

# 1



## Real Numbers

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### Lesson at a Glance

1. Numbers are divided into two parts: real numbers and imaginary numbers.
2. Real numbers can be divided into two parts: rationals and irrationals.
3. Any rational number can be put in the form  $\frac{P}{Q}$ , where  $P$  and  $Q$  are integers with  $Q \neq 0$ . But irrational number cannot be put in this form.
4. Every integer is a rational number.
5. Zero (0) is a rational number.
6.  $\pi$  is irrational as  $\frac{22}{7}$  is not an exact value of  $\pi$ .
7. Square root of every prime number is irrational.
8.  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{101}$  are examples of irrationals.
9. If a prime  $p$  divides  $a^2$ , then  $p$  divides  $a$ , where  $a$  is a positive integer.
10. Each of prime numbers has only two factors: one factor is 1 and other one is number itself.
11. Two numbers are called coprime if they have no common factor other than 1. *e.g.*, 2 and 5.
12. Rational numbers have either a terminating decimal expansion or a non-terminating repeating (recurring) decimal expansion.
13. The denominator of a rational number having terminating decimal expansion has only powers of 2, or powers of 5, or both.
14.  $0.\overline{428571}$ ,  $-21.\overline{037}$  and 23.3408 are examples of rationals.

15. The sum or difference of a rational and an irrational is irrational.
16. The product or quotient of a non-zero rational and an irrational is irrational.
17. If  $a$  and  $b$  be two positive integers, then  

$$a \times b = \text{HCF}(a, b) \times \text{LCM}(a, b).$$
18. Every composite number has more than two factors.
19. Every composite number can be expressed as a product of primes.
20. 0.120012000120..... is an irrational number.
21. If two positive numbers  $a$  and  $b$  are given, then there exist two unique whole numbers  $q$  and  $r$  satisfying the relation  

$$a = b \times q + r; 0 \leq r < b.$$

## TEXTBOOK QUESTIONS SOLVED

### Exercise 1.1 (Page – 7)

1. Use Euclid's division algorithm to find the HCF of:

(i) 135 and 225

(ii) 196 and 38220

(iii) 867 and 255

**Sol.** (i) Applying Euclid's division algorithm on 135 and 225, to get

$$225 = 1 \times 135 + 90$$

Again, applying Euclid's division algorithm on 90 and 135, to get

$$135 = 1 \times 90 + 45$$

Again, applying Euclid's division algorithm on 45 and 90, to get

$$90 = 2 \times 45 + 0.$$

Now  $\text{HCF}(225, 135) = \text{HCF}(135, 90) = \text{HCF}(90, 45) = 45.$

(ii) Applying Euclid's division algorithm on 196 and 38220, to get

$$38220 = 195 \times 196 + 0$$

$\therefore \text{HCF}(38220, 196) = 196.$

(iii) Applying Euclid's division algorithm on 867 and 225, to get

$$867 = 3 \times 255 + 102$$

Again, applying Euclid's division algorithm on 102 and 255, to get

$$255 = 2 \times 102 + 51$$

Again, applying Euclid's division algorithm on 51 and 102, to get

$$102 = 2 \times 51 + 0$$

Now  $\text{HCF}(867, 255) = \text{HCF}(255, 102)$

$$= \text{HCF}(102, 51) = 51.$$

2. Show that any positive odd integer is of the form  $6q + 1$ , or  $6q + 3$ , or  $6q + 5$ , where  $q$  is some integer.

**Sol.** Consider  $a$ , a positive integer, we apply division algorithm with  $q$  and  $b = 6$ ,  $0 \leq r < 6$  such that

$$a = 6q + r$$

As  $0 \leq r < 6$ . Therefore, possible remainders are 0, 1, 2, 3, 4, 5.

Number  $a$  can be  $6q$ ,  $6q + 1$ ,  $6q + 2$ ,  $6q + 3$ ,  $6q + 4$  or  $6q + 5$ .

$$6q = 2(3q) = 2m; \quad 6q + 1 = 2(3q) + 1 = 2m + 1;$$

$$6q + 2 = 2(3q + 1) = 2n; \quad 6q + 3 = 2(3q + 1) + 1 = 2n + 1;$$

$$6q + 4 = 2(3q + 2) = 2t; \quad 6q + 5 = 2(3q + 2) + 1 = 2t + 1.$$

We note  $6q$ ,  $6q + 2$  and  $6q + 4$  are of the form  $2r$ ,  $r \in \mathbb{N}$ , which are even numbers; and  $6q + 1$ ,  $6q + 3$  and  $6q + 5$  are of the form  $2r + 1$ ,  $r \in \mathbb{N}$ , which are odd numbers.

Hence,  $6q + 1$ ,  $6q + 3$  and  $6q + 5$  are positive and odd integers.

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

**Sol.** To find maximum number of columns, we have to find HCF of 616 and 32.

Using Euclid's division algorithm, we have

$$616 = 19 \times 32 + 8$$

Now applying Euclid's division algorithm on 32 and 8, we have

$$32 = 4 \times 8 + 0$$

$$\therefore \text{HCF}(616, 32) = \text{HCF}(32, 8) = 8$$

Hence, maximum number of columns is 8.

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form  $3m$  or  $3m + 1$  for some integer  $m$ .

[Hint: Let  $x$  be any positive integer then it is of the form  $3q$ ,  $3q + 1$  or  $3q + 2$ . Now square each of these and show that they can be rewritten in the form  $3m$  or  $3m + 1$ .]

**Sol.** Consider positive integer  $x$ , with  $q$  and  $b = 3$ ,  $0 \leq r < 3$   
 $x$  can be of the form  $3q$ ,  $3q + 1$  and  $3q + 2$  because  
 $r = 0, 1, 2$

There exist three cases:

**Case I:**  $x = 3q$

$$\Rightarrow x^2 = (3q)^2 = 9q^2 = 3m, m = 3q^2 \quad \dots(i)$$

**Case II:**  $x = 3q + 1$

$$\begin{aligned} \Rightarrow x^2 &= (3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 \\ &= 3m + 1, m = 3q^2 + 2q \quad \dots(ii) \end{aligned}$$

**Case III:**  $x = 3q + 2$

$$\begin{aligned} \Rightarrow x^2 &= (3q + 2)^2 = 9q^2 + 12q + 4 \\ &= 3(3q^2 + 4q + 1) + 1 \\ &= 3m + 1, m = 3q^2 + 4q + 1 \quad \dots(iii) \end{aligned}$$

Hence, from (i), (ii) and (iii), the square of any positive integer is either of the form  $3m$  or  $3m + 1$ .

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form  $9m$ ,  $9m + 1$  or  $9m + 8$ .

**Sol.** Consider positive integer  $a$ , with  $q$  and  $b = 3$ ,  $0 \leq r < 3$ .  
 $a$  can be of the form  $3q$ ,  $3q + 1$  or  $3q + 2$  because  $r = 0, 1, 2$ .

Consider  $a = 3q$

$$\Rightarrow a^3 = 27q^3 = 9(3q^3) = 9m, m = 3q^3.$$

Consider  $a = 3q + 1$

$$\begin{aligned} \Rightarrow a^3 &= (3q + 1)^3 = 27q^3 + 27q^2 + 9q + 1 \\ &= 9(3q^3 + 3q^2 + q) + 1 = 9m + 1, \end{aligned}$$

$$m = 3q^3 + 3q^2 + q$$

And consider  $a = 3q + 2$

$$\begin{aligned} \Rightarrow a^3 &= (3q + 2)^3 = 27q^3 + 54q^2 + 36q + 8 \\ &= 9(3q^3 + 6q^2 + 4q) + 8 = 9m + 8, \\ m &= 3q^3 + 6q^2 + 4q \end{aligned}$$

Hence, cube of any positive integer is of the form  $9m, 9m + 1$  or  $9m + 8$ .

**Exercise 1.2 (Page – 11)**

1. Express each number as a product of its prime factors:

(i) 140

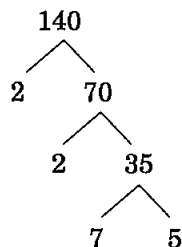
(ii) 156

(iii) 3825

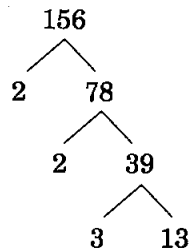
(iv) 5005

(v) 7429

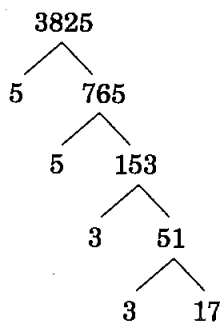
Sol. (i)  $140 = 2 \times 2 \times 7 \times 5 = 2^2 \times 5 \times 7$ .



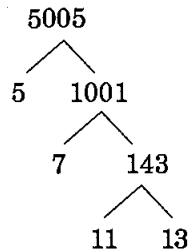
(ii)  $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$ .



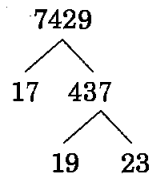
(iii)  $3825 = 5 \times 5 \times 3 \times 3 \times 17$   
 $= 5^2 \times 3^2 \times 17$



(iv)  $5005 = 5 \times 7 \times 11 \times 13$ .



(v)  $7429 = 17 \times 19 \times 23$ .



2. Find the LCM and HCF of the following pairs of integers and verify that  $LCM \times HCF = \text{product of the two numbers}$ .

- (i) 26 and 91      (ii) 510 and 92      (iii) 336 and 54

**Sol.** (i)  $26 = 2 \times 13$ ,  $91 = 7 \times 13$

$\therefore$  HCF = 13 and LCM =  $2 \times 7 \times 13 = 182$

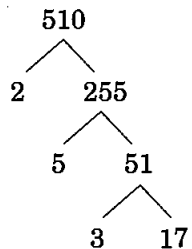
Now, HCF  $\times$  LCM =  $13 \times 182 = 2366$  ...*(i)*

Product of the numbers =  $26 \times 91 = 2366$  ...*(ii)*

From *(i)* and *(ii)*, we get the result.

(ii)  $510 = 2 \times 5 \times 3 \times 17$  ...*(i)*

$92 = 2^2 \times 23$  ...*(ii)*



From *(i)* and *(ii)*, we have

HCF = 2

LCM =  $2^2 \times 5 \times 3 \times 17 \times 23 = 23460$

Now, HCF  $\times$  LCM

=  $2 \times 23460 = 46920$  ...*(iii)*

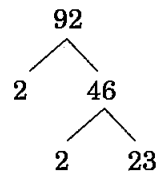
Product of the numbers =  $510 \times 92$

= 46920 ...*(iv)*

From *(iii)* and *(iv)*, we get the result.

(iii)  $336 = 2^4 \times 3 \times 7$  ...*(i)*

$54 = 2 \times 3^3$  ...*(ii)*



From (i) and (ii), we get

$$\text{HCF} = 2 \times 3 = 6$$

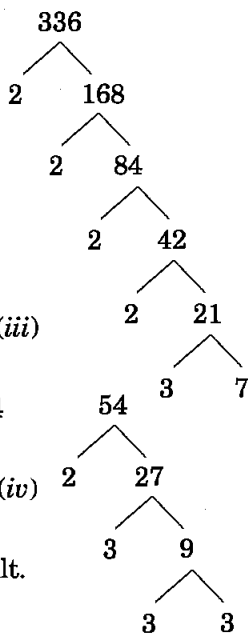
$$\begin{aligned} \text{LCM} &= 2^4 \times 3^3 \times 7 = 16 \times 27 \times 7 \\ &= 3024 \end{aligned}$$

$$\begin{aligned} \text{Now, HCF} \times \text{LCM} &= 6 \times 3024 \\ &= 18144 \quad \dots(iii) \end{aligned}$$

$$\text{Product of the numbers} = 336 \times 54$$

$$= 18144 \quad \dots(iv)$$

From (iii) and (iv), we get the result.



3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

- (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Sol.

$$\begin{aligned} (i) \quad 12 &= 2^2 \times 3 \\ 15 &= 3 \times 5 \\ 21 &= 3 \times 7 \end{aligned}$$

$$\therefore \text{HCF} (12, 15, 21) = 3;$$

$$\text{LCM} (12, 15, 21) = 2^2 \times 3 \times 5 \times 7 = 420.$$

$$\begin{aligned} (ii) \quad 17 &= 1 \times 17 \\ 23 &= 1 \times 23 \\ 29 &= 1 \times 29 \end{aligned}$$

$$\therefore \text{HCF} (17, 23, 29) = 1,$$

$$\text{LCM} (17, 23, 29) = 1 \times 17 \times 23 \times 29 = 11339.$$

$$\begin{aligned} (iii) \quad 8 &= 2^3 \\ 9 &= 3^2 \\ 25 &= 5^2 \end{aligned}$$

$$\therefore \text{HCF} (8, 9, 25) = 1,$$

$$\text{LCM} (8, 9, 25) = 2^3 \times 3^2 \times 5^2 = 1800.$$

4. Given that  $HCF(306, 657) = 9$ , find  $LCM(306, 657)$ .

**Sol.** We know for two numbers  $a$  and  $b$   $HCF(a, b) \times LCM(a, b) = a \times b$

$$\begin{aligned} \therefore LCM(306, 657) &= \frac{306 \times 657}{HCF(306, 657)} \\ &= \frac{306 \times 657}{9} = 22338. \end{aligned}$$

5. Check whether  $6^n$  can end with the digit 0 for any natural number  $n$ .

**Sol.** If  $6^n$  for  $n \in N$ , ends with digit zero, then

$$6^n = 5 \times m + 0, \text{ for some } m \in N.$$

$\Rightarrow$  Prime factorisation of  $6^n$  contains 5 as a prime factor.

Only prime factorisation of 6 is 3 and 2 and by Uniqueness of Fundamental Theorem of Arithmetic, 5 is not a prime factor.

Therefore, it contradicts our supposition that 5 is a factor of  $6^n$ . Hence, 5 is not a factor of  $6^n$ , i.e.,  $6^n$  does not end with digit zero.

6. Explain why  $7 \times 11 \times 13 + 13$  and  $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$  are composite numbers.

**Sol.** Composite numbers have more than two factors.

Number  $7 \times 11 \times 13 + 13 = 13(7 \times 11 + 1)$  has factors  $7 \times 11 + 1$ , 1 and 13. As it has more than 2 factors it is a composite number, therefore, the number is a composite number.

Similarly,  $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5(7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$\Rightarrow$  The number has factors 1, 5 and  $(7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$ .

As it has more than 2 factors, therefore, the number is a composite number.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?



**Sol.** If we have to find after how many minutes they again meet, we have to find the least common multiple of 18 and 12.

$$\text{Now, } 18 = 2 \times 3^2 \text{ and } 12 = 2^2 \times 3$$

$$\therefore \text{LCM} = 2^2 \times 3^2 = 4 \times 9 = 36.$$

Hence, after 36 minutes they will meet again at the starting point.

### Exercise 1.3 (Page – 14)

1. Prove that  $\sqrt{5}$  is irrational.

**Sol.** Let, if possible,  $\sqrt{5}$  is a rational number.

$$\therefore \sqrt{5} = \frac{p}{q}, p, q \in \mathbb{Z}, q \neq 0; p \text{ and } q \text{ are coprime.}$$

$$\Rightarrow p^2 = 5q^2 \quad \dots(i)$$

$$\Rightarrow 5 \text{ divides } p^2 \quad \Rightarrow 5 \text{ divides } p.$$

$$\text{Let } p = 5m, m \in \mathbb{Z}. \quad \dots(ii)$$

$$\therefore (5m)^2 = 5q^2 \quad \Rightarrow q^2 = 5m^2 \quad [\text{From (i)}]$$

$$\Rightarrow 5 \text{ divides } q^2 \quad \Rightarrow 5 \text{ divides } q$$

$$\therefore q = 5n, n \in \mathbb{Z} \quad \dots(iii)$$

From (ii) and (iii), we notice

$p$  and  $q$  have 5 as common factor.

$\therefore p$  and  $q$  are not co-prime.

Hence, our supposition,  $\sqrt{5}$  is rational, is wrong.

Hence,  $\sqrt{5}$  is an irrational number.

2. Prove that  $3 + 2\sqrt{5}$  is irrational.

**Sol.** Let if possible,  $3 + 2\sqrt{5}$  is a rational number.

$$\therefore 3 + 2\sqrt{5} = \frac{a}{b}, a, b \in \mathbb{Z}, b \neq 0; a \text{ and } b \text{ are coprime.}$$

$$\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3 = \frac{a - 3b}{b}$$

$$\Rightarrow \sqrt{5} = \frac{a-3b}{2b} \quad \dots(i)$$

From (i), we notice R.H.S. is a rational number, using properties of rational numbers.

$\Rightarrow$  L.H.S., i.e.,  $\sqrt{5}$  is also a rational number which is not true, as  $\sqrt{5}$  is an irrational number. Hence, our supposition is wrong. Hence,  $3 + 2\sqrt{5}$  is an irrational number.

3. Prove that the following are irrationals:

$$(i) \frac{1}{\sqrt{2}} \qquad (ii) 7\sqrt{5} \qquad (iii) 6 + \sqrt{2}$$

**Sol.** (i) Let, if possible,  $\frac{1}{\sqrt{2}}$  is a rational number.

$$\therefore \frac{1}{\sqrt{2}} = \frac{a}{b}, a, b \in \mathbb{Z}; a, b \neq 0.$$

$$\Rightarrow \sqrt{2} = \frac{b}{a} \quad \dots(i)$$

From equation (i), we notice RHS is a rational number but LHS, i.e.,  $\sqrt{2}$  is not rational. This contradicts the fact that  $\sqrt{2}$  is an irrational. So, our supposition is wrong.

Hence,  $\frac{1}{\sqrt{2}}$  is an irrational number.

(ii) Let, if possible,  $7\sqrt{5}$  is a rational number.

$$\therefore 7\sqrt{5} = \frac{a}{b}, a, b \in \mathbb{Z}; b \neq 0.$$

$$\Rightarrow \sqrt{5} = \frac{a}{7b}. \text{ Now LHS is an irrational number,}$$

whereas R.H.S. is a rational number. Therefore, LHS  $\neq$  RHS So, our supposition is wrong.

Hence,  $7\sqrt{5}$  is an irrational number.

(iii) Let, if possible,  $6 + \sqrt{2}$  is a rational number.

$$\therefore 6 + \sqrt{2} = \frac{a}{b}, a, b \in \mathbb{Z}, b \neq 0.$$

$\Rightarrow \sqrt{2} = \frac{a}{b} - 6 = \frac{a-6b}{b}$ . Now LHS is an irrational number and RHS is a rational number using properties of rational numbers. Therefore,  $LHS \neq RHS$ . So, our supposition is wrong.

Hence,  $6 + \sqrt{2}$  is an irrational number.

**Exercise 1.4 (Page – 17-18)**

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i)  $\frac{13}{3125}$

(ii)  $\frac{17}{8}$

(iii)  $\frac{64}{455}$

(iv)  $\frac{15}{1600}$

(v)  $\frac{29}{343}$

(vi)  $\frac{23}{2^3 5^2}$

(vii)  $\frac{129}{2^2 5^7 7^6}$

(viii)  $\frac{6}{15}$

(ix)  $\frac{35}{50}$

(x)  $\frac{77}{210}$

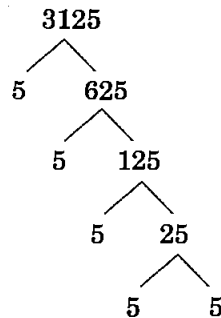
**Sol.** (i)  $\frac{13}{3125} = \frac{13}{5^5}$

As denominator can be represented as power of prime factor 5. Hence,

$\frac{13}{3125}$  has terminating decimal expansion.

(ii)  $\frac{17}{8} = \frac{17}{2^3}$

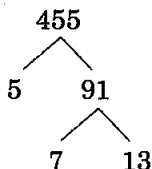
As denominator has factors of the form  $2^m 5^n$ .



Hence,  $\frac{17}{8}$  has terminating decimal expansion.

$$(iii) \frac{64}{455} = \frac{64}{5 \times 7 \times 13}$$

As denominator cannot be represented in the form  $2^m 5^n$ ,  $m, n \in \mathbb{N}$ .



Hence,  $\frac{64}{455}$  has non-terminating but repeating decimal expansion.

$$(iv) \frac{15}{1600} = \frac{3}{320} = \frac{3}{2^6 5^1}$$

As denominator has factors of the form  $2^m 5^n$ . Hence,

$\frac{15}{1600}$  has terminating decimal expansion.

$$(v) \frac{29}{343} = \frac{29}{7^3}$$

As denominator has factors other than 2 and 5. Hence,

$\frac{29}{343}$  has a non-terminating but repeating decimal expansion.

(vi)  $\frac{23}{2^3 5^2}$  is a terminating decimal expansion. As denominator is of the form  $2^m 5^n$ ,  $m, n \in \mathbb{N}$ .

(vii)  $\frac{129}{2^2 5^7 7^6}$  has a non-terminating but a repeating decimal expansion. As denominator has factors other than 2 and 5.

$$(viii) \frac{6}{15} = \frac{2}{5}$$

As denominator has factors of the term  $2^m 5^n$ . Hence,

$\frac{6}{15}$  has terminating decimal expansion.

$$(ix) \frac{35}{50} = \frac{7}{10} = \frac{7}{2 \times 5}$$

As denominator has factors of the form  $2^m 5^n$ . Hence,

$\frac{35}{50}$  has terminating decimal expansion.

$$(x) \frac{77}{210} = \frac{77}{2 \times 3 \times 5 \times 7} = \frac{11}{2 \times 3 \times 5}$$

As denominator has factors other than 2 and 5. Therefore, it has non-terminating but repeating decimal expansion.

2. Write down the decimal expansions of those rational numbers in Question 1 above which have a terminating decimal expansions.

**Sol.** (i)  $\frac{13}{3125} = 0.00416$

(ii)  $\frac{17}{8} = 2.125$

(iv)  $\frac{15}{1600} = 0.009375$

(vi)  $\frac{23}{2^3 5^2} = \frac{23}{200} = 0.115$

(viii)  $\frac{6}{15} = 0.4$

(ix)  $\frac{35}{50} = \frac{7}{10} = 0.7$ .

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not.

If they are rational, and of the form  $\frac{p}{q}$ , what can you say

about the prime factors of  $q$ ?

(i)  $43.123456789$       (ii)  $0.120120012000120000\dots$

(iii)  $43.\overline{123456789}$

**Sol.** (i)  $43.123456789$  is rational. It is terminating decimal, so

it can be represented as  $\frac{p}{q}$ ,  $p, q \in \mathbb{Z}$ ,  $q \neq 0$  and denominator will be of form  $2^m 5^n$ ,  $m, n \in \mathbb{N}$ .

(ii)  $0.120120012000\dots$  is not rational. It has a non-terminating, non-repeating decimal expansion. It cannot

be represented in the form  $\frac{p}{q}$ .

(iii)  $43.\overline{123456789}$  is rational. It has non-terminating but repeating decimal expansion. It can be represented in

the form  $\frac{p}{q}$ ,  $q \neq 0$  but denominator cannot be represented in the form  $2^m 5^n$ ,  $m, n \in \mathbb{N}$ . Such denominator will have factors other than 2 and 5 also.

