

10

Constructions

EXERCISE 10.1

Choose the correct answer from the given four options:

Q1. To divide a line segment AB in the ratio 5 : 7, first a ray AX is drawn so that $\angle BAX$ is an acute angle and then at equal distances points are marked on the ray AX such that the minimum number of these points is

- (a) 8 (b) 10 (c) 11 (d) 12

Sol. (d): Minimum number of the points marked = $5 + 7 = 12$ verifies option (d).

Q2. To divide a line segment AB in ratio 4 : 7, a ray AX is drawn first such that $\angle BAX$ is an acute angle and then points A_1, A_2, A_3, \dots are located at equal distances on the ray AX and the point B is joined to

- (a) A_{12} (b) A_{11} (c) A_{10} (d) A_9

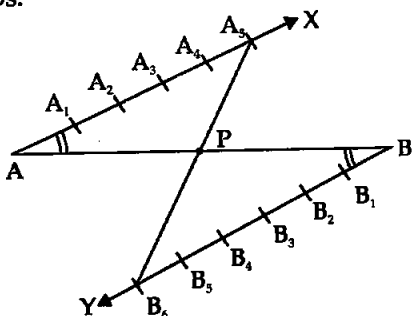
Sol. (b): We have to divide the constructed line into $7 + 4 = 11$ equal parts and 11th part will be joined to B. Verifies the option (b).

Q3. To divide a line segment AB in the ratio 5 : 6, draw a ray AX such that $\angle BAX$ is an acute angle, then draw a ray BY parallel to AX, and the points, A_1, A_2, A_3, \dots and B_1, B_2, B_3, \dots are located at equal distances on ray AX and BY, respectively. Then the points joined are

- (a) A_5 and B_6 (b) A_6 and B_5 (c) A_4 and B_5 (d) A_5 and B_4

Sol. (a): In the figure, segment AB of given length is divided into 2 parts of ratio 5 : 6 in following steps:

- Draw a line-segment AB of given length.
- Draw an acute angle BAX as shown in figure either up side or down side.
- Draw angle $\angle ABY = \angle BAX$ on other side of AX, i.e., down side.
- Divide AX into 5 equal parts by using compass.
- Divide BX into same distance in 6 equal parts as AX was divided.
- Now, join A_5 and B_6 which meet AB at P. P divides AB in ratio $AP : PB = 5 : 6$.



Q4. To construct a triangle similar to a given $\triangle ABC$ with its sides $\frac{3}{7}$ of the corresponding sides of $\triangle ABC$, first draw a ray BX such that $\angle CBX$

is an acute angle and X lies on the opposite side of A with respect to BC . Then locate points B_1, B_2, B_3, \dots on BX at equal distances and next step is to join

- (a) B_{10} to C (b) B_3 to C (c) B_7 to C (d) B_4 to C

Sol. (c): Here, ratio is $\frac{3}{7} < 1$ so resultant figure will be smaller than original so, last 7th part is to be joined to C , so that parallel line from third part of BX meet on BC without producing. So, verifies the option (c).

Q5. To construct a triangle similar to a given $\triangle ABC$ with its sides $\frac{8}{5}$ of the corresponding sides of $\triangle ABC$ draw a ray BX such that $\angle CBX$ is an acute angle and X is on the opposite side of A with respect to BC . The minimum number of points to be located at equal distances on the ray BX

- (a) 5 (b) 8 (c) 13 (d) 3

Sol. (b): To construct a triangle similar to a given triangle ABC with its sides $\frac{8}{5}$ of the corresponding sides of $\triangle ABC$, the minimum number of parts in which BX is divided in 8 equal parts. Verifies the option (b).

Q6. To draw a pair of tangents to a circle which are inclined to each other at an angle of 60° , it is required to draw tangents at end points of those two radii of the circle, the angle between them should be

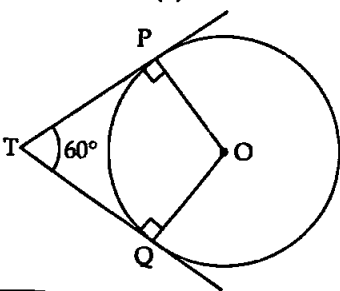
- (a) 135° (b) 90° (c) 60° (d) 120°

Sol. (d): We know that tangent and radius at contact point are perpendicular to each other.

So, $\angle P$ and $\angle Q$ in quadrilateral $TPOQ$ formed by tangents and radii will be of 90° each. So, the sum of $\angle T + \angle O = 180^\circ$ as $T = 60^\circ$ (Given)

$$\therefore \angle O = 180^\circ - 60^\circ = 120^\circ$$

Verifies the option (d).



EXERCISE 10.2

Write True or False and give reason for your answer in each of the following:

Q1. By geometrical construction, it is possible to divide a line segment in ratio $\sqrt{3} : \frac{1}{\sqrt{3}}$.

Sol. True: On multiplying or dividing a given ratio by a real number, the ratio remains same.

On multiplying the given ratio by $\sqrt{3}$ we get $\sqrt{3} \cdot \sqrt{3} : \frac{1}{\sqrt{3}} \cdot \sqrt{3}$ or $3 : 1$

Hence, the given ratio $\sqrt{3} : \frac{1}{\sqrt{3}}$ is possible to divide a line in ratio $3 : 1$ in

place of $\sqrt{3} : \frac{1}{\sqrt{3}}$.

Q2. To construct a triangle similar to a given ΔABC with its sides $\frac{7}{3}$ of the corresponding sides of ΔABC , draw a ray BX making acute angle with BC and X lies on the opposite side of A with respect to BC . The points B_1, B_2, \dots, B_7 are located at equal distances on BX , B_3 is joined to C and then a line segment B_6C' is drawn parallel to B_3C where C' lies on BC produced. Finally, the line segment $A'C'$ is drawn parallel to AC .

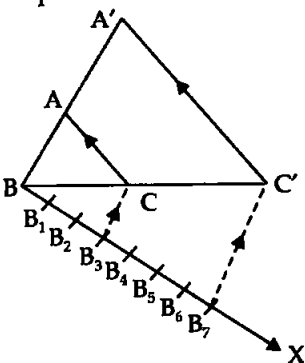
Sol. False: Given ratio is $\frac{7}{3} > 1$ so, the resulting triangle will be larger than given as $B_7C' \parallel B_3C$ and BX is equally divided into 7 parts as $(7 > 3)$.

Construction: (i) Draw given triangle with given specifications.

- (ii) Draw an acute angle CBX .
- (iii) Divide BX into 7 equal parts and mark them $B_1, B_2, B_3, \dots, B_7$.
- (iv) Produce BC and BA as shown in figure.
- (v) Join B_3C .
- (vi) Draw $B_7C' \parallel B_3C$, C' is on BC produced.
- (vii) Draw $C'A' \parallel AC$, A' on BA produced $\Delta A'BC'$ is required triangle i.e.,

$$\frac{\Delta A'BC'}{\Delta ABC} = \frac{3}{7}$$

Here, $B_7C' \parallel B_3C$. But in Question $B_6C' \parallel B_3C$, which is false.



Q3. A pair of tangents can be constructed from a point P to a circle of radius 3.5 cm situated at a distance of 3 cm from the centre.

Sol. False: Any tangent on a circle can be drawn only if the distance of point to draw tangent is equal to or more than radius of circle. Here, radius of circle is 3.5 cm and point is at 3 cm from centre which is inside the circle. So, no tangent can be drawn if point is inside the circle.

Q4. A pair of tangents can be constructed to a circle inclined at an angle of 170° .

Sol. True: A pair of tangents can be constructed if the angle between the tangents is between zero and less than 180° . Because the sum of angles between tangents and radii on tangent are supplementary.

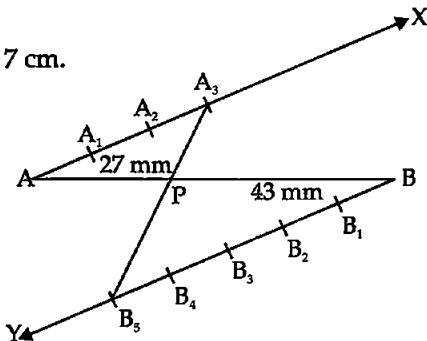
So, a pair of tangents can be constructed to circle inclined at an angle of 170° .

EXERCISE 10.3

Q1. Draw a line segment of length 7 cm. Find a point P on it which divides it in the ratio 3 : 5.

Sol. Steps of construction:

- (i) Draw a line-segment $AB = 7$ cm.
- (ii) Draw $AX \parallel BY$ such that $\angle A$ and $\angle B$ are acute angles.
- (iii) Divide AX and BY in 3 and 5 parts equally by compass and mark $A_1, A_2, A_3, B_1, B_2, B_3, B_4$ and B_5 respectively.
- (iv) Join A_3B_5 which intersect AB at P and divides $AP : PB = 3 : 5$.



Hence, P is the required point on AB which divide it in 3 : 5.

Verification (Justification): In $\triangle AA_3P$ and $\triangle BB_5P$

$AX \parallel BY$

[By construction]

$\angle A = \angle B$

[Alt. angles]

$\angle A_3PA = \angle B_5PB$

[Vertically opp. angles]

$\therefore \triangle AA_3P \sim \triangle BB_5P$

[By AA criterion of similarity]

$$\Rightarrow \frac{AA_3}{BB_5} = \frac{AP}{BP}$$

[Let each equal part = x cm]

$\therefore AA_1 = A_1A_2 = B_1B_2 \dots = x$

$$\Rightarrow \frac{3x}{5x} = \frac{AP}{BP}$$

$$\Rightarrow AP : BP = 3 : 5.$$

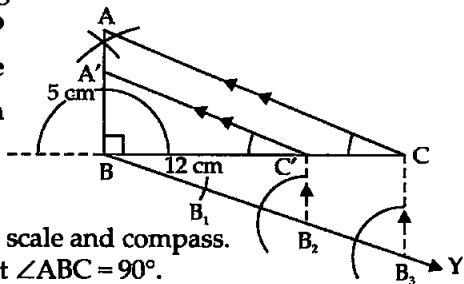
Hence, verified.

Q2. Draw a right angled $\triangle ABC$ in which $BC = 12$ cm, $AB = 5$ cm, and $\angle B = 90^\circ$. Construct a triangle similar to it and of scale factor $\frac{2}{3}$. Is the new triangle also a right triangle?

Sol. Here, scale factor or ratio factor is $\frac{2}{3} < 1$, so triangle to be constructed will be smaller than given $\triangle ABC$.

Steps of construction:

- (i) Draw $BC = 12$ cm.
- (ii) Draw $\angle CBA = 90^\circ$ with scale and compass.
- (iii) Cut $BA = 5$ cm such that $\angle ABC = 90^\circ$.
- (iv) Join AC . $\triangle ABC$ is the given triangle.
- (v) Draw an acute $\angle CBY$ such that A and Y are in opposite direction with respect to BC .



- (vi) Divide BY in 3 equal segments by marking arc at same distance at B_1, B_2 and B_3 .
- (vii) Join B_3C .
- (viii) Draw $B_2C' \parallel B_3C$ by making equal alternate angles at B_2 and B_3 .
- (ix) From point C' , draw $C'A' \parallel CA$ by making equal alternate angles at C and C' .

$\Delta A'BC'$ is the required triangle of scale factor $\frac{2}{3}$. This triangle is also a right triangle.

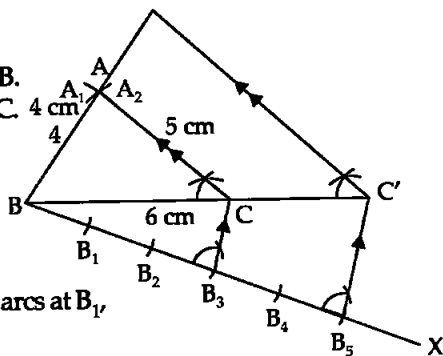
Q3. Draw a ΔABC in which $BC = 6$ cm, $CA = 5$ cm and $AB = 4$ cm.

Construct a triangle similar to it and of scale factor $\frac{5}{3}$.

Sol. Here, scale factor is $\frac{5}{3} > 1$, so the resulting figure will be larger.

Steps of construction:

- (i) Draw $BC = 6$ cm.
- (ii) Draw arc $BA_1 = 4$ cm from B.
- (iii) Draw arc $CA_2 = 5$ cm from C. A_1A_2 intersect at A.
- (iv) Join AB and AC.
- (v) Draw acute angle CBX below BC.
- (vi) Cut BX into equal parts by arcs at B_1, B_2, B_3, B_4 and B_5 .
- (vii) Join B_3C .
- (viii) Draw $B_2C' \parallel B_3C$ by making alternate angles. C' is on BC produced.
- (ix) Draw $C'A' \parallel CA$ which meet BA produced at A' . Now, $\Delta A'BC'$ is the required triangle.



Justification:

$$\Delta ABC \sim \Delta B_3CB_5$$

[By AA criterion of similarity]

$$\therefore \frac{BB_3}{BB_5} = \frac{BC}{BC'}$$

$$[BB_1 = B_1B_2 = \dots = x]$$

$$\therefore BB_3 = 3x \text{ and } BB_5 = 5x$$

$$\Rightarrow \frac{3x}{5x} = \frac{BC}{BC'}$$

$$\Delta ABC \sim \Delta A'BC'$$

[By AA criterion of similarity]

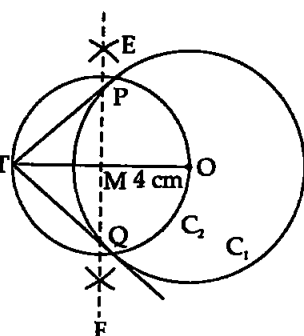
$$\Rightarrow \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC}{BC'}$$

Q4. Construct a pair of tangents to a circle of radius 4 cm from a point which is at a distance of 6 cm from the centre of circle.

Sol. The distance of point from which tangents to be drawn should be more than radius so that tangents can be drawn.

Steps of construction:

- (i) Draw a line-segment $OT = 6$ cm.
- (ii) Draw a circle of radius 4 cm taking O as centre.
- (iii) Draw perpendicular bisector EF of OT which meets OT at M .
- (iv) Taking MT as radius and M as centre draw a circle C_2 which intersect C_1 at P and Q . Join TP and TQ . Then, TP and TQ are the required tangents.

**EXERCISE 10.4**

Q1. Two line-segments AB and AC include an angle of 60° , where $AB = 5$ cm and $AC = 7$ cm. Locate points P and Q on AB and AC respectively such that $AP = \frac{3}{4} AB$ and $AQ = \frac{1}{4} AC$. Join P and Q and measure the length PQ .

Sol. (i) Draw $\angle BAC = 60^\circ$ such that $AB = 5$ cm and $AC = 7$ cm.

(ii) Draw acute angle CAX and mark X_1, X_2, X_3 and X_4 equally spaced.

(iii) Join X_4C .

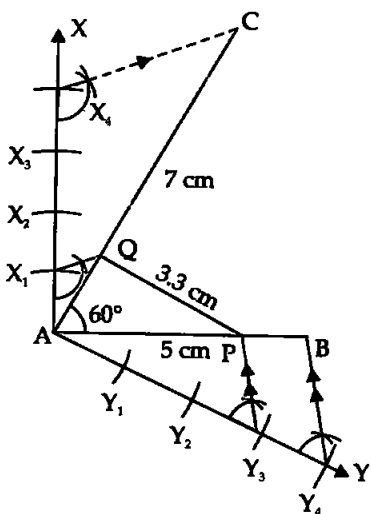
(iv) Draw $X_1Q \parallel X_4C$.

(v) Similarly, draw $\angle BAY$ and divide AY in 4 equal parts i.e., Y_1, Y_2, Y_3 and Y_4 .

(vi) Join Y_4B and draw $Y_3P \parallel Y_4B$.

(vii) Join PQ and measure it.

(viii) PQ is equal to 3.3 cm.



Q2. Draw a parallelogram $ABCD$ in which $BC = 5$ cm, $AB = 3$ cm and $\angle ABC = 60^\circ$. Divide it into triangles BCD and $\triangle ABD$, by diagonal BD . Construct the triangle $BD'C'$ similar to $\triangle BDC$ with scale factor $\frac{4}{3}$. Draw the line segment $D'A'$ parallel to DA ,

where A' lies on extended side BA . Is $A'BC'D'$ a parallelogram?

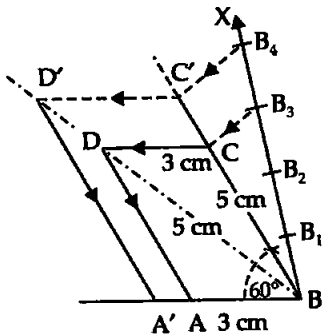
Sol. Steps of construction:

(i) Draw a line segment $AB = 3$ cm.

(ii) Make $\angle ABC = 60^\circ$ such that $BC = 5$ cm.

(iii) Draw $CD \parallel AB$ and $AD \parallel BC$, $\square ABCD$ is the required parallelogram.

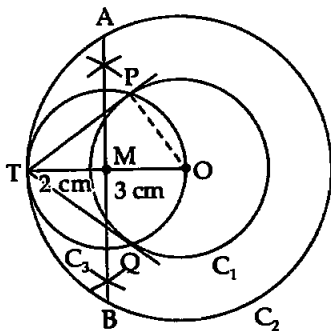
- (iv) Join diagonal BD and produce it.
- (v) Make acute angle CBX on opposite of D with respect to BC.
- (vi) Mark (equi spaced) B_1, B_2, B_3, B_4 by compass.
- (vii) Join B_3C and draw $B_3C \parallel B_4C'$ on BC produced.
- (viii) Again, draw $C'D' \parallel CD$, where D' is on BD produced.
- (ix) Now, draw $D'A' \parallel DA$ where A' is on BA produced. Parallelogram $A'B'C'D'$ is similar to parallelogram ABCD with scale factor $\frac{4}{3}$.



Q3. Draw two concentric circles of radii 3 cm and 5 cm. Taking a point on outer circle construct the pair of tangents to the other. Measure the length of a tangent and verify it by actual calculation.

Sol. Steps of construction:

- (i) Draw two concentric circles C_1, C_2 of radii 3 cm and 5 cm respectively taking 'O' as centre.
- (ii) Draw perpendicular bisector AB of OT. T is any point on C_2 .
- (iii) Draw circle C_3 taking radius $TM = OM$ and M as centre.
- (iv) Circle C_3 intersect the circle C_1 at P and Q. Join TP and TQ. These are the required tangents. $TP = TQ = 4.1$ cm by measuring.



Mathematically length of tangent: Join OP. OP and TP are radius and tangent respectively at contact point P. So, $\angle TPO = 90^\circ$.

By Pythagoras theorem in ΔTPO ,

$$PT^2 = OT^2 - OP^2 = 5^2 - 3^2 = 25 - 9 = 16$$

$$\Rightarrow PT = 4 \text{ cm}$$

Difference in measurement and by mathematical calculation

$$PT = 4.1 \text{ cm} - 4 \text{ cm} = 0.1 \text{ cm.}$$

Q4. Draw an isosceles ΔABC in which $AB = AC = 6$ cm and $BC = 5$ cm. Construct a triangle PQR similar to ΔABC in which $PQ = 8$ cm. Also justify the construction.

Sol. We have to draw

$$\Delta PQR \sim \Delta ABC$$

$PQ = 8 \text{ cm}$

$\therefore \frac{PQ}{AB} = \frac{8}{6} = \frac{4}{3} \quad (\because AB = 6 \text{ cm})$

So, $PQ = QR = 8 \text{ cm}$

So, we have to draw $\Delta PQR \sim \Delta ABC$ with scale

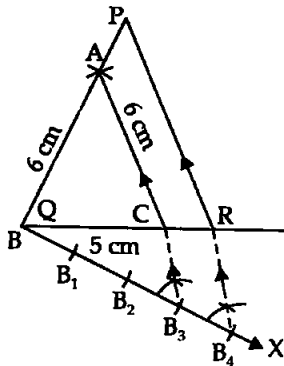
factor $\frac{4}{3} > 1$ resulting ΔPQR will be larger than

ΔABC .

Steps of Construction:

- (i) Draw $BC = 5 \text{ cm}$
- (ii) Draw two arcs of 6 cm each from B and C in same direction let it be upside.
- (iii) Join AB and AC .
- (iv) Draw acute $\angle CBX$ and mark B, B_1, B_2, B_3, B_4 with compass.
- (v) Join B_3C and draw $B_4R \parallel B_3C$, R is on BC produced.
- (vi) Again, draw $RP \parallel CA$, P is on BA produced.

Therefore, $\Delta PQR \sim \Delta ABC$ with $PQ = PR = 8 \text{ cm}$. It's scale factor is $\frac{4}{3}$.



Q5. Draw a ΔABC in which $AB = 5 \text{ cm}$, $BC = 6 \text{ cm}$ and $\angle ABC = 60^\circ$.

Construct a triangle similar to ΔABC with scale factor $\frac{5}{7}$. Justify the construction.

Sol. Scale factor $\frac{5}{7} < 1$, so the resulting Δ will be smaller than ΔABC .

Steps of construction:

- (i) Draw $AB = 5 \text{ cm}$.
- (ii) Draw $\angle ABC = 60^\circ$, cut $BC = 6 \text{ cm}$ and join AC .
- (iii) Draw acute $\angle BAX$ and mark it equispaced marks A_1, A_2, \dots, A_7 as shown in figure.
- (iv) Join A_7B and draw $A_5B' \parallel A_7B$. B' is on segment AB .

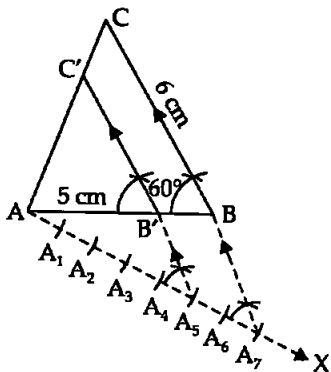
Draw $B'C' \parallel BC$, point C' is on AC .

$\Delta AB'C' \sim \Delta ABC$ with scale factor $\frac{5}{7}$.

Justification: In $\Delta AA_5B'$ and ΔAA_7B ,
 $A_7B \parallel A_5B'$

$\therefore \angle A_5 = \angle A_7$
 $\angle BAA_5 = \angle BAA_7$

[Corresponding \angle s]
 [Common]



$\therefore \Delta AA_5B' \sim \Delta AA_7B$ [By AA criterion of similarity]

$$\Rightarrow \frac{AB'}{AB} = \frac{AA_5}{AA_7} = \frac{5x}{7x} = \frac{5}{7} \dots(i)$$

where $x = AA_1 = A_1A_2 = \dots A_6A_7$

Similarly, $\Delta AB'C' \sim \Delta ABC$ [By AA criterion of similarity]

$$\Rightarrow \frac{AB'}{AB} = \frac{AC'}{AC} = \frac{B'C'}{BC}$$

$$\Rightarrow \frac{5}{7} = \frac{AC'}{AC} = \frac{B'C'}{BC}$$

Hence, $\Delta AB'C' \sim \Delta ABC$ with scale factor $\frac{5}{7}$.

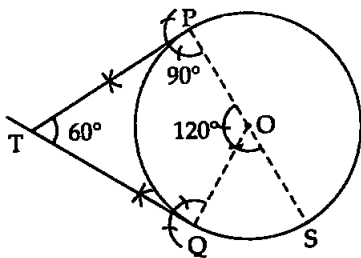
Q6. Draw a circle of radius 4 cm. Construct a pair of tangents to it, the angle between which is 60° . Also, justify the construction. Measure the distance between the centre of the circle and the point of intersection of tangents.

Sol. Angle between tangents is 60° . So angles between their radii is $180^\circ - 60^\circ = 120^\circ$.

As the angles between tangents and their corresponding radii are supplementary.

Steps of construction:

- (i) Draw a circle of radius 4 cm.
- (ii) Draw any diameter POS.
- (iii) Draw OQ making $\angle AOC = 120^\circ$.
- (iv) Draw tangent at P by drawing $\angle OPT = 90^\circ$.
- (v) Similarly, draw $\angle OQT$ equal to 90° to draw tangent.
- (vi) Both PT, QT tangents intersect at T and make angle of 60° .



Hence, the two tangents on circle are TP and TQ inclined at 60° .

Justification: Because the radius OP and tangent PT at contact point makes angle $\angle TPO = 90^\circ$.

Similarly, $\angle TQO = 90^\circ$

In quadrilateral TPOQ,

$$\angle T + \angle P + \angle O + \angle Q = 360^\circ$$

$$\Rightarrow \angle T + 90^\circ + 120^\circ + 90^\circ = 360^\circ$$

$$\Rightarrow \angle T = 360^\circ - 300^\circ$$

$$\Rightarrow \angle T = 60^\circ.$$

[$\because \angle O = 120^\circ$ by construction]

Hence, verified.

Q7. Draw a ΔABC in which $AB = 4$ cm, $BC = 6$ cm, and $AC = 9$ cm.

Construct a triangle similar to ΔABC with scale factor $\frac{3}{2}$. Justify the

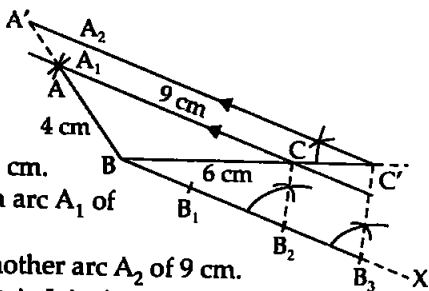
construction. Are the two triangles congruent? Note that, all the three angles and two sides of the two triangles are equal.

Sol. Scale factor $\frac{3}{2} > 1$

So, the resulting figure will be greater than $\triangle ABC$.

Steps of construction:

- (i) Draw line segment $BC = 6$ cm.
- (ii) From B as centre, draw an arc A_1 of 6 cm.
- (iii) From C as centre, draw another arc A_2 of 9 cm.
- (iv) Arcs A_1 and A_2 intersect at A. Join A to B and C.
- (v) Make an acute angle of $\angle CBX$ on other side of A.
- (vi) Make the equispaced marks B_1, B_2, B_3 with compass.
- (vii) Join B_2C and draw $B_3C' \parallel B_2C$, where C' is on BC produced.
- (viii) Draw $CA \parallel C'A'$, where A' is on BA produced.



$$\therefore \triangle A'BC' \sim \triangle ABC \text{ with scale factor } \frac{3}{2}.$$

Justification: In $\triangle BB_3C'$ and $\triangle BB_2C$

$$\angle B = \angle B$$

[Common]

$$B_3C' \parallel B_2C$$

[By construction]

$$\therefore \angle BB_2C = \angle BB_3C'$$

[Corresponding angles]

$$\therefore \triangle BB_3C' \sim \triangle BB_2C$$

[By AA criterion of similarity]

$$\Rightarrow \frac{BC'}{CB} = \frac{BB_3}{BB_2} = \frac{3x}{2x} = \frac{3}{2}$$

$$[\because BB_1 = B_1B_2 = B_2B_3 = x]$$

$$\Rightarrow \frac{BC'}{BC} = \frac{3}{2}$$

In $\triangle ABC$ and $\triangle A'BC'$,

$$\angle B = \angle B$$

[Common]

$$\therefore A'C' \parallel AC$$

$$\therefore \angle A'CB = \angle ACB$$

[Corresponding angles]

$$\therefore \triangle ABC \sim \triangle A'BC'$$

[By AA criterion of similarity]

$$\Rightarrow \frac{A'C'}{AC} = \frac{A'B}{AB} = \frac{C'B}{BC}$$

$$\Rightarrow \frac{A'C'}{AC} = \frac{A'B}{AB} = \frac{3}{2}$$

Hence, proved.

□□□