.

Sol. Consider an A.P. whose first term, common difference and last term are a, d and a_n

Given: $a_{26} = 0$, $a_{11} = 3$ and $a_n = -\frac{1}{5}$

$a_{26} = 0$ [Given]

$\Rightarrow \quad a + (26-1)d = 0$

$\Rightarrow \quad a + 25d = 0$ ...(*i*)

$a_{11} = 3$ [Given]

$\Rightarrow \quad a + (11-1)d = 3$

$\Rightarrow \quad a + 10d = 3$ ...(*ii*)

$a_n = -\frac{1}{5}$

$\Rightarrow \quad a + (n-1)d = -\frac{1}{5}$ ...(*iii*)

On subtracting eqn. (*ii*) from eqn. (*i*), we get

$$15d = -3$$

$$\Rightarrow \quad d = \frac{-3}{15} = \frac{-1}{5}$$

From (*ii*), $a + 10d = 3$

$$\Rightarrow \quad a + 10\left(-\frac{1}{5}\right) = 3$$

$$\Rightarrow \quad a - 2 = 3 \quad \Rightarrow \quad a = 3 + 2$$

$$\Rightarrow \quad a = 5$$

$\therefore$ From (*iii*), $a + (n-1)d = -\frac{1}{5}$

$$\Rightarrow \quad 5 + (n-1)\left(\frac{-1}{5}\right) = \frac{-1}{5}$$

$$\Rightarrow \quad 25 - (n-1) = -1$$

$$\Rightarrow \quad 25 + 1 = (n-1)$$

$$\Rightarrow \quad n - 1 = 26$$

$$\Rightarrow \quad n = 27$$

Hence, the common difference and number of terms in A.P. are $-\frac{1}{5}$ and 27 respectively.

Q7. The sum of the 5th and the 7th terms of an A.P. is 52, and the 10th term is 46. Find the A.P.

Sol. Consider an A.P. whose Ist term and common difference are *a* and *d* respectively. According to the question,

$$a_5 + a_7 = 52$$

$$\Rightarrow \quad a + (5-1)d + a + (7-1)d = 52 \qquad [\because a_n = a + (n-1)d]$$

$$\Rightarrow \quad 2a + 4d + 6d = 52$$

$$\Rightarrow \quad 2a + 10d = 52$$

$$\Rightarrow \quad a + 5d = 26 \qquad ...(i)$$

Also, $a_{10} = 46$ [Given]

$$\Rightarrow \quad a + (10-1)d = 46$$

$$\Rightarrow \quad a + 9d = 46 \qquad ...(ii)$$

$$a + 5d = 26 \qquad \text{[From (i)]}$$

$$a + 9d = 46 \qquad \text{[From (ii)]}$$

$$- \quad - \quad -$$

$$-4d = -20 \qquad \text{[Subtract (ii) from (i)]}$$

$$\Rightarrow \quad d = \frac{20}{4}$$

$$\Rightarrow \quad d = 5$$

Now, $a + 5d = 26$ [From (*i*)]

$\Rightarrow \quad a + 5 \times 5 = 26$

$\Rightarrow \quad a = 26 - 25$

$\Rightarrow \quad a = 1$

A.P. is given by $a, a + d, a + 2d, \ldots$

Hence, the required A.P. is given by 1, 6, 11, 16, ...

Q8. Find the 20th term of an A.P. whose 7th term is 24 less than the 11th term, first term being 12.

Sol. Consider an A.P. whose first term and common difference are 'a' and 'd' respectively.

According to the question, we have

$$a_7 = a_{11} - 24$$

$\Rightarrow \quad a + (7-1)d + 24 = a + (11-1)d \quad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + 6d + 24 - a = 10d$

$\Rightarrow \quad 6d - 10d = -24$

$\Rightarrow \quad -4d = -24$

$\Rightarrow \quad d = \frac{24}{4} = 6$

Now, $\quad a_n = a + (n-1)d$

$\therefore \quad a_{20} = 12 + (20-1)6 \quad [\because n = 20, a = 12, d = 6]$

$= 12 + 19 \times 6 = 12 + 114$

$\Rightarrow \quad a_{20} = 126$

Hence, the 20th term of A.P. is 126.

Q9. If the 9th term of an A.P. is zero, prove that its 29th term is twice its 19th term.

Sol. Consider an A.P. whose first term and common difference are 'a' and 'd' respectively.

$a_9 = 0 \quad$ [Given]

$\therefore \quad a + (9-1)d = 0 \quad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + 8d = 0$

$\Rightarrow \quad a = -8d \quad \ldots(i)$

We have to prove that $a_{29} = 2a_{19}$

So, $\quad a_{29} = a + (29-1)d$

$= -8d + 28d \quad$ [Using equation (i)]

$\Rightarrow \quad a_{29} = 20d \quad \ldots(ii)$

Now, $\quad a_{19} = a + (19-1)d$

$\Rightarrow \quad a_{19} = -8d + 18d \quad$ [Using (i)]

$\Rightarrow \quad a_{19} = 10d$

But, $\quad a_{29} = 20d \quad$ [From (ii)]

$$= 2 \times 10d$$
$$= 2 \times a_{19} \qquad [\because a_{19} = 10d]$$
$$= 2a_{19}$$
$$\therefore \quad a_{29} = 2a_{19}$$

Hence, the 29th term is twice the 19th term in the given A.P.

Q10. Find whether 55 is a term of the A.P.: 7, 10, 13, ... or not. If yes, find which term it is.

Sol. Main concept used: 55 will be nth term of the given A.P. if value of n is only natural number.

Here, $a = 7$, $d = 10 - 7 = 3$

Let 55 is the nth term of the given A.P.

$$\therefore \quad a_n = 55 \qquad \text{[Given]}$$
$$\Rightarrow \quad 7 + (n-1)3 = 55 \qquad [\because a_n = a + (n-1)d]$$
$$\Rightarrow \quad (n-1)3 = 55 - 7$$
$$\Rightarrow \quad (n-1) = \frac{48}{3}$$
$$\Rightarrow \quad n - 1 = 16$$
$$\Rightarrow \quad n = 17, \text{ which is a natural number}$$

Hence, 55 is the 17th term of the given A.P.

Q11. Determine k so that $(k^2 + 4k + 8)$, $(2k^2 + 3k + 6)$ and $3k^2 + 4k + 4$ are three consecutive terms of an A.P.

Sol. Main concept used: Given numbers will be in A.P. if $d_1 = d_2 = d$

Here, $d_1 = a_2 - a_1 = 2k^2 + 3k + 6 - (k^2 + 4k + 8)$
$$= 2k^2 + 3k + 6 - k^2 - 4k - 8$$
$$\Rightarrow \quad d_1 = k^2 - k - 2$$

Now, $d_2 = a_3 - a_2 = 3k^2 + 4k + 4 - (2k^2 + 3k + 6)$
$$= 3k^2 + 4k + 4 - 2k^2 - 3k - 6$$
$$= 3k^2 - 2k^2 + 4k - 3k - 6 + 4$$
$$\Rightarrow \quad d_2 = k^2 + k - 2$$

As the given terms are in A.P.

$$\therefore \quad d_2 = d_1$$
$$\Rightarrow \quad k^2 + k - 2 = k^2 - k - 2$$
$$\Rightarrow \quad 2k = -2 + 2$$
$$\Rightarrow \quad 2k = 0 \quad \Rightarrow \quad k = \frac{0}{2} \quad \Rightarrow \quad k = 0$$

Hence, for $k = 0$, the given sequence of numbers will be in A.P.

Q12. Split 207 into three parts such that these are in A.P. and the product of the two smaller parts is 4623.

Sol. Main concept used: Sum of three terms is given so terms can be considered as $(a - d)$, a, $(a + d)$.

Consider an A.P. whose three consecutive terms are $(a-d), a, (a+d)$.
According to the question,

$$(a-d)+a+(a+d) = 207$$
$$\Rightarrow \quad 3a = 207$$
$$\Rightarrow \quad a = \frac{207}{3} \quad \Rightarrow \quad a = 69$$

Also, $$(a-d)(a) = 4623$$
$$\Rightarrow \quad (69-d)69 = 4623 \qquad [\because a = 69]$$
$$\Rightarrow \quad 69-d = \frac{4623}{69}$$
$$\Rightarrow \quad 69-d = 67$$
$$\Rightarrow \quad d = 69-67$$
$$\Rightarrow \quad d = 2$$

So, $$\text{A.P.} = (a-d), a, (a+d)$$
$$= (69-2), 69, (69+2)$$
$$= 67, 69, 71$$

Hence, 207 can be divided into 67, 69, 71 which form three terms of an A.P.

Q13. The angles of a triangle are in A.P. The greatest angle is twice the least. Find all the angles of the triangle.

Sol. Main concept used: (*i*) Sum of interior angles of a triangle is 180°.
(*ii*) So, 180° is divided into three parts which are in A.P. Hence, the terms of A.P. are $(a-d), a\ (a+d)$.

$$\therefore \quad a-d+a+a+d = 180°$$
[Angle sum property of a triangle]
$$\Rightarrow \quad 3a = 180°$$
$$\Rightarrow \quad a = \frac{180°}{3} = 60°$$

Also, the greatest angle is twice of the smallest. [Given]

$$\Rightarrow \quad a+d = 2(a-d)$$
$$\Rightarrow \quad a+d = 2a-2d$$
$$\Rightarrow \quad a+d-2a+2d = 0 \quad \Rightarrow \quad -a+3d = 0$$
$$\Rightarrow \quad 3d = a \quad \Rightarrow \quad d = \frac{60°}{3} \Rightarrow d = 20° \ [\because a = 60°]$$

$\therefore$ Required parts are $a-d, a, a+d$
$$= 60°-20°, 60°, 60°+20°$$
$$= 40°, 60°, 80°$$

Hence, the angles of triangle are 40°, 60° and 80°.

Q14. If nth terms of two A.P.s: 9, 7, 5, ... and 24, 21, 18. ... are same, then find the value of n. Also find that term.

Sol. First A.P. series is 9, 7, 5, ...

Here, $a_1 = 9, \quad d = 7 - 9 = -2$

Now, $a_n = a + (n-1)d$

$= 9 + (n-1)(-2) = 9 - 2(n-1)$

$= 9 - 2n + 2$

$\Rightarrow \quad a_n = 11 - 2n$

Second A.P. series is 24, 21, 18, ...

Here, $a'_1 = 24, \quad d'_1 = 21 - 24 = -3$

$\therefore \quad a'_n = a'_1 + (n-1)d'$

$\Rightarrow \quad a'_n = 24 + (n-1)(-3)$

$\Rightarrow \quad a'_n = 24 - 3n + 3$

$\Rightarrow \quad a'_n = 27 - 3n$

According to the question, we have

$a_n = a'_n$

$\Rightarrow \quad 11 - 2n = 27 - 3n$

$\Rightarrow \quad 3n - 2n = 27 - 11$

$\Rightarrow \quad n = 16$

So, 16th term of Ist A.P., i.e., $a_{16} = a_1 + (n-1)d$

$\Rightarrow \quad a_{16} = 9 + (16-1)(-2)$

$= 9 - 2 \times 15 = 9 - 30$

$\Rightarrow \quad a_{16} = -21$

16th term of IInd A.P., i.e., $a'_{16} = a'_1 + (n-1)d'$

$\Rightarrow \quad a'_{16} = 24 + (16-1)(-3)$

$= 24 - 15 \times 3 = 24 - 45$

$\Rightarrow \quad a'_{16} = -21$

Hence, the 16th term of both A.P.s is equal to -21.

Q15. If the sum of 3rd and the 8th terms of an A.P. is 7 and the sum of 7th and 14th terms is -3, find the 10th term.

Sol. Consider an A.P. whose Ist term and common difference are a and d, respectively.

According to the question,

$a_3 + a_8 = 7$ [Given]

$\Rightarrow \quad a + (3-1)d + a + (8-1)d = 7$ $[\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + 2d + a + 7d = 7$

$\Rightarrow \quad 2a + 9d = 7$ ...(i)

Also, $a_7 + a_{14} = -3$ [Given]

$\Rightarrow \quad a + (7-1)d + a + (14-1)d = -3$

$\Rightarrow \quad a + 6d + a + 13d = -3$

$\Rightarrow \quad 2a + 19d = -3$ (ii)

Now, subtracting (*ii*) from (*i*), we get

$$2a + 19d = -3 \quad \text{...(ii)}$$
$$2a + 9d = 7 \quad \text{...(i)}$$
$$- \quad - \quad -$$
$$10d = -10$$

$\Rightarrow$ $d = -1$

Now, $2a + 9d = 7$ [Using (*i*)]

$\Rightarrow$ $2a + 9(-1) = 7$

$\Rightarrow$ $2a = 7 + 9 \Rightarrow a = \frac{16}{2} \Rightarrow a = 8$

$\therefore$ $a_{10} = a + (10-1)d = 8 + 9(-1)$

$\Rightarrow$ $a_{10} = -1$

Hence, the 10th term of A.P. is -1.

Q16. Find the 12th term from the end of the A.P.: $-2, -4, -6, \dots -100$.

Sol. Main concept used: To find the term from end, consider the given A.P. in reverse order and find the term.

To find the term from the end consider the given A.P. in reverse order i.e., $-100, \dots -6, -4, -2$.

Now, $a = -100$

$d = a_{n+1} - a_n = -4 - (-6) = -4 + 6 = 2$

$n = 12$

$\therefore$ $a_{12} = a + (n-1)d$

$\Rightarrow$ $a_{12} = -100 + (12-1)(2)$

$= -100 + 11 \times 2 = -100 + 22$

$\Rightarrow$ $a_{12} = -78$

Hence, the 12th term from the last of A.P. $-2, -4, -6, \dots -100$ is -78.

Q17. Which term of the A.P.: 53, 48, 43, ... is the first negative term?

Sol. Given A.P. is 53, 48, 43, ...

$\therefore$ $a = 53, \quad d = 48 - 53 = -5$

Let the *n*th term of A.P. is the first negative term.

Then, $a_n < 0$

$\Rightarrow$ $a + (n-1)d < 0 \Rightarrow 53 + (n-1)(-5) < 0$

$\Rightarrow$ $-5(n-1) < -53 \Rightarrow 5(n-1) > 53$

$\Rightarrow$ $5n - 5 > 53 \Rightarrow 5n > 53 + 5$

$\Rightarrow$ $n > \frac{58}{5} \Rightarrow n > 11.6$

$\therefore$ $n = 12$

Hence, the first negative term of A.P. is 12th term, i.e.,

$a_{12} = a + (n-1)d$

$= 53 + (12-1)(-5) = 53 - 5 \times 11$

$= 53 - 55 = -2$

Q18. How many numbers lie between 10 and 300, which when divided by 4 leave remainder 3?

Sol. Main concept used: Find the least and the largest required number between 10 and 300 and make an A.P.

The least number between 10 and 300 which leaves remainder 3 after dividing by 4 is 11. The largest number between 10 and 300 which leaves remainder 3 on dividing by 4 is 296 + 3 = 299.

So, Ist term or number = 11, IInd term or number = 15

IIIrd term or number = 19, last term or number = 299

$\therefore$ A.P. becomes 11, 15, 19, ..., 299

Here, $a_n = 299, \quad a = 11, \quad d = 15 - 11 = 4, \quad n = ?$

Now, $a + (n-1)d = 299 \Rightarrow 11 + (n-1)4 = 299$

$$\Rightarrow \quad (n-1)4 = 299 - 11 \Rightarrow n - 1 = \frac{288}{4}$$

$$\Rightarrow \quad n = 72 + 1 \Rightarrow n = 73$$

Hence, the required numbers between 10 and 300 are 73.

Q19. Find the sum of the two middle most terms of an A.P. $\frac{-4}{3}, -1, \frac{-2}{3}, \ldots 4\frac{1}{3}$.

Sol. Main concept used: (*i*) Finding the number of terms, i.e., *n* (*ii*) median of *n*.

Given A.P. is $\frac{-4}{3}, -1, \frac{-2}{3}, \ldots, +\frac{13}{3}$

Here, $a = \frac{-4}{3}, \quad d = \frac{-1}{1} - \left(\frac{-4}{3}\right) = \frac{-1}{1} + \frac{4}{3}$

$$\Rightarrow d = \frac{-3+4}{3} = \frac{1}{3}$$

And, $a_n = \frac{13}{3}$

$$\Rightarrow \quad a + (n-1)d = \frac{13}{3}$$

$$\Rightarrow \quad \frac{-4}{3} + (n-1)\left(\frac{1}{3}\right) = \frac{13}{3}$$

$$\Rightarrow \quad -4 + (n-1) = 13$$

$$\Rightarrow \quad n - 1 = 13 + 4$$

$$\Rightarrow \quad n = 17 + 1$$

$$\Rightarrow \quad n = 18$$

So, the middle most terms in 18 terms $= \left(\frac{18}{2}\right)$th and $\left(\frac{18}{2}\right)$th + 1

= 9th and 10th terms are middle most

So, the required sum $= a_9 + a_{10}$

$$= a + (9-1)d + a + (10-1)d$$

$$= 2a + 8d + 9d = 2a + 17d$$

$$= 2\left(\frac{-4}{3}\right) + 17\left(\frac{1}{3}\right) = \frac{-8+17}{3}$$

$$= \frac{9}{3} = 3$$

Hence, the sum of two middle most terms, i.e., $a_9 + a_{10} = 3$.

Q20. The first term of an A.P. is -5 and last term is 45. If the sum of the terms of A.P. is 120, then find the number of terms and the common difference.

Sol. Let us consider an A.P. whose first term and common difference are a and d respectively.

Here, $a = -5, \quad a_n = 45, \quad S_n = 120$

Now, $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + a + (n-1)d]$

$\Rightarrow \quad S_n = \frac{n}{2}[a + a_n] \qquad [a_n = \text{last term}]$

$\Rightarrow \quad 120 = \frac{n}{2}[-5 + 45] \quad \Rightarrow \quad 120 = \frac{n}{2} \times 40$

$\Rightarrow \quad n = \frac{120 \times 2}{40} = 6 \quad \Rightarrow \quad n = 6$

Hence, the number of terms = 6

Now, $a_n = a + (n-1)d \quad \Rightarrow \quad 45 = -5 + (6-1)d$

$\Rightarrow \quad 45 + 5 = 5d \quad \Rightarrow \quad 5d = 50$

$\Rightarrow \quad d = \frac{50}{5} \quad \Rightarrow \quad d = 10$

Hence, the common difference and the number of terms in A.P. are 10 and 6 respectively.

Q21. Find the sum:

(*i*) $1 + (-2) + (-5) + (-8) + \ldots + (-236)$

(*ii*) $\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \cdots$ upto n terms

(*iii*) $\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \cdots$ upto 11 terms.

Sol. (*i*) From the given series,

$$a = 1, \quad a_n = -236$$

$$d_1 = -2 - 1 = -3, \quad d_2 = -5 - (-2) = -5 + 2 = -3$$

$$d_3 = -8 - (-5) = -8 + 5 = -3$$

$\therefore$ $$d = d_1 = d_2 = d_3 = -3$$

Now, $$a + (n-1)d = a_n$$

$\Rightarrow$ $$1 + (n-1)(-3) = -236 \quad \Rightarrow \quad -3(n-1) = -236 - 1$$

$\Rightarrow$ $$-3(n-1) = -237 \quad \Rightarrow \quad -(n-1) = \frac{-237}{3}$$

$\Rightarrow$ $$n - 1 = +79 \quad \Rightarrow \quad n = 79 + 1 \quad \Rightarrow \quad n = 80$$

Now, $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow$ $$S_{80} = \frac{80}{2}[2(1) + (80-1)(-3)]$$

$$= 40[2 - 79 \times 3] = 40[2 - 237]$$

$$= 40[-235] = -9400$$

Hence, the sum of all terms $= -9400$

(*ii*) From the given series, we have

$$a = \left(4 - \frac{1}{n}\right) \quad \text{and} \quad n = n$$

$$d_1 = \left(4 - \frac{2}{n}\right) - \left(4 - \frac{1}{n}\right) = 4 - \frac{2}{n} - 4 + \frac{1}{n} = -\frac{1}{n}$$

$$d_2 = \left(4 - \frac{3}{n}\right) - \left(4 - \frac{2}{n}\right) = 4 - \frac{3}{n} - 4 + \frac{2}{n} = -\frac{1}{n}$$

Now, $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}\left[2\left(4 - \frac{1}{n}\right) + (n-1)\left(-\frac{1}{n}\right)\right]$$

$$= \frac{n}{2}\left[8 - \frac{2}{n} - \frac{(n-1)}{n}\right] = \frac{n}{2}\left[8 - \frac{2}{n} - 1 + \frac{1}{n}\right]$$

$$= \frac{n}{2}\left[7 - \frac{1}{n}\right] = -\left[\frac{7 \quad 1}{\quad}\right]$$

$\Rightarrow$ $$S_n = \frac{7n - 1}{2}$$

(*iii*) From the given series, we have

$$a \text{ (Ist term)} = \frac{a-b}{a+b}, \quad n = 11$$

$$d = \frac{(3a-2b)}{(a+b)} - \frac{(a-b)}{(a+b)}$$

$$= \frac{3a - 2b - (a-b)}{a+b} = \frac{3a - 2b - a + b}{a+b}$$

$$\Rightarrow \quad d = \frac{2a-b}{a+b}$$

Now, $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow \quad S_{11} = \frac{11}{2}\left[\frac{2(a-b)}{(a+b)} + (11-1)\frac{(2a-b)}{(a+b)}\right]$$

$$= \frac{11}{2(a+b)}[2a-2b+10(2a-b)]$$

$$= \frac{11}{2(a+b)}[2a-2b+20a-10b]$$

$$= \frac{11}{2(a+b)}[22a-12b]$$

$$= \frac{11}{2}\frac{(22a-12b)}{(a+b)} = \frac{11\times 2(11a-6b)}{2(a+b)}$$

$$= \frac{11(11a-6b)}{(a+b)}$$

Q22. Which term of the A.P., – 2, – 7, – 12, ... will be – 77? Find the sum of this A.P. upto the term – 77.

Sol. Given A.P. is – 2, – 7, –12 ... – 77

Here, $a = -2, \quad a_n = -77$

$$d_1 = -7-(-2) = -7+2 = -5$$

$$d_2 = -12-(-7) = -12+7 = -5$$

Now, $$a_n = -77$$

$$\Rightarrow \quad a+(n-1)d = -77 \quad \Rightarrow \quad -2+(n-1)(-5) = -77$$

$$\Rightarrow \quad -[2+(n-1)5] = -77 \quad \Rightarrow \quad (2+5n-5) = 77$$

$$\Rightarrow \quad 5n-3 = 77 \quad \Rightarrow \quad 5n = 77+3$$

$$\Rightarrow \quad n = \frac{80}{5} \quad \Rightarrow \quad n = 16$$

So, the 16th term will be –77.

Now, $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow \quad S_{16} = \frac{16}{2}[2(-2)+(16-1)(-5)]$$

$$= 8[-4-15\times 5] = 8[-4-75]$$

$$= 8[-79] = -632$$

Hence, the sum of the given A.P. upto – 77 terms is – 632.

Q23. If $a_n = 3 - 4n$, then show that $a_1, a_2, a_3, \ldots$ form an A.P. Also find S_{20}.

Sol. $a_n = 3 - 4n$ [Given]

$\therefore$ $a_1 = 3 - 4(1) = 3 - 4 = -1$

$a_2 = 3 - 4(2) = 3 - 8 = -5$

$a_3 = 3 - 4(3) = 3 - 12 = -9$

$a_4 = 3 - 4(4) = 3 - 16 = -13$

Now, $d_1 = a_2 - a_1 = -5 - (-1) = -5 + 1 = -4$

$d_2 = a_3 - a_2 = -9 - (-5) = -9 + 5 = -4$

$d_3 = a_4 - a_3 = -13 - (-9) = -13 + 9 = -4$

As $d_1 = d_2 = d_3 = -4$ so $a_1, a_2, a_3 \ldots a_n$ are in A.P.

So, $a = -1, \quad d = -4, \quad n = 20$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $S_{20} = \frac{20}{2}[2 \times (-1) + (20-1)(-4)]$

$= 10[-2 - 76] = 10[-78]$

$\Rightarrow$ $S_{20} = -780$

Hence, $a_1, a_2, a_3, \ldots a_n$ are in A.P. and $S_{20} = -780$.

Q24. In an A.P., if $S_n = n(4n + 1)$ then find the A.P.

Sol. Main concept used: $a_1 = S_1, \quad a_2 = S_2 - S_1, \quad a_3 = S_3 - S_2$

$S_n = n(4n + 1) = 4n^2 + n$ [Given]

$a_n = S_n - S_{n-1}$

$\Rightarrow$ $a_n = [4n^2 + n] - [4(n-1)^2 + (n-1)]$

$= 4n^2 + n - [4(n^2 + 1 - 2n) + n - 1]$

$= 4n^2 + n - [4n^2 + 4 - 8n + n - 1]$

$= 4n^2 + n - [4n^2 - 7n + 3]$

$= 4n^2 + n - 4n^2 + 7n - 3$

$\Rightarrow$ $a_n = 8n - 3$

$\therefore$ $a_1 = 8(1) - 3 = 8 - 3 = 5$

$a_2 = 8(2) - 3 = 16 - 3 = 13$

$a_3 = 8(3) - 3 = 24 - 3 = 21$

$a_4 = 8(4) - 3 = 32 - 3 = 29$

Hence, the required A.P. is 5, 13, 21, 29, ...

Q25. In an A.P. if $S_n = 3n^2 + 5n$ and $a_k = 164$, then find the value of k.

Sol. Main concept used: $a_n = S_n - S_{n-1}$

$S_n = 3n^2 + 5n$

$\therefore$ $S_{n-1} = 3(n-1)^2 + 5(n-1)$

$\Rightarrow \quad S_{n-1} = 3(n^2 + 1 - 2n) + 5n - 5$

$\quad = 3n^2 + 3 - 6n + 5n - 5$

$\Rightarrow \quad S_{n-1} = 3n^2 - n - 2$

Now, $\quad a_n = S_n - S_{n-1}$

$\Rightarrow \quad a_n = 3n^2 + 5n - (3n^2 - n - 2)$

$\Rightarrow \quad a_n = 3n^2 + 5n - 3n^2 + n + 2$

$\Rightarrow \quad a_n = 6n + 2 \quad \Rightarrow \quad a_k = 6k + 2$

$\Rightarrow \quad 164 = 6k + 2 \quad \Rightarrow \quad 6k = 164 - 2$

$\Rightarrow \quad k = \frac{162}{6} \quad \Rightarrow \quad k = 27$

Q26. If S_n denotes the sum of first n terms of an A.P., then prove that $S_{12} = 3(S_8 - S_4)$.

Sol. Consider an A.P. whose first term and common difference are 'a' and 'd' respectively.

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \Rightarrow \quad S_{12} = \frac{12}{2}[2a + (12-1)d]$$

$\Rightarrow \quad S_{12} = 6[2a + 11d] \quad \ldots(i)$

and $\quad S_8 = \frac{8}{2}[2a + (8-1)d]$

$\Rightarrow \quad S_8 = 4[2a + 7d] \quad \ldots(ii)$

and $\quad S_4 = \frac{4}{2}[2a + (4-1)d]$

$\Rightarrow \quad S_4 = 2[2a + 3d] \quad \ldots(iii)$

Now, $\quad 3(S_8 - S_4) = 3[4(2a + 7d) - 2(2a + 3d)]$ [Using eqns. (*ii*) and (*iii*)]

$\quad = 3[8a + 28d - 4a - 6d]$

$\quad = 3[4a + 22d]$

$\quad = 3 \times 2[2a + 11d]$

$\quad = 6[2a + 11d] = S_{12}$ [Using eqn. (*i*)]

$\therefore \quad$ RHS = LHS

Hence, proved.

Q27. Find the sum of first 17 terms of an A.P. whose 4th and 9th terms are -15, and -30 respectively.

Sol. $a_4 = -15, \quad a_9 = -30, \quad S_{17} = ?$

Consider an A.P. whose Ist term and common difference are a and d respectively.

$\quad a_4 = -15$ [Given]

$\Rightarrow \quad a + (4-1)d = -15$ [$\because a_n = a + (n-1)d$]

$\Rightarrow \quad a + 3d = -15$ ...(i)

Also, $\quad a_9 = -30$ [Given]

$\Rightarrow \quad a + (9-1)d = -30$ $[\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + 8d = -30$ ...(ii)

Subtracting (i) from (ii), we get

$$\begin{array}{r} a + 8d = -30 \quad \text{...(ii)} \\ a + 3d = -15 \quad \text{[From (i)]} \\ - \quad - \quad + \\ \hline 5d = -15 \end{array}$$

$\Rightarrow \quad d = \dfrac{-15}{5} = -3$

Now, $\quad a + 3d = -15$ [From (i)]

$\Rightarrow \quad a + 3(-3) = -15$

$\Rightarrow \quad a = -15 + 9$

$\Rightarrow \quad a = -6$

$S_{17} = ?$

We know that $\quad S_n = \dfrac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{17} = \dfrac{17}{2}[2(-6) + (17-1)(-3)]$

$= \dfrac{17}{2}[-12 + 16(-3)] = \dfrac{17}{2}[-12 - 48]$

$= \dfrac{17}{2}(-60) = -17 \times 30$

$\Rightarrow \quad S_{17} = -510$

Q28. If sum of first 6 terms of an A.P. is 36 and that of the first 16 terms is 256, find the sum of the first 10 terms.

Sol. Consider the A.P. whose first term and common difference are '*a*' and '*d*' respectively.

$S_6 = 36$ [Given]

$\therefore \quad \dfrac{6}{2}[2a + (6-1)d] = 36$ $\left[\because S_n = \dfrac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow \quad 2a + 5d = \dfrac{36}{3}$

$\Rightarrow \quad 2a + 5d = 12$ ...(i)

Also, $\quad S_{16} = 256$ [Given]

$\Rightarrow \quad \dfrac{16}{2}[2a + (16-1)d] = 256$

$\Rightarrow \quad 2a + 15d = \dfrac{256}{8}$

$\Rightarrow \quad 2a + 15d = 32 \quad ...(ii)$

Subtracting (*i*) from (*ii*), we get

$$\begin{array}{r} 2a + 15d = 32 \\ 2a + 5d = 12 \\ \underline{- \quad - \qquad -} \\ 10d = 20 \end{array} \qquad \begin{array}{r} ...(ii) \\ [\text{From } (i)] \end{array}$$

$\Rightarrow \quad d = 2$

Now, $\quad 2a + 5d = 12 \quad$ [From (*i*)]

$\Rightarrow \quad 2a + 5(2) = 12$

$\Rightarrow \quad 2a = 12 - 10 \Rightarrow a = \frac{2}{2} \Rightarrow a = 1$

So, $\quad S_{10} = \frac{10}{2}[2a + (10-1)d]$

$= 5[2(1) + 9(2)] = 5[2 + 18] = 5[20] = 100$

$\Rightarrow \quad S_{10} = 100$

Hence, the sum of first 10 terms is 100.

Q29. Find the sum of all the 11 terms of an A.P. whose middle most term is 30.

Sol. Number of terms are 11, so $n = 11$

$$\text{Middle term} = \frac{11+1}{2} = \frac{12}{2} = 6\text{th term}$$

Also, middle term $= 30$ [Given]

$\therefore \quad a_6 = 30$ [Given]

$\Rightarrow \quad a + (6-1)d = 30 \quad [\because \; a_n = a + (n-1)d]$

$\Rightarrow \quad a + 5d = 30 \quad ...(i)$

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{11} = \frac{11}{2}[2a + (11-1)d] = \frac{11}{2}[2a + 10d]$

$= \frac{11 \times 2}{2}[a + 5d] \quad$ [Using (*i*)]

$= 11 \times 30$

$\Rightarrow \quad S_{11} = 330$

Hence, the sum of all 11 terms is 330.

Q30. Find the sum of last 10 terms of the A.P. 8, 10, 12, ..., 126.

Sol. To find out the sum of last 10 terms, we will reverse the order of the given A.P. and get 126, ..., 12, 10, 8

So, $\quad a = 126, \quad d = 10 - 12 = -2, \quad n = 10$

$\therefore \quad S_{10} = \frac{10}{2}[2(126) + (10-1)(-2)] \quad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$$= 5[252 + 9(-2)] = 5[252 - 18]$$
$$= 5 \times 234$$
$$\Rightarrow \quad S_{10} = 1170$$

Hence, the sum of 10 terms from the end of A.P. 8, 10, 12, ..., 126 is 1170.

Q31. Find the sum of first seven numbers which are multiples of 2 as well as of 9. [**Hint:** Take the L.C.M. of 2 and 9]

Sol. The numbers which are multiples of 2 as well as of 9 are 18, 36, 54, ... 7 terms

So, $n = 7, \quad a = 18, \quad d = 36 - 18 = 18$

$$\therefore \quad S_7 = \frac{7}{2}[2(18) + (7-1)(18)] \qquad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$
$$= \frac{7 \times 18}{2}[2 + 6]$$
$$= 7 \times 9 \times 8 = 7 \times 72$$
$$\Rightarrow \quad S_7 = 504$$

Hence, the sum of first 7 numbers which are multiple of 2 as well as 9, i.e., multiples of 18 is 504.

Q32. How many terms of the A.P.: –15, –13, –11, ... are needed to make the sum –55? Explain the reason for double answer.

Sol. Given A.P. is – 15, – 13, – 11, ...

$\therefore \quad S_n = -55, \quad a = -15, \quad n = ?$

$$d = -13 - (-15) = -13 + 15 = 2 \quad \Rightarrow \quad d = 2$$

But, $$S_n = -55$$
$$\Rightarrow \quad \frac{n}{2}[2a + (n-1)d] = -55$$
$$\Rightarrow \quad n[2(-15) + (n-1)(2)] = -55 \times 2$$
$$\Rightarrow \quad n[-30 + 2(n-1)] = -110$$
$$\Rightarrow \quad n[-30 + 2n - 2] + 110 = 0$$
$$\Rightarrow \quad -30n + 2n^2 - 2n + 110 = 0$$
$$\Rightarrow \quad 2n^2 - 32n + 110 = 0$$
$$\Rightarrow \quad n^2 - 16n + 55 = 0$$
$$\Rightarrow \quad n^2 - 11n - 5n + 55 = 0$$
$$\Rightarrow \quad n(n-11) - 5(n-11) = 0$$
$$\Rightarrow \quad (n-11)(n-5) = 0$$
$$\Rightarrow \quad n - 11 = 0 \text{ or } n - 5 = 0$$
$$\Rightarrow \quad n = 11 \text{ or } n = 5$$

So, 5 or 11 terms of A.P. are needed to make the sum – 55.

Q33. The sum of first n terms of an A.P. whose first term is 8 and the common difference is 20 is equal to sum of first $2n$ terms of another A.P. whose first term is -30, and common difference is 8. Find n.

Sol. For AP I: $a = 8, d = +20$ | For AP II: $a' = -30, d' = 8$

According to the question, $S_n = S'_{2n}$

$$\Rightarrow \quad \frac{n}{2}[2a+(n-1)d] = \frac{2n}{2}[2a'+(2n-1)d']$$

$$\Rightarrow \quad [2(8)+(n-1)20] = 2[2(-30)+(2n-1)8]$$

$$\Rightarrow \quad 2\times 8+n\times 20-20 = 2[-60+16n-8]$$

$$\Rightarrow \quad 16+20n-20 = 2[-68+16n]$$

$$\Rightarrow \quad 20n-4 = -136+32n$$

$$\Rightarrow \quad -32n+20n = -136+4$$

$$\Rightarrow \quad -12n = -132$$

$$\Rightarrow \quad n = \frac{132}{12} = 11$$

Hence, the required value of n is 11.

Q34. Kanika was given her pocket money on Jan. 1, 2008. She puts ₹ 1 on day 1, ₹ 2 on day 2, ₹ 3 on day 3, and continued doing so till the end of the month, from this money into her piggy bank. She also spent ₹ 204 of her pocket money, and found that at the end of the month she still had ₹ 100 with her. How much was her pocket money for the month?

Sol. Let the pocket money of Kanika for the month be ₹ x.

Out of x, the money which she deposited in piggy bank and spent = ₹ 204

Money put in piggy bank from Jan. 1 to Jan. 31 = $1+2+3+4+\cdots+31$

So, $a = 1, \quad d = 1, \quad n = 31$

Now, $$S_{31} = \frac{31}{2}[2(1)+(31-1)(1)] \qquad \left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$$

$$= \frac{31}{2}[2+30]$$

$$\Rightarrow \quad S_{31} = \frac{31\times 32}{2} = 31\times 16 \quad \Rightarrow \quad S_{31} = 496$$

$\therefore$ Money deposited in piggy bank = ₹ 496

Money spent = ₹ 204

Money which she still have = ₹ 100

$$\therefore \quad x-496-204 = 100$$

$$\Rightarrow \qquad x = 100 + 496 + 204 = 800$$

Hence, her monthly pocket money is ₹ 800.

Q35. Yasmeen saves ₹ 32 during the first month, ₹ 36 in the second month and ₹ 40 in 3rd month. If she continues to save in this manner, in how many months will she save ₹ 2000?

Sol. During Ist month, savings of Yasmeen = ₹ 32

During IInd month, savings of Yasmeen = ₹ 36

During IIIrd month, savings of Yasmeen = ₹ 40

During IVth month, savings of Yasmeen = ₹ 44

$$\therefore \qquad 32 + 36 + 40 + 44 + \ldots = 2000$$

Also, $a = 32$, $\quad d = 36 - 32 = 4$

Now, $S_n = 2000$

$$\Rightarrow \quad \frac{n}{2}[2a + (n-1)d] = 2000 \quad \Rightarrow \quad n[2(32) + (n-1)4] = 2000 \times 2$$

$$\Rightarrow \quad n[64 + 4n - 4] = 4000 \quad \Rightarrow \quad n[4n + 60] = 4000$$

$$\Rightarrow \quad 4n[n + 15] = 4000 \quad \Rightarrow \quad n[n + 15] = \frac{4000}{4}$$

$$\Rightarrow \quad n^2 + 15n - 1000 = 0 \quad \Rightarrow \quad n^2 + 40n - 25n - 1000 = 0$$

$$\Rightarrow \quad n[n + 40] - 25[n + 40] = 0 \quad \Rightarrow \quad (n + 40)(n - 25) = 0$$

$$\Rightarrow \quad n + 40 = 0 \quad \text{or} \quad n - 25 = 0$$

$$\Rightarrow \quad n = -40 \quad \text{or} \quad n = 25$$

Rejecting $n = -40$, we have $n = 25$.

Hence, in 25 months she saves ₹ 2000.

EXERCISE 5.4

Q1. The sum of the first five terms of an A.P. and the sum of the first seven terms of the same A.P. is 167. If the sum of the first ten terms of this A.P. is 235, find the sum of its first twenty terms.

Sol. Consider an A.P. whose first term and the common difference are a and d respectively.

According to the question:

$$S_5 + S_7 = 167 \qquad \text{[Given]}$$

$$\Rightarrow \quad \frac{5}{2}[2a + (5-1)d] + \frac{7}{2}[2a + (7-1)d] = 167$$

$$\Rightarrow \quad 5\{2a + 4d\} + 7\{2a + 6d\} = 167 \times 2$$

On multiplying both sides by $\frac{1}{2}$, we get

$$\frac{1}{2}[10a + 20d + 14a + 42d] = 167$$

$$\Rightarrow \quad \frac{1}{2}[24a + 62d] = 167$$

$$\Rightarrow \quad \frac{1}{2} \times 2[12a + 31d] = 167$$

$$\Rightarrow \quad 12a + 31d = 167 \quad \text{...}(i)$$

Also, $S_{10} = 235$ [Given]

$$\Rightarrow \quad \frac{10}{2}[2a + (10-1)d] = 235$$

$$\Rightarrow \quad 5[2a + 9d] = 235$$

$$\Rightarrow \quad 2a + 9d = \frac{235}{5}$$

$$\Rightarrow \quad 2a + 9d = 47 \quad \text{...}(ii)$$

Multiplying (*ii*) by 6, we have

$$12a + 54d = 282 \quad \text{...}(iii)$$

Now, subtracting (*i*) from (*iii*), we get

$$12a + 54d = 282 \quad (iii)$$

$$12a + 31d = 167 \quad [\text{From } (i)]$$

$$- \quad - \quad\quad -$$

$$23d = 115$$

$$\Rightarrow \quad d = \frac{115}{23} \quad \Rightarrow \quad d = 5$$

Now, $2a + 9d = 47$ [From (*ii*)]

$$\Rightarrow \quad 2a + 9 \times 5 = 47$$

$$\Rightarrow \quad 2a = 47 - 45 \quad \Rightarrow \quad 2a = 2 \quad \Rightarrow \quad a = 1$$

$$\therefore \quad S_{20} = \frac{20}{2}[2a + (20-1)d] \quad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$= 10[2 \times (1) + 19(5)] = 10[2 + 95] = 10 \times 97$$

$$\Rightarrow \quad S_{20} = 970$$

Hence, the sum of first twenty terms is 970.

Q2. Find the

(*i*) sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

(*ii*) sum of those integers from 1 to 500 which are multiples of 2 as well as of 5.

(*iii*) sum of those integers from 1 to 500 which are multiples of 2 or 5.

[**Hint:** These numbers will be: multiples of 2 + multiples of 5 – multiples of 2 as well as of 5.]

Sol. (*i*) Integers which are multiples of 2 as well as 5 are multiples of 10, i.e., 10, 20, 30, ..., 490. [∵ Between 1 and 500]

$\therefore\ a = 10, \quad d = 10, \quad a_n = 490$

Now, $a_n = 490$ $\qquad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + (n-1)d = 490$

$\Rightarrow \quad 10 + (n-1)10 = 490$

$\Rightarrow \quad 1 + (n-1) = \frac{490}{10}$

$\Rightarrow \quad n = 49$

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{49} = \frac{49}{2}[2 \times 10 + (49-1)10]$

$= \frac{49}{2} \times 10[2 + 48] = 49 \times 5 \times 50$

$\Rightarrow \quad S_{49} = 12250$

(*ii*) Multiple of 2 as well as of 5 are multiples of $2 \times 5 = 10$. Multiples of 10 from (not between) 1 to 500 are 10, 20, 30, 40, ..., 500.

$\therefore\ a = 10, \quad d = 10, \quad a_n = 500$

Now, $a_n = a + (n-1)d = 500$

$\Rightarrow \quad 10 + (n-1)10 = 500$

$\Rightarrow \quad 1 + n - 1 = 50$

$\Rightarrow \quad n = 50$

So, $S_{50} = \frac{50}{2}[2 \times 10 + (50-1)10]$

$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= \frac{50 \times 10}{2}[2 + 49] = 50 \times 5 \times 51$

$\Rightarrow \quad S_{50} = 12750$

(*iii*) Sum of integers which are multiples of 2 or 5 only (not of 10)

= Sum of integers which are multiples of 2 + Sum of integers which are multiples of 5 − Sum of integers which are multiples of 10

$= (2 + 4 + 6 + \ldots + 500) + (5 + 10 + 15 + 20 + \ldots + 500) - (10 + 20 + 30 + \ldots + 500)$

$= S_1 + S_2 - S_3$

For $S_1 = 2 + 4 + 6 + \ldots + 500$, we have

$a = 2, \quad d = 2, \quad a_n = 500$

$\therefore \quad a + (n-1)d = 500 \Rightarrow 2 + (n-1)2 = 500$

$\Rightarrow \quad 2[1 + (n-1)] = 500 \Rightarrow 2n = 500$

$\Rightarrow \quad n = 250$

$\therefore \quad S_1 = S_{250}$

$\Rightarrow \quad S_1 = S_{250} = \frac{250}{2}[2 \times 2 + (250-1)(2)]$

$$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$= 125[4 + 249 \times 2]$

$\Rightarrow \quad S_1 = 125\,[4 + 498]$

$\Rightarrow \quad S_1 = 125 \times 502 = 62750$

For $S_2 = 5 + 10 + 15 + 20 + \cdots + 500$, we have

$a = 5, \quad d = 5, \quad a_n = 500$

$\therefore \quad a + (n-1)d = 500 \quad \Rightarrow \quad 5 + (n-1)5 = 500$

$\Rightarrow \quad 5[1 + n - 1] = 500 \quad \Rightarrow \quad n = 100$

$\therefore \quad S_2 = S_{100}$

$\Rightarrow \quad S_2 = S_{100} = \frac{100}{2}[2a + (n-1)d]$

$= 50[2(5) + (100-1)5] = 50[10 + 99 \times 5]$

$= 50[10 + 495] = 50 \times 505$

$\Rightarrow \quad S_2 = 25250$

For $S_3 = 10 + 20 + 30 + \ldots + 500$, we have

$a = 10, \quad d = 10, \quad a_n = 500$

$\therefore \quad a + (n-1)d = 500 \qquad [\because \; a_n = a + (n-1)d]$

$\Rightarrow \quad 10 + (n-1)10 = 500$

$\Rightarrow \quad 10[1 + n - 1] = 500$

$\Rightarrow \quad n = \frac{500}{10} = 50$

Now, $\quad S_3 = S_{50} = \frac{50}{2}[2a + (n-1)d]$

$= 25[2(10) + (50-1)\,10]$

$= 25[20 + 490]$

$= 25 \times 510$

$\Rightarrow \quad S_3 = 12750$

Hence, the sum of the required integers $= S_1 + S_2 - S_3$

$= 62750 + 25250 - 12750$

$= 88000 - 12750 = 75250$

Q3. The 8th term of an A.P. is half its second term and 11th term exceeds one third of its fourth term by '1'. Find the 15th term.

Sol. Consider the first term and common difference as a and d respectively.

$$a_8 = \frac{1}{2}a_2 \qquad \text{[Given]}$$

$$\Rightarrow \quad a+(8-1)d = \frac{1}{2}[a+(2-1)d] \qquad [\because a_n = a+(n-1)d]$$

$$\Rightarrow \quad 2(a+7d) = a+d$$

$$\Rightarrow \quad 2a+14d-a-d = 0$$

$$\Rightarrow \quad a+13d = 0 \qquad \text{...(i)}$$

Now,

$$a_{11} = \frac{1}{3}a_4+1 \qquad \text{[Given]}$$

$$\Rightarrow \quad a+(11-1)d = \frac{1}{3}[a+(4-1)d]+1$$

$$\Rightarrow \quad (a+10d) = \frac{1}{3}(a+3d)+1$$

$$\Rightarrow \quad 3(a+10d) = a+3d+3$$

$$\Rightarrow \quad 3a+30d-a-3d = 3$$

$$\Rightarrow \quad 2a+27d = 3 \qquad \text{...(ii)}$$

Multiplying (*i*) by 2, we have

$$2a+26d = 0 \qquad \text{...(iii)}$$

Now, subtraction (*iii*) from (*ii*), we get

$$\begin{aligned} 2a+27d &= 3 \qquad \text{...(ii)} \\ 2a+26d &= 0 \qquad \text{(iii)} \\ - \quad - \quad &\quad - \\ \hline d &= 3 \end{aligned}$$

Now,

$$a+13d = 0 \qquad \text{[From (i)]}$$

$$\Rightarrow \quad a+13\times 3 = 0$$

$$\Rightarrow \quad a = -39$$

Now, we know that $a_n = a+(n-1)d \Rightarrow a_{15} = -39+(15-1)3$

$$= -39+14\times 3 = -39+42$$

$$\Rightarrow \quad a_{15} = 3$$

Q4. An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the A.P.

Sol. Consider an A.P. whose first term and common difference are 'a' and 'd' respectively.

Total terms = 37

The middle most term $= \frac{37+1}{2} = \frac{38}{2} = 19\text{th term}$

So, the sum of the three middle most terms $= a_{18}+a_{19}+a_{20}$

$$= a + (18 - 1)d + a + (19 - 1)d + a + (20 - 1)d$$
$$= 3a + 17d + 18d + 19d$$
$$\Rightarrow \quad 225 = 3a + 54d \quad ...(i)$$
$$\Rightarrow \quad a + 18d = 75$$

The sum of the last three terms $= a_{37} + a_{36} + a_{35} = 429$ [Given]

$$= a + (37 - 1)d + a + (36 - 1)d + a + (35 - 1)d = 429$$
$$\Rightarrow \quad 3a + 36d + 35d + 34d = 429$$
$$\Rightarrow \quad 3a + 105d = 429$$
$$\Rightarrow \quad a + 35d = 143 \quad ...(ii)$$

Now, subtracting (*i*) from (*ii*), we get

$$a + 35d = 143 \quad ...(ii)$$
$$a + 18d = 75 \quad (i)$$
$$- \quad - \quad -$$
$$17d = 68$$
$$\Rightarrow \quad d = 4$$

Now, $a + 18d = 75$ [Using (*i*)]

$$\Rightarrow \quad a + 18 \times 4 = 75$$
$$\Rightarrow \quad a = 75 - 72 = 3$$

$\therefore$ $a = 3$ and $d = 4$

Hence, the required A.P. is $a, a + d, a + 2d, a + 3d... = 3, 7, 11, 15 ...$

Q5. Find the sum of the integers between 100 and 200 that are (*i*) divisible by 9 (*ii*) not divisible by 9.

[**Hint (*ii*):** These numbers will be: Total numbers – Total numbers divisible by 9.]

Sol. (*i*) Numbers between 100 – 200 divisible by 9 are 108, 117, 125, ...198

Here, $a = 108$, $d = 117 - 108 = 9$ and $a_n = 198$

$$\Rightarrow \quad a + (n - 1)d = 198 \quad [a_n = a + (n - 1)d]$$
$$\Rightarrow \quad 108 + (n - 1)9 = 198 \quad \Rightarrow \quad 9[12 + n - 1] = 198$$
$$\Rightarrow \quad 11 + n = \frac{198}{9} \quad \Rightarrow \quad n = 22 - 11 \quad \Rightarrow \quad n = 11$$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$\Rightarrow \quad S_{11} = \frac{11}{2}[2(108) + (11 - 1)(9)]$$
$$= \frac{11}{2}[216 + 99 - 9] = \frac{11}{2}[216 + 90]$$
$$= \frac{11}{2} \times 306$$
$$\Rightarrow \quad S_{11} = 1683$$

(*ii*) Numbers between 100 and 200 = 101, 102, 103, ...199

Here, $a = 101,\quad d = 1,\quad a_n = 199$

$$\Rightarrow \quad a + (n-1)d = 199 \quad \Rightarrow \quad 101 + (n-1)(1) = 199$$

$$\Rightarrow \quad (n-1) = 199 - 101 = 98$$

$$\Rightarrow \quad n = 99$$

Now, $$S_{99} = \frac{99}{2}[2 \times 101 + (99-1)(1)]$$

$$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$= \frac{99}{2}[202 + 98] = \frac{99}{2} \times 300 = 99 \times 150 = 14850$$

So, the sum of integers between 100 and 200 which are not divisible by 9 = 14850 – 1683 = 13167.

Q6. The ratio of the 11th term to the 18th term of an A.P. is 2 : 3. Find the ratio of the 5th term to the 21st term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms.

Sol. Consider an A.P. whose first term and common difference are a and d respectively.

$$a_{11} : a_{18} = 2 : 3 \qquad \text{[Given]}$$

$$\Rightarrow \quad \frac{a + 10d}{a + 17d} = \frac{2}{3} \qquad [\therefore a_n = a + (n-1)d]$$

$$\Rightarrow \quad 3a + 30d = 2a + 34d$$

$$\Rightarrow \quad 3a - 2a = 34d - 30d$$

$$\Rightarrow \quad a = 4d$$

To find:

$$\frac{a_5}{a_{21}} = \frac{a + 4d}{a + 20d} = \frac{4d + 4d}{4d + 20d} = \frac{8d}{24d} = \frac{1}{3}$$

$$\therefore \quad a_5 : a_{21} = 1 : 3$$

Now, $$\frac{S_5}{S_{21}} = \frac{\frac{5}{2}[2a + (5-1)d]}{\frac{21}{2}[2a + (21-1)d]} = \frac{5[2(4d) + 4d]}{21[2(4d) + 20d]} = \frac{5[8d + 4d]}{21[8d + 20d]}$$

$$= \frac{5 \times 12d}{21 \times 28d} = \frac{5}{7 \times 7} = \frac{5}{49} = 5 : 49$$

$$\therefore \quad S_5 : S_{21} = 5 : 49$$

Q7. Show that the sum of an A.P. whose first term is a, the second term b and the last term c, is equal to $\dfrac{(a+c)(b+c-2a)}{2(b-a)}$

Sol. Here, a (Ist term) $= a$, $\quad d = (b - a)$, $\quad a_n = c$

As $a_n = c$

$\Rightarrow \quad a + (n-1)d = c \qquad [\because \; a_n = a + (n-1)d]$

$\Rightarrow \quad (n-1)(b-a) = c - a$

$$\Rightarrow \quad (n-1) = \frac{(c-a)}{b-a}$$

$$\Rightarrow \quad n = \frac{c-a}{b-a} + 1 = \frac{c-a+b-a}{b-a}$$

$$\Rightarrow \quad n = \frac{(b+c-2a)}{b-a} \qquad ...(i)$$

Now,
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{(b+c-2a)}{2(b-a)}\left[2a + \left\{\frac{b+c-2a}{b-a} - 1\right\}(b-a)\right] \qquad \text{[Using (i)]}$$

$$= \frac{(b+c-2a)}{2(b-a)}\left[2a + \left\{\frac{b+c-2a-b+a}{(b-a)}\right\} \times (b-a)\right]$$

$$= \frac{(b+c-2a)}{2(b-a)}[2a + c - a]$$

$$\Rightarrow \quad S_n = \frac{(b+c-2a)}{2(b-a)}(a+c)$$

Hence proved.

Q8. Solve the equation $-4 + (-1) + 2 + \ldots + x = 437$.

Sol. Given series is $-4 + (-1) + 2 + \ldots + x$

So, $\quad d_1 = -1 - (-4) = -1 + 4 = 3$, $d_2 = 2 - (-1) = 2 + 1 = 3$

$\therefore$ Given list of numbers are in A.P. $\qquad [\because \; d = d_1 = d_2 = 3]$

Here, $\qquad a = -4$ and $a_n = x$

As $a_n = x$

$\Rightarrow \quad a + (n-1)d = x \qquad [\because \; a_n = a + (n-1)d]$

$\Rightarrow \quad -4 + (n-1)(3) = x$

$\Rightarrow \quad (n-1)3 = x + 4$

$$\Rightarrow \quad (n-1) = \frac{x+4}{3}$$

$$\Rightarrow \quad n = \frac{x+4}{3} + 1 = \frac{x+4+3}{3}$$

$$\Rightarrow \quad n = \frac{x+7}{3} \quad \ldots(i)$$

$$\therefore \quad S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow \quad S_n = \frac{(x+7)}{(2\times 3)}\left[2(-4)+\frac{(x+4)3}{3}\right] \quad \text{[Using (i)]}$$

$$= \frac{(x+7)}{6}[-8+x+4] \Rightarrow S_n = \frac{(x+7)(x-4)}{6}$$

But, $S_n = 437 \Rightarrow \frac{(x+7)(x-4)}{6} = 437$

$$\Rightarrow \quad x^2+3x-28 = 437\times 6$$

$$\Rightarrow \quad x^2+3x-28-2622 = 0$$

$$\Rightarrow \quad x^2+3x-2650 = 0$$

$$\Rightarrow \quad x^2+53x-50x-2650 = 0$$

$$\Rightarrow \quad x(x+53)-50(x+53) = 0$$

$$\Rightarrow \quad (x+53)(x-50) = 0$$

$$\Rightarrow \quad x = -53 \text{ or } x = 50$$

Rejecting the negative value $x = -53$, we have $x = 50$.

So, $x = 50$ is the required value as forward terms are positive.

Q9. Jaspal Singh repays his total loan of ₹ 118000 by paying every month starting with the first instalment of ₹ 1000. If he increases the instalment by ₹ 100 every month, what amount will be paid by him in the 30th instalment? What amount of loan does he still have to pay after the 30th instalment?

Sol. Monthly instalment paid by Jaspal Singh are 1000, 1100, 1200, ... 30 terms

$\therefore a = 1000, \quad d = 100, \quad a_n = ?, \quad n = 30$

$$\Rightarrow \quad a_n = a+(n-1)d = 1000+(30-1)\,100$$

$$= 100\,[10+29] = 3900$$

So, the amount paid by him in 30th instalment = ₹ 3900.

Total amount of all 30 instalments paid

$$= 1000+1100+1200+\ldots+3900$$

Here, $a = 1000, \quad d = 100, \quad n = 30$

$$\therefore \quad S_n = \frac{n}{2}[2a+(n-1)d] \Rightarrow S_{30} = \frac{30}{2}[2\times 1000+(30-1)100]$$

$$= 15\,[2000+2900]$$

$\Rightarrow \quad S_{30} = 15 \times 4900 = ₹\,73500$

So, the loan amount left after 30th instalment

$= ₹\,118000 - ₹73500 = ₹\,44500$

Hence, he has to pay ₹ 44500 after 30th instalment.

Q10. The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags.

Ruchi kept her books where the flags were stored. She could carry only one flag at a time. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag?

Sol. 27 flags are to be fixed at intervals of 2 m.

Position of the middle most flag $= \frac{27+1}{2}$th flag $= \frac{28}{2}$ th flag = 14th flag

This means that 13 flags are to be fixed before the middle most 14th flag and 13 flags are to be fixed after the 14th flag.

Distance between flags = 2 m

Distance covered by placing a first flag = 2 + 2 = 4 m

Distance covered to place IInd flag = 4 + 4 = 8 m

Distance covered to place IIIrd flag = 6 + 6 = 12 m

So, the total distance covered to place 13 flags on either side is given by

$$S_{13} = 4 + 8 + 12 + \ldots \text{ 13 terms}$$

Here, $a = 4, \quad d = 4, \quad n = 13$

$$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d] \quad \Rightarrow \quad S_{13} = \frac{13}{2}[2(4) + (13-1)(4)]$$

$$= \frac{13}{2}[8 + 48] = \frac{13}{2} \times 56 = 13 \times 28$$

$$\Rightarrow \quad S_{13} = 364$$

Distance covered by Ruchi for other side 13 flags = 364 m

Hence, the total distance to place 27 flags and pickup her books

$= 364 \times 2 = 728$ m

Maximum distance which she travelled carrying a flag = Distance covered in fixing Ist or 27th flag

$= (13 \times 2)$ m = 26 m.

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