3 Pair of Linear Equations in Two Variables

Lesson at a Glance

1. A pair of two linear equations is either consistent or inconsistent.
2. Consistency provides either a unique solution or infinitely many solutions.
3. The two lines represented by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

   (*i*) intersect, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

   In this case, the pair of equations is consistent with unique solution.

   (*ii*) coincident, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

   In this case, the pair of equations is consistent with infinitely many solutions.

   (*iii*) are parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

   In this case, the pair of equations is inconsistent and has no solution.
4. A pair of linear equations in two variables can be solved by graphical method or algebraic method.
5. Algebraic method provides the better results than graphical method.

TEXTBOOK QUESTIONS SOLVED

Exercise 3.1 *(Page – 44)*

1. *Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall*

be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Sol. Let Aftab's present age = x years

Daughter's present age = y years

We have,

$$x - 7 = 7(y - 7) \quad ...(i)$$

And $$x + 3 = 3(y + 3) \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$x - 7y = -42 \quad ...(iii)$$

$$x - 3y = 6 \quad ...(iv)$$

The system of equations (*iii*) and (*iv*) represents the situation algebraically.

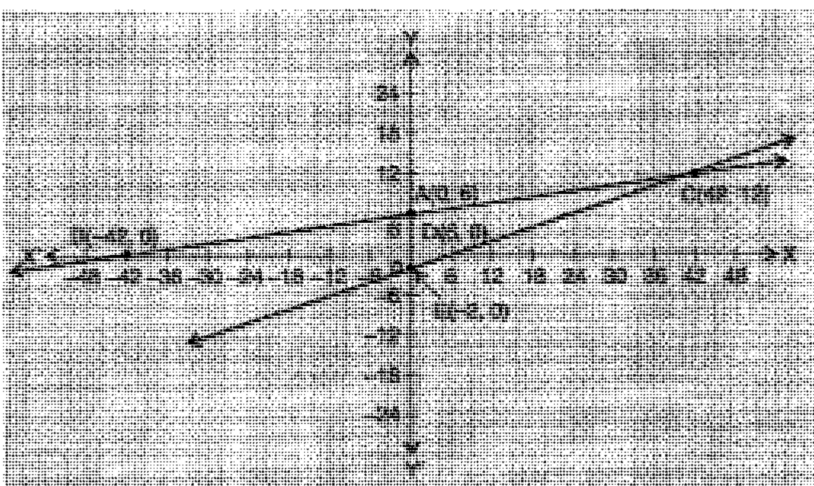
Take some points on line (*iii*) and (*iv*) and plot on the graph. Two lines will meet at (42, 12).

Some points on line (*iii*) are:

x	0	– 42	42
y	6	0	12

Some points on line (*iv*) are:

x	0	6	42
y	– 2	0	12



To represent the given situation graphically, we plot the points A(0, 6), B(– 42, 0) and C(42, 12) to get the line AB and the points C(42, 12), D(6, 0) and E(0, – 2) to get the line CD as shown:

2. *The coach of a cricket team buys 3 bats and 6 balls for ₹ 3900. Later, she buys another bat and 3 more balls of the same kind for ₹ 1300. Represent this situation algebraically and geometrically.*

Sol. Let cost of a bat = ₹ x and cost of a ball = ₹ y.

Therefore, algebraic situation is the system of equations as given below:

$$3x + 6y = 3900 \text{ and } x + 3y = 1300 \quad ...(i)$$

i.e., $$x + 2y = 1300 \text{ and } x + 3y = 1300 \quad ...(ii)$$

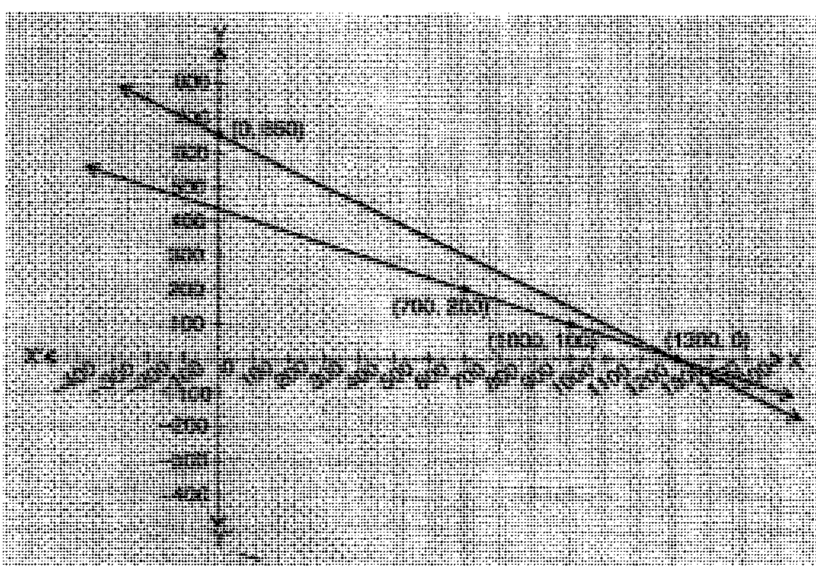
Take some points on line (*i*) and (*ii*) and plot on the graph.

Some points on line $3x + 6y = 3900$ are:

x	0	1300
y	650	0

Some points on line $x + 3y = 1300$ are:

x	1000	700
y	100	200



3. *The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹ 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹ 300. Represent the situation algebraically and geometrically.*

Sol. Let cost of 1 kg of apples = ₹ x

and cost of 1 kg of grapes = ₹ y

According to the given condition,

$$2x + y = 160 \quad \text{...}(i)$$

and $$4x + 2y = 300 \quad \text{...}(ii)$$

The system of equations (*i*) and (*ii*) represents the situation algebraically.

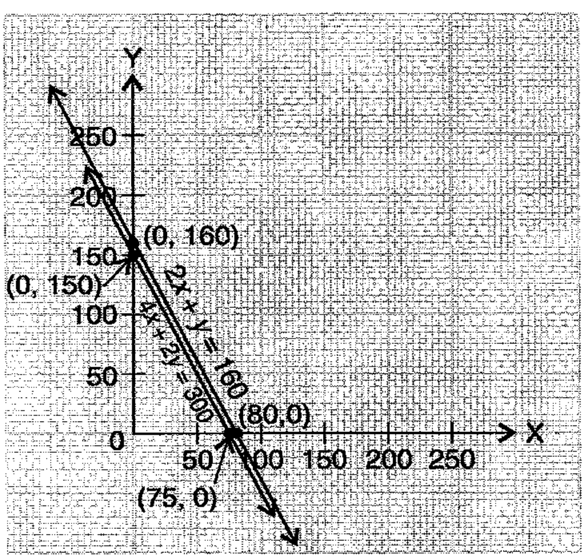
Some points on line (*i*) are:

x	80	0	40
y	0	160	80

Some points on line (*ii*) are:

x	75	0	50
y	0	150	50

Plotting these points on the graph we notice that the two equations represent the parallel lines.



Exercise 3.2 *(Page – 49-50)*

1. *Form the pair of linear equations in the following problems, and find their solutions graphically.*

(i) *10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.*

(ii) *5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.*

Sol. (i) Let number of boys = x and number of girls = y

We have, $x + y = 10$ ...(i)

and $y = x + 4$ ...(ii)

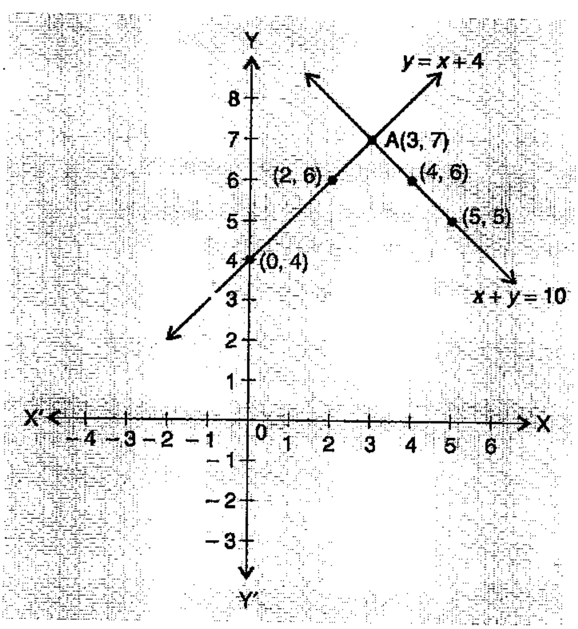
Let us plot the two equations on the graph.

Some points on line (i) are:

x	4	3	5
y	6	7	5

Some points on line (ii) are:

x	0	3	2
y	4	7	6



The two lines meet at A (3, 7), *i.e.*, $x = 3, y = 7$

Hence, number of boys = 3, number of girls = 7.

(*ii*) Let cost of one pencil = ₹ x

and cost of one pen = ₹ y

According to the given condition, we have

$$5x + 7y = 50 \quad ...(i)$$

and

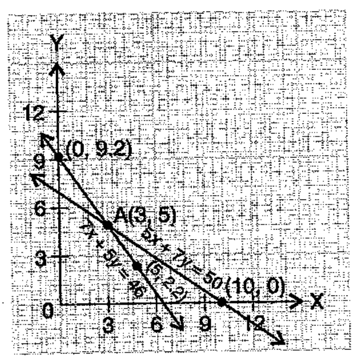
$$7x + 5y = 46 \quad ...(ii)$$

Some points on line (*i*) are:

x	10	3
y	0	5

Some points on line (*ii*) are:

x	0	5
y	9.2	2.2



Plotting the two equations on the graph we find that two lines intersect at the point A(3, 5), *i.e.*, $x = 3$, $y = 5$.

Hence, cost of one pencil = ₹ 3

and cost of one pen = ₹ 5.

2. *On comparing the ratios* $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ *and* $\frac{c_1}{c_2}$, *find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:*

(*i*) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

(*ii*) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

(*iii*) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Sol. (*i*) Consider equation $5x - 4y + 8 = 0$, here, $a_1 = 5$,

$$b_1 = -4,\ c_1 = 8$$

and $7x + 6y - 9 = 0$, here, $a_2 = 7$, $b_2 = 6$, $c_2 = -9$

Clearly, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, *i.e.*, $\frac{5}{7} \neq \frac{-4}{6} \neq \frac{8}{-9}$

Hence, lines representing the equations intersect at a point.

(*ii*) Consider equation $9x + 3y + 12 = 0$, here,

$$a_1 = 9,\ b_1 = 3,\ c_1 = 12$$

and $18x + 6y + 24 = 0$,

here, $a_2 = 18$, $b_2 = 6$, $c_2 = 24$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$,

i.e., $\frac{9}{18} = \frac{3}{6} = \frac{12}{24} = \frac{1}{2}$

Hence, two lines representing the equations are coincident.

(*iii*) Consider equation $6x - 3y + 10 = 0$,

here, $a_1 = 6$, $b_1 = -3$, $c_1 = 10$

and $2x - y + 9 = 0$, here, $a_2 = 2$, $b_2 = -1$, $c_2 = 9$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, *i.e.*, $\frac{6}{2} = \frac{-3}{-1} \neq \frac{10}{9}$

Hence, two lines representing the equations are parallel.

3. *On comparing the ratios* $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ *and* $\frac{c_1}{c_2}$, *find out whether the following pair of linear equations are consistent, or inconsistent.*

(*i*) $3x + 2y = 5;\ 2x - 3y = 7$

(*ii*) $2x - 3y = 8;\ 4x - 6y = 9$

(*iii*) $\frac{3}{2}x + \frac{5}{3}y = 7;\ 9x - 10y = 14$

(iv) $5x - 3y = 11; -10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8; 2x + 3y = 12$

Sol. (i) Consider equation $3x + 2y = 5$ or $3x + 2y - 5 = 0$

Here, $a_1 = 3, b_1 = 2, c_1 = -5$

And $2x - 3y = 7$ or $2x - 3y - 7 = 0$

Here, $a_2 = 2, b_2 = -3, c_2 = -7$

Clearly, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, i.e., $\frac{3}{2} \neq \frac{2}{-3} \neq \frac{-5}{-7}$

Hence, the pair of linear equations is consistent.

(ii) Consider equation $2x - 3y = 8$ or $2x - 3y - 8 = 0$

Here, $a_1 = 2, b_1 = -3, c_1 = -8$

And $4x - 6y = 9$ or $4x - 6y - 9 = 0$

Here, $a_2 = 4, b_2 = -6, c_2 = -9$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, i.e., $\frac{2}{4} = \frac{-3}{-6} \neq \frac{-8}{-9}$

Hence, the pair of linear equations is inconsistent.

(iii) Consider equation $\frac{3}{2}x + \frac{5}{3}y = 7$

or $9x + 10y - 42 = 0$

Here, $a_1 = 9, b_1 = 10, c_1 = -42$

And $9x - 10y = 14$ or $9x - 10y - 14 = 0$

Here, $a_2 = 9, b_2 = -10, c_2 = -14$

Clearly, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, i.e., $\frac{9}{9} \neq \frac{10}{-10} \neq \frac{-42}{-14}$

Hence, the pair of linear equations is consistent.

(iv) Consider equations $5x - 3y = 11$

or $5x - 3y - 11 = 0$

Here, $a_1 = 5, b_1 = -3, c_1 = -11$

And $-10x + 6y = -22$ or $-10x + 6y + 22 = 0$

Here, $a_2 = -10, b_2 = 6, c_2 = 22$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, *i.e.*, $\frac{5}{-10} = \frac{-3}{6} = \frac{-11}{22}$

Hence, the pair of linear equations is dependent (consistent).

(*v*) Consider equations $\frac{4}{3}x + 2y = 8$

or $4x + 6y - 24 = 0$

Here, $a_1 = 4, b_1 = 6, c_1 = -24$

And $2x + 3y = 12$ or $2x + 3y - 12 = 0$

Here, $a_2 = 2, b_2 = 3, c_2 = -12$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, *i.e.*, $\frac{4}{2} = \frac{6}{3} = \frac{-24}{-12}$

Hence, the pair of linear equations is dependent (consistent).

4. *Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:*

(*i*) $x + y = 5, 2x + 2y = 10$

(*ii*) $x - y = 8, 3x - 3y = 16$

(*iii*) $2x + y - 6 = 0, 4x - 2y - 4 = 0$

(*iv*) $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$

Sol. (*i*) Consider equations

$x + y = 5 \Rightarrow a_1 = 1, \quad b_1 = 1, \quad c_1 = -5$

and $2x + 2y = 10 \Rightarrow a_2 = 2, \quad b_2 = 2, \quad c_2 = -10$

We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$.

Therefore, the pair of linear equations is consistent.

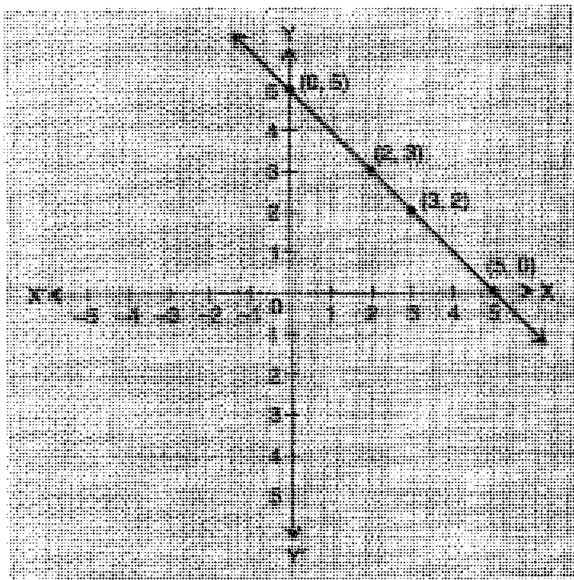
Let us draw the graph to obtain the solution.

Some points for $x + y = 5$ are:

x	0	5	2
y	5	0	3

Some points for $2x + 2y = 10$ are:

x	2	3	0
y	3	2	5



From the graph, the lines are overlapping.

Hence, the pair of linear equations has infinite number of solutions, *e.g.*, $x = 2, y = 3$ and $x = 0, y = 5$.

(*ii*) Consider equations

$$x - y = 8 \Rightarrow a_1 = 1, \quad b_1 = -1, \quad c_1 = -8$$

and $3x - 3y = 16 \Rightarrow a_2 = 3, \quad b_2 = -3, \quad c_2 = -16$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Hence, the pair of equations is inconsistent.

(*iii*) Consider equation $2x + y - 6 = 0$, here, $a_1 = 2, b_1 = 1, c_1 = -6$

and $4x - 2y - 4 = 0$, here, $a_2 = 4, b_2 = -2, c_2 = -4$

We notice $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

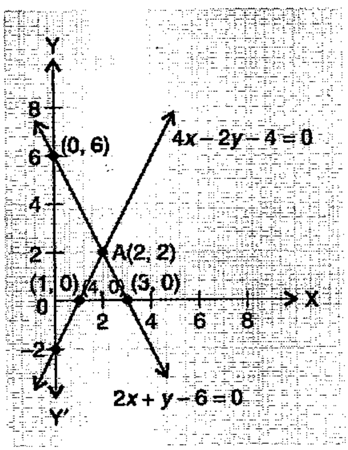
Hence, the pair of linear equations is consistent and has a unique solution.

Some points for $2x + y - 6 = 0$ are:

x	0	2	3
y	6	2	0

Some points for $4x - 2y - 4 = 0$ are:

x	0	1	3
y	– 2	0	4



Plotting the two equations on the graph, we notice two lines meeting at A(2, 2), *i.e.*, $x = 2, y = 2$

Hence, solution is $x = 2, y = 2$.

(*iv*) Consider equation $2x - 2y - 2 = 0$,

here, $a_1 = 2,\ b_1 = -2,\ c_1 = -2$

and $4x - 4y - 5 = 0$,

here, $a_2 = 4,\ b_2 = -4,\ c_2 = -5$

We notice $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the pair of linear equations is inconsistent and so, it has no solution.

5. *Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.*

Sol. Let length of the garden = x m

and breadth = y m; half the perimeter = 36 m

We have $x + y = 36$ ...(*i*)

and $x = y + 4$ ...(*ii*)

Solving (*i*) and (*ii*), we get $x = 20$, $y = 16$

Therefore, length and breadth of the garden are 20 m and 16 m respectively.

6. *Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:*

(*i*) *intersecting lines* (*ii*) *parallel lines*

(*iii*) *coincident lines*

Sol. Given linear equation: $2x + 3y - 8 = 0$ ...(*i*)

(*i*) If the geometrical representation is intersecting lines, then

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Consider the equation $5x + 3y - 1 = 0$

Hence, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, *i.e.*, $\frac{2}{5} \neq \frac{3}{3}$ [From (*i*)]

(*ii*) If the geometrical representation is parallel lines, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Consider the equation $4x + 6y + 1 = 0$.

Hence, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, *i.e.*, $\frac{2}{4} = \frac{3}{6} \neq \frac{-8}{1}$ [From (*i*)]

(*iii*) If the geometrical representation is coincident lines, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

Consider the equation $4x + 6y - 16 = 0$

Hence, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, *i.e.*, $\frac{2}{4} = \frac{3}{6} = \frac{-8}{-16} = \frac{1}{2}$.

7. ***Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.***

Sol. Given equation: $x - y + 1 = 0$

$\Rightarrow \quad y = x + 1$

Some points on graph are:

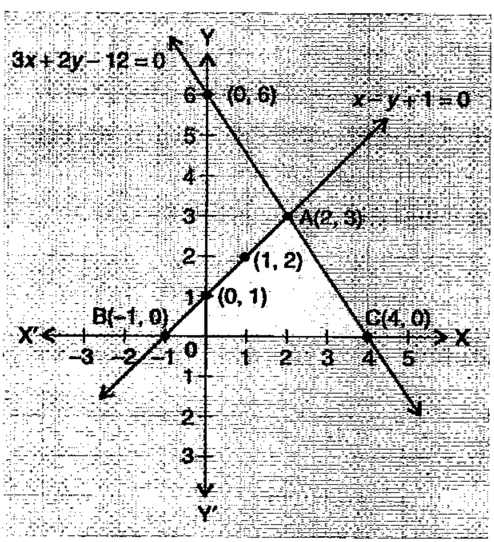
x	0	1	2
y	1	2	3

Another given equation : $3x + 2y - 12 = 0$

$\Rightarrow \quad y = \frac{12 - 3x}{2}$

Some points on graph are:

x	0	2	4
y	6	3	0



On plotting these points on a graph paper, we get the coordinates of the vertices of a triangle ABC are A(2, 3), B(– 1, 0), C(4, 0).

Exercise 3.3 *(Page – 53-54)*

1. *Solve the following pair of linear equations by the substitution method.*

(*i*) $x + y = 14$ (*ii*) $s - t = 3$

$x - y = 4$ $\frac{s}{3} + \frac{t}{2} = 6$

(*iii*) $3x - y = 3$ (*iv*) $0.2x + 0.3y = 1.3$

$9x - 3y = 9$ $0.4x + 0.5y = 2.3$

(*v*) $\sqrt{2}x + \sqrt{3}y = 0$ (*vi*) $\frac{3x}{2} - \frac{5y}{3} = -2$

$\sqrt{3}x - \sqrt{8}y = 0$ $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Sol. (*i*) Consider equations: $x + y = 14$ ...(*i*)

and $x - y = 4$ ...(*ii*)

From (*ii*), we have, $x = y + 4$

Substituting for x in (*i*), we get

$y + 4 + y = 14 \Rightarrow 2y = 10 \Rightarrow y = 5$

Substituting this value of y in (*i*), we get $x = 9$

$\therefore$ Solution is $x = 9$, $y = 5$.

(*ii*) Consider equations $s - t = 3$ ...(*i*)

and $\frac{s}{3} + \frac{t}{2} = 6$ ...(*ii*)

Substituting value of s from (*i*) in (*ii*), we get

$$\frac{t+3}{3} + \frac{t}{2} = 6 \Rightarrow 2t + 6 + 3t = 36$$

$$\Rightarrow 5t = 30 \Rightarrow t = 6$$

Substituting $t = 6$ in (*i*), we get $s = 9$

H... solution is $s = 9$, $t = 6$.

(*iii*) Consider equations $3x - y = 3$ ...(*i*)

and $9x - 3y = 9$ ...(*ii*)

From (*i*), we get

$$y = 3x - 3$$

Substituting for y in (*ii*), we get

$$9x - 3(3x - 3) = 9 \Rightarrow 9x - 9x + 9 = 9$$

$\Rightarrow$ $9 = 9$, which is always true.

Hence, the pair of linear equations represents infinite number of solutions. One such solution is $x = 1, y = 0$.

General solution is $y = 3x - 3$, where x can take any real number value.

(*iv*) Consider equation $0.2x + 0.3y = 1.3$

$$\Rightarrow \quad y = \frac{1.3 - 0.2x}{0.3} \quad ...(i)$$

Substituting this value for y in equation $0.4x + 0.5y = 2.3$, we get

$$0.4x + 0.5\left(\frac{1.3 - 0.2x}{0.3}\right) = 2.3$$

$$\Rightarrow \quad 0.12x + 0.65 - 0.1x = 0.69$$

$$\Rightarrow \quad 0.02x = 0.04 \Rightarrow x = 2$$

Substituting $x = 2$ in (*i*), we get

$$y = \frac{1.3 - 0.4}{0.3} = \frac{0.9}{0.3} = 3$$

Therefore, solution is $x = 2, \; y = 3$.

(*v*) Consider equation $\sqrt{2}\,x + \sqrt{3}\,y = 0$

$$\Rightarrow \quad y = -\frac{\sqrt{2}}{\sqrt{3}}x \quad ...(i)$$

Substituting this value for y in equation

$\sqrt{3}\,x - \sqrt{8}\,y = 0$, we get

$$\sqrt{3}\,x - \sqrt{8}\,.\left(\frac{-\sqrt{2}}{\sqrt{3}}x\right) = 0 \Rightarrow 3x + 4x = 0$$

$$\Rightarrow \quad 7x = 0 \Rightarrow x = 0.$$

Substituting this value of x in (*i*), we get $y = 0$
Hence, solution is $x = 0, y = 0$.

(*vi*) Consider equations $\frac{3x}{2} - \frac{5y}{3} = -2$ ...(*i*)

and $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ ...(*ii*)

From (*ii*), we get

$$y = 2\left(\frac{13}{6} - \frac{x}{3}\right) \quad ...(iii)$$

Substituting this value of y in (*i*), we get

$$\frac{3x}{2} - \frac{5}{3} \times 2\left(\frac{13}{6} - \frac{x}{3}\right) = -2$$

$$\Rightarrow \quad \frac{3x}{2} - \frac{65}{9} + \frac{10}{9}x = -2$$

$$\Rightarrow \quad 27x - 130 + 20x = -36$$

$\Rightarrow \quad 47x = 94 \quad \Rightarrow \quad x = 2$

Substituting the value of x in (*iii*), we get

$$y = 2\left(\frac{13}{6} - \frac{2}{3}\right) = 2\left(\frac{9}{6}\right) = 3$$

Hence, solution is $x = 2, y = 3$.

2. *Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.*

Sol. Consider equation $2x + 3y = 11 \quad \Rightarrow \quad y = \frac{11 - 2x}{3}$ ...(*i*)

Substituting this value for y in equation $2x - 4y = -24$, we get

$$2x - 4\left(\frac{11 - 2x}{3}\right) = -24$$

$$\Rightarrow \quad 6x - 44 + 8x = -72$$

$$\Rightarrow \quad 14x = -28$$

$$\Rightarrow \quad x = -2$$

Substituting this value of x in (*i*), we get

$$y = \frac{11 + 4}{3} = \frac{15}{3} = 5$$

Hence solution is $x = -2, y = 5$.

Substituting the values of x and y in $y = mx + 3$, we get $m = -1$.

3. *Form the pair of linear equations for the following problems and find their solution by substitution method.*

(i) *The difference between two numbers is 26 and one number is three times the other. Find them.*

(ii) *The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.*

(iii) *The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.*

(iv) *The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?*

(v) *A fraction becomes* $\frac{9}{11}$, *if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and denominator, it becomes* $\frac{5}{6}$. *Find the fraction.*

(vi) *Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?*

Sol. (i) Let numbers be x and y, $(x > y)$.

According to given conditions,

$$x - y = 26 \quad ...(i)$$

$$x = 3y \quad ...(ii)$$

From (i) and (ii), we get

$$3y - y = 26 \Rightarrow 2y = 26 \Rightarrow y = 13.$$

From (ii), we get $x = 39$.

Thus, numbers are 39 and 13.

(*ii*) Let larger angle = x and smaller angle = y

We have $x + y = 180°$ and $x - y = 18°$

Solving, we get $x = 99°$, $y = 81°$.

Hence, larger angle = 99°, smaller angle = 81°.

(*iii*) Let cost of each bat = ₹ x

and cost of each ball = ₹ y

According to the given conditions, we get

$$7x + 6y = 3800 \quad ...(i)$$

$$3x + 5y = 1750 \quad ...(ii)$$

From (*ii*), we have $y = \dfrac{1750 - 3x}{5}$ ...(*iii*)

Substituting this value for y in (*i*), we get

$$7x + 6\left(\frac{1750 - 3x}{5}\right) = 3800$$

$\Rightarrow \quad 35x + 10500 - 18x = 19000$

$\Rightarrow \quad 17x = 8500 \quad \Rightarrow \quad x = 500$

Substituting this value of x in (*iii*), we get

$$y = \frac{1750 - 1500}{5} = \frac{250}{5} = 50$$

Hence, cost of each bat = ₹ 500

and cost of each ball = ₹ 50.

(*iv*) Let fixed charges be ₹ x and charge per km = ₹ y.

We have $\left.\begin{array}{l} x + 10y = 105 \\ x + 15y = 155 \end{array}\right\}$ Solving, we get $x = 5$, $y = 10$

Hence, fixed charge = ₹ 5 and charge per km = ₹ 10.

Charge of a distance of 25 km = ₹ (5 + 25 × 10) = ₹ 255.

(*v*) Let fraction be $\dfrac{x}{y}$

According to the given conditions,

we have $\dfrac{x + 2}{y + 2} = \dfrac{9}{11} \Rightarrow 11x - 9y = -4$ ...(*i*) and $\frac{x + 3}{y + 3} = \frac{5}{6} \Rightarrow 6x - 5y = -3$ ...(ii)

Solving (i) and (ii), we get $x = 7, y = 9$

Hence, the required fraction is $\frac{7}{9}$.

(vi) Let Jacob's present age = x years, son's present age = y years

According to the given conditions,

we have $x + 5 = 3(y + 5)$, i.e., $x - 3y = 10$ ...(i)

and $x - 5 = 7(y - 5)$, i.e., $x - 7y = -30$ ...(ii)

Solving equations (i) and (ii) for x and y, we obtain

$$x = 40, y = 10.$$

Hence, Jacob's and son's present ages are 40 years and 10 years respectively.

Exercise 3.4 *(Page – 56-57)*

1. *Solve the following pair of linear equations by the elimination method and the substitution method:*

(i) $x + y = 5$ *and* $2x - 3y = 4$

(ii) $3x + 4y = 10$ *and* $2x - 2y = 2$

(iii) $3x - 5y - 4 = 0$ *and* $9x = 2y + 7$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ *and* $x - \frac{y}{3} = 3$.

Sol. (i) **Elimination Method:**

Consider $x + y = 5$ ...(i)

and $2x - 3y = 4$ ...(ii)

Multiplying (i) by 3 and adding (ii) to the result, we get

$$3x + 3y = 15$$
$$2x - 3y = 4$$
$$5x = 19$$

$\Rightarrow$ $x = \frac{19}{5}$

Substituting the value of x in (i), we get

$$y = 5 - \frac{19}{5} = \frac{6}{5}$$

Hence, the solution is $x = \frac{19}{5}, y = \frac{6}{5}$.

Substitution Method:

Consider $x + y = 5$ ...(i)

and $2x - 3y = 4$ ...(ii)

From equation (i), we have $y = 5 - x$

Substituting this value of y in equation (ii), we get

$$2x - 3(5 - x) = 4 \Rightarrow 5x = 19$$

$$\Rightarrow \quad x = \frac{19}{5}$$

Substituting this value of x in equation (i), we get

$$\frac{19}{5} + y = 5 \Rightarrow y = 5 - \frac{19}{5} = \frac{6}{5}$$

Hence, the solution is $x = \frac{19}{5}, y = \frac{6}{5}$.

(ii) **Elimination Method:** Consider equations

$3x + 4y = 10$ ...(i)

and $2x - 2y = 2$ ...(ii)

Multiplying (ii) by 2 and adding the result to (i), we get

$$\begin{array}{r} 3x + 4y = 10 \\ 4x - 4y = 4 \\ \hline 7x = 14 \end{array}$$

$$\Rightarrow \quad x = 2$$

Substituting this value for x in (i), we get

$$6 + 4y = 10 \Rightarrow 4y = 4 \Rightarrow y = 1$$

Hence, the solution is $x = 2, y = 1$.

(iii) **Elimination Method:** Consider equations

$3x - 5y - 4 = 0$ ...(i)

and $9x = 2y + 7$ ...(ii)

Multiplying (i) by 3 and subtracting (ii) from the result, we get

$$\begin{array}{r} 9x - 15y - 12 = 0 \\ 9x - 2y - 7 = 0 \\ - \quad + \quad + \\ \hline -13y - 5 = 0 \end{array}$$

$$\Rightarrow \qquad y = -\frac{5}{13}$$

Substituting this value of y in (i), we get

$$3x + \frac{25}{13} - 4 = 0 \Rightarrow 3x = 4 - \frac{25}{13} = \frac{27}{13}$$

$$\Rightarrow \qquad x = \frac{9}{13}$$

Hence, the solution is $x = \frac{9}{13}, y = -\frac{5}{13}$.

(*iv*) **Elimination Method:**

Consider equations

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$\Rightarrow \quad 3x + 4y = -6 \qquad ...(i)$$

and $\quad x - \frac{y}{3} = 3$

$$\Rightarrow \quad 3x - y = 9 \qquad ...(ii)$$

Subtracting (ii) from (i), we get

$$\begin{array}{r} 3x + 4y = -6 \\ 3x - y = 9 \\ - \quad + \qquad - \\ \hline 5y = -15 \end{array}$$

$$\Rightarrow \qquad y = -3$$

Substituting $y = -3$ in (ii), we get

$$3x + 3 = 9 \Rightarrow 3x = 6 \Rightarrow x = 2$$

Hence, the solution is $x = 2, y = -3$.

Substitution Method:

Given equation can be represented by

$$3x + 4y = -6 \qquad ...(i)$$

and $3x - y = 9$ ...(*ii*)

From equation (*ii*), we have

$$y = 3x - 9$$

Substituting this value of y in (*i*), we get

$$3x + 4(3x - 9) = -6$$

$\Rightarrow \quad 3x + 12x = -6 + 36 \quad \Rightarrow \quad 15x = 30 \quad \Rightarrow \quad x = 2$

Substituting this value of x in equation (*ii*), we get

$$6 - y = 9 \quad \Rightarrow \quad y = -3$$

Hence, the required solution is $x = 2, y = -3$.

2. *Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:*

   (*i*) *If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes* $\frac{1}{2}$ *if we only add 1 to the denominator. What is the fraction?*

   (*ii*) *Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?*

   (*iii*) *The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.*

   (*iv*) *Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.*

   (*v*) *A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.*

Sol. (*i*) Let fraction be $\frac{x}{y}$

According to given conditions,

$$\frac{x+1}{y-1} = 1 \quad \Rightarrow \quad x + 1 = y - 1$$

$\Rightarrow \quad x - y = -2$ ...(i)

and $\dfrac{x}{y+1} = \dfrac{1}{2} \quad \Rightarrow \quad 2x = y + 1$

$\Rightarrow \quad 2x - y = 1$ ...(ii)

Solving (i) and (ii), we get

$x = 3, y = 5$

Hence, required fraction is $\dfrac{3}{5}$.

(ii) Let present age of Nuri = x years

and present age of Sonu = y years

Five years ago,

Nuri's age = $(x - 5)$ years and Sonu's age

= $(y - 5)$ years

$\therefore \quad x - 5 = 3(y - 5) \quad \Rightarrow \quad x - 3y = -10$ ...(i)

Ten years later,

Nuri's age = $(x + 10)$ years and Sonu's age

= $(y + 10)$ years

$\therefore \quad x + 10 = 2(y + 10) \quad \Rightarrow \quad x - 2y = 10$ ...(ii)

Solving (i) and (ii), we get

$x = 50, y = 20$

Hence, Nuri's age = 50 years and Sonu's age

= 20 years.

(iii) Let unit's digit be y and ten's digit be x.

Then number is $10x + y$.

And number obtained on reversing the digits is

$10y + x$

Given $\quad x + y = 9$ ...(i)

and $9(10x + y) = 2(10y + x) \Rightarrow 90x + 9y = 20y + 2x$

$\Rightarrow \quad 88x - 11y = 0 \quad \Rightarrow \quad 8x - y = 0$ ...(ii)

Adding (i) and (ii), we get

$x + y + 8x - y = 9 + 0 \quad \Rightarrow \quad 9x = 9 \quad \Rightarrow \quad x = 1$

Substituting the value of x in (i), we get $y = 8$

Hence, number is 18.

(*iv*) Let the number of ₹ 50 notes be x and that of ₹ 100 notes be y

We have $x + y = 25$ ...(*i*)

and $50x + 100y = 2000 \Rightarrow x + 2y = 40$ ...(*ii*)

Subtracting (*i*) from (*ii*), we get

$$x + 2y - x - y = 40 - 25 \Rightarrow y = 15$$

Substituting the value of y in (*i*), we get $x = 10$.

∴ The number of ₹ 50 notes = 10

and that of ₹ 100 notes = 15.

(*v*) Let fixed charge for first three days = ₹ x

and additional charge per day thereafter = ₹ y

We have $x + 4y = 27$ ...(*i*)

$x + 2y = 21$ ...(*ii*)

Subtracting (*ii*) from (*i*), we get

$$x + 4y - x - 2y = 27 - 21$$

$\Rightarrow$ $2y = 6 \Rightarrow y = 3$.

Substituting the value for y in (*i*), we get $x = 15$

Hence, fixed charge for first three days = ₹ 15 and additional charge per day thereafter = ₹ 3.

Exercise 3.5 *(Page – 62-63)*

1. *Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross-multiplication method.*

(*i*) $x - 3y - 3 = 0$
$3x - 9y - 2 = 0$

(*ii*) $2x + y = 5$
$3x + 2y = 8$

(*iii*) $3x - 5y = 20$
$6x - 10y = 40$

(*iv*) $x - 3y - 7 = 0$
$3x - 3y - 15 = 0$

Sol. (*i*) Consider equations $x - 3y - 3 = 0$

$\Rightarrow a_1 = 1, b_1 = -3, c_1 = -3$

and $3x - 9y - 2 = 0, \Rightarrow a_2 = 3, b_2 = -9, c_2 = -2$

We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. Inconsistent, no solution.

(*ii*) Consider equations $2x + y = 5$ or $2x + y - 5 = 0$

$\Rightarrow a_1 = 2, b_1 = 1, c_1 = -5$

and $\quad 3x + 2y = 8$ or $3x + 2y - 8 = 0$

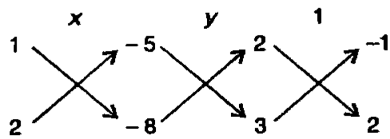
$\Rightarrow \quad a_2 = 3,\ b_2 = 2,\ c_2 = -8$

We have $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$.

So, the pair of linear equations is consistent with a unique solution.

Let us solve the pair of linear equations by using cross- multiplication method.

First, we draw the diagram as given below:



$$\frac{x}{(1)(-8)-(2)(-5)} = \frac{y}{(-5)(3)-(-8)(2)} = \frac{1}{(2)(2)-(3)(1)}$$

$$\Rightarrow \quad \frac{x}{-8+10} = \frac{y}{-15+16} = \frac{1}{4-3}$$

$$\Rightarrow \quad \frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

$$\Rightarrow \quad \frac{x}{2} = \frac{1}{1} \text{ and } \frac{y}{1} = \frac{1}{1}$$

$\Rightarrow \quad x = 2$ and $y = 1$.

Hence, the solutions are $x = 2$ and $y = 1$.

(*iii*) Consider equations $3x - 5y = 20$, *i.e.*, $3x - 5y - 20 = 0$

$\Rightarrow \quad a_1 = 3,\ b_1 = -5,\ c_1 = -20$

and $\quad 6x - 10y = 40$, *i.e.*, $6x - 10y - 40 = 0$

$\Rightarrow \quad a_2 = 6,\ b_2 = -10,\ c_2 = -40$

We have $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$.

So, the pair of equations is consistent with infinitely many solutions.

We can consider one equation say $3x - 5y = 20$.

Let $y = k$

$\therefore \quad 3x - 5k = 20 \quad \Rightarrow \quad x = \dfrac{5k + 20}{3}$

$\therefore$ Solutions are $x = \dfrac{5k + 20}{3}$ and $y = k$ for different values of k. We get different solutions.

(iv) **Hint:** Consider equations $x - 3y - 7 = 0$

$\Rightarrow \quad a_1 = 1, b_1 = -3, c_1 = -7$

and $\quad 3x - 3y - 15 = 0$

$\Rightarrow \quad a_2 = 3, b_2 = -3, c_2 = -15$

We have $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$.

So, the pair of linear equations is consistent and has a unique solution.

Solve for x and y by using cross-multiplication method to get $x = 4$ and $y = -1$.

2. (i) *For which values of a and b does the following pair of linear equations have an infinite number of solutions?*

$2x + 3y = 7$

$(a - b)x + (a + b)y = 3a + b - 2$

(ii) *For which value of k will the following pair of linear equations have no solution?*

$3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

Sol. (i) For $2x + 3y = 7$, we have $a_1 = 2, b_1 = 3, c_1 = -7$

and for $(a - b)x + (a + b)y = 3a + b - 2$, we have

$a_2 = a - b, b_2 = a + b, c_2 = -(3a + b - 2)$

For infinite number of solutions $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

$$\Rightarrow \quad \frac{2}{a - b} = \frac{3}{a + b} = \frac{-7}{-(3a + b - 2)}$$

Consider $\quad \dfrac{2}{a - b} = \dfrac{3}{a + b}$

$\Rightarrow \quad 2a + 2b = 3a - 3b$

$\Rightarrow \quad a = 5b \quad$ ...(i)

and $\frac{2}{a-b} = \frac{7}{3a+b-2}$

$\Rightarrow$ $6a + 2b - 4 = 7a - 7b$

$\Rightarrow$ $-a + 9b = 4$

$\Rightarrow$ $-5b + 9b = 4$ [From (*i*)]

$\Rightarrow$ $4b = 4$ $\Rightarrow$ $b = 1.$

From (*i*), $a = 5 \times 1 = 5$

$\therefore$ For $a = 5$, $b = 1$, the pair of equations has infinitely many solutions.

(*ii*) For no solution, $\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{-1}{-(2k+1)}$

Consider $\frac{3}{2k-1} = \frac{1}{k-1}$

$\Rightarrow$ $3k - 3 = 2k - 1$ $\Rightarrow$ $k = 2.$

$\therefore$ For $k = 2$, the given pair of linear equations has no solution.

3. *Solve the following pair of linear equations by the substitution and cross-multiplication method:*

$8x + 5y = 9$

$3x + 2y = 4$

Sol. Consider equations $8x + 5y - 9 = 0$

and $3x + 2y - 4 = 0$

Here, we are using cross-multiplication method only.

$$\frac{x}{-20+18} = \frac{-y}{-32+27} = \frac{1}{16-15}$$

$$\Rightarrow \quad \frac{x}{-2} = \frac{-y}{-5} = \frac{1}{1}$$

$\Rightarrow$ $x = -2$, $y = 5$ is the required solution.

4. *Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:*

(*i*) *A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay ₹ 1000 as hostel charges* *whereas a student B, who takes food for 26 days, pays ₹ 1180 as hostel charges. Find the fixed charges and the cost of food per day.*

(ii) *A fraction becomes* $\frac{1}{3}$ *when 1 is subtracted from the numerator and it becomes* $\frac{1}{4}$ *when 8 is added to its denominator. Find the fraction.*

(iii) *Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?*

(iv) *Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?*

(v) *The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.*

Sol. (i) Let fixed charges = ₹ x and cost of food / day = ₹ y

We have $x + 20y = 1000$ ...(i)

and $x + 26y = 1180$ ...(ii)

solving (i) and (ii), we get $y = 30$ and $x = 400$

Hence, fixed charges = ₹ 400 and cost of food / day = ₹ 30.

(ii) Let fraction = $\frac{x}{y}$

We have $\frac{x-1}{y} = \frac{1}{3} \Rightarrow 3x - 3 = y$

$\Rightarrow \quad 3x - y = 3$ ...(i)

and $\frac{x}{y+8} = \frac{1}{4} \Rightarrow 4x = y + 8$

$\Rightarrow \quad 4x - y = 8$ ...(*ii*)

Solving (*i*) and (*ii*) simultaneously, we get

$x = 5, y = 12$

Therefore, fraction is $\frac{5}{12}$.

(*iii*) Let number of Yash's right answers = x

and that of wrong answers = y

Total number of questions = $x + y$

We have $3x - y = 40$ ...(*i*)

and $4x - 2y = 50$, *i.e.*, $2x - y = 25$ ...(*ii*)

Solving (*i*) and (*ii*), we get $x = 15, y = 5$

$\therefore$ Total number of questions = 20.

(*iv*) Let speed of car starting from A = x km / hr and speed of car starting from B = y km / hr.

We have $5x - 5y = 100 \Rightarrow x - y = 20$ ...(*i*)

and $x + y = 100$ ...(*ii*)

Solving (*i*) and (*ii*), we get $x = 60, y = 40$.

Hence, speed of car starting from place A = 60 km / hr

and speed of car starting from place B = 40 km / hr.

(*v*) Let length of the rectangle = x units

and breadth of the rectangle = y units.

According to given conditions, we get

$(x - 5)(y + 3) = xy - 9$

$\Rightarrow \quad xy + 3x - 5y - 15 = xy - 9$

$\Rightarrow \quad 3x - 5y = 6$ ...(*i*)

and $(x + 3)(y + 2) = xy + 67$

$\Rightarrow \quad xy + 3y + 2x + 6 = xy + 67$

$\Rightarrow \quad 2x + 3y = 61$ ...(*ii*)

Multiplying (*i*) by 3 and (*ii*) by 5 and adding the results, we get

$9x - 15y = 18$

$$10x + 15y = 305$$
$$19x = 323$$
$$\Rightarrow \quad x = 17$$

Substituting the value for x in (*ii*), we get

$$34 + 3y = 61$$
$$\Rightarrow \quad 3y = 27 \quad \Rightarrow \quad y = 9$$

Hence, length of the rectangle = 17 units

and breadth of the rectangle = 9 units.

Exercise 3.6 *(Page – 67)*

1. *Solve the following pairs of equations by reducing them to a pair of linear equations:*

(*i*) $\frac{1}{2x} + \frac{1}{3y} = 2$

$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

(*ii*) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

(*iii*) $\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

(*iv*) $\frac{5}{x-1} + \frac{1}{y-2} = 2$

$\frac{6}{x-1} - \frac{3}{y-2} = 1$

(*v*) $\frac{7x-2y}{xy} = 5$

$\frac{8x+7y}{xy} = 15$

(*vi*) $6x + 3y = 6xy$

$2x + 4y = 5xy$

(*vii*) $\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{5}{x-y} = -2$

(*viii*) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

Sol. (*i*) Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$, we get

$$\frac{a}{2} + \frac{b}{3} = 2 \quad \Rightarrow \quad 3a + 2b = 12 \qquad ...(i)$$

and $\frac{a}{3} + \frac{b}{2} = \frac{13}{6} \Rightarrow 2a + 3b = 13$ ...(*ii*)

Solving (*i*) and (*ii*), we get

$a = 2, b = 3$

$\therefore \frac{1}{x} = 2, \frac{1}{y} = 3 \Rightarrow x = \frac{1}{2}, y = \frac{1}{3}$

is the solution.

(*ii*) Let $\frac{1}{\sqrt{x}} = a, \frac{1}{\sqrt{y}} = b$, we have

$2a + 3b = 2$ ...(*i*)

and $4a - 9b = -1$ ...(*ii*)

Solving (*i*) and (*ii*), we get $a = \frac{1}{2}, b = \frac{1}{3}$

Now $a = \frac{1}{2} \Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2}$

$\Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$

and $b = \frac{1}{3} \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3}$

$\Rightarrow \sqrt{y} = 3 \Rightarrow y = 9$

Hence, solution is $x = 4, y = 9$.

(*iii*) Let $\frac{1}{x} = a$, we get

$4a + 3y = 14$ and $3a - 4y = 23$.

Solving for a and y, we get

$a = 5$ and $y = -2 \Rightarrow \frac{1}{x} = 5$ and $y = -2$

$\Rightarrow x = \frac{1}{5}$ and $y = -2$ is the solutions.

(*iv*) Let $\frac{1}{x-1} = a$ and $\frac{1}{y-2} = b$,

then $5a + b = 2$ ...(*i*)

and $6a - 3b = 1$ ...(*ii*)

Solving (*i*) and (*ii*), we get

$a = \frac{1}{3}, b = \frac{1}{3}$

$\therefore \quad \dfrac{1}{x-1} = \dfrac{1}{3}$ and $\dfrac{1}{y-2} = \dfrac{1}{3}$

$\Rightarrow \quad x = 4,\ y = 5$ is the solution.

(*v*) Consider $\dfrac{7x-2y}{xy} = 5 \Rightarrow \dfrac{7}{y} - \dfrac{2}{x} = 5$ ...(*i*)

and $\dfrac{8x+7y}{xy} = 15 \Rightarrow \dfrac{8}{y} + \dfrac{7}{x} = 15$ ...(*ii*)

Let $\dfrac{1}{x} = a$ and $\dfrac{1}{y} = b$, then we get

$$7b - 2a = 5$$

and $$8b + 7a = 15.$$

Solving for a, b, we get

$a = 1$ and $b = 1$ $\quad \Rightarrow \dfrac{1}{x} = 1$ and $\dfrac{1}{y} = 1$

$\Rightarrow \quad x = 1$ and $y = 1$ is the solutions.

(*vi*) One solution is $x = 0,\ y = 0$

For other solution,

consider

$6x + 3y = 6xy$

$\Rightarrow \quad \dfrac{6}{y} + \dfrac{3}{x} = 6,\ x \neq 0,\ y \neq 0$ ...(*i*)

and $\quad 2x + 4y = 5xy$

$\Rightarrow \quad \dfrac{2}{y} + \dfrac{4}{x} = 5$ ...(*ii*)

Let $\dfrac{1}{x} = a,\ \dfrac{1}{y} = b.$

From (*i*) and (*ii*), we get

$6b + 3a = 6$ ...(*iii*)

and $\quad 2b + 4a = 5$ ...(*iv*)

Solving (*iii*) and (*iv*), we get $a = 1,\ b = \dfrac{1}{2}$

Now $a = 1 \Rightarrow \dfrac{1}{x} = 1 \Rightarrow x = 1$

and $b = \frac{1}{2} \Rightarrow \frac{1}{y} = \frac{1}{2} \Rightarrow y = 2$

Hence, $x = 1, y = 2$ is the solution.

(*vii*) Let $\frac{1}{x+y} = a, \frac{1}{x-y} = b$, we have

$10a + 2b = 4$ ...(*i*)

$15a - 5b = -2$ ...(*ii*)

Solving (*i*) and (*ii*), we get $a = \frac{1}{5}, b = 1$

$a = \frac{1}{5} \Rightarrow \frac{1}{x+y} = \frac{1}{5} \Rightarrow x + y = 5$ ...(*iii*)

$b = 1 \Rightarrow \frac{1}{x-y} = 1 \Rightarrow x - y = 1$ ...(*iv*)

Solving (*iii*) and (*iv*), we get $x = 3, y = 2$.

(*viii*) Let $\frac{1}{3x+y} = a$ and $\frac{1}{3x-y} = b$

We have $a + b = \frac{3}{4} \Rightarrow 4a + 4b = 3$ ...(*i*)

and $\frac{a}{2} - \frac{b}{2} = -\frac{1}{8} \Rightarrow 4a - 4b = -1$ ...(*ii*)

Solving (*i*) and (*ii*), we get $a = \frac{1}{4}, b = \frac{1}{2}$

Now, $a = \frac{1}{4} \Rightarrow \frac{1}{3x+y} = \frac{1}{4}$

$\Rightarrow 3x + y = 4$ ...(*iii*)

and $b = \frac{1}{2} \Rightarrow \frac{1}{3x-y} = \frac{1}{2}$

$\Rightarrow 3x - y = 2$ ...(*iv*)

Solving (*iii*) and (*iv*), we get $x = 1, y = 1$.

2. *Formulate the following problems as a pair of equations, and hence find their solutions:*

(*i*) *Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.*

(*ii*) *2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.*

(*iii*) *Roohi travels 300 km to her home partly by train and party by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.*

Sol. (*i*) Let speed of rowing in still water = x km / hr

and speed of current = y km / hr

$\therefore$ Speed along downstream = $(x + y)$ km / hr

and speed along upstream = $(x - y)$ km / hr

we have $\frac{20}{x+y} = 2 \Rightarrow x + y = 10$ ...(*i*)

and $\frac{4}{x-y} = 2 \Rightarrow x - y = 2$ ...(*ii*)

Solving (*i*) and (*ii*), we get $x = 6, y = 4$.

Therefore, speed of rowing in still water = 6 km / hr

and speed of stream(current) = 4 km / hr

(*ii*) Let 1 man finishes the work in x days

and 1 woman finishes the work in y days.

So, 1 man's 1 day's work = $\frac{1}{x}$

$\therefore$ 5 men's 1 day's work = $\frac{5}{x}$

And 1 woman's one day's work = $\frac{1}{y}$

$\therefore$ 2 women's one day's work = $\frac{2}{y}$

$\therefore$ 2 women's and 5 men's 1 day's work = $\frac{2}{y} + \frac{5}{x}$

Since 2 women and 5 men finish the work in 4 days.

$\therefore 4\left(\frac{5}{x} + \frac{2}{y}\right) = 1 \Rightarrow \frac{20}{x} + \frac{8}{y} = 1$ ...(*i*)

Similarly, we can find out that $3\left(\frac{6}{x}+\frac{3}{y}\right)=1$

$$\Rightarrow \quad \frac{18}{x}+\frac{9}{y}=1 \qquad ...(ii)$$

Solving for x and y, we get $x = 36, y = 18$

Hence, 1 man can finish the work in 36 days

and 1 woman can finish the work in 18 days.

(*iii*) Let speed of the train = x km / hr

and speed of the bus = y km / hr

According to given conditions,

$$\frac{60}{x}+\frac{240}{y}=4$$

$$\Rightarrow \quad \frac{15}{x}+\frac{60}{y}=1 \qquad ...(i)$$

$$\text{and} \quad \frac{100}{x}+\frac{200}{y}=4\frac{10}{60}=\frac{25}{6}$$

$$\Rightarrow \quad \frac{4}{x}+\frac{8}{y}=\frac{1}{6} \qquad ...(ii)$$

Multiplying (*i*) by 2 and (*ii*) by 15 and then subtracting, we get

$$\frac{30}{x}+\frac{120}{y}=2$$

$$\frac{60}{x}+\frac{120}{y}=\frac{15}{6}$$

$$\underline{-\qquad -\qquad\qquad -}$$

$$-\frac{30}{x}=-\frac{3}{6} \quad \Rightarrow \quad x=60$$

Substituting the value of x in (*i*), we get

$$\frac{15}{60}+\frac{60}{y}=1 \quad \Rightarrow \quad \frac{60}{y}=1-\frac{1}{4}=\frac{3}{4}$$

$$\Rightarrow \quad y=80$$

Hence, speed of the train = 60 km / hr

and speed of the bus = 80 km / hr.

Exercise 3.7 (OPTIONAL) *(Page – 68)*

1. *The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.*

Sol. Let Ani's age = x years and Biju's age = y years.

we have $x - y = \pm 3$ ...(*i*)

$\Rightarrow$ Either $x - y = 3$

or $x - y = -3$ ...(*ii*)

Ani's father's age = $2x$ years

Biju's sister Cathy's age = $\frac{y}{2}$ years

$$2x - \frac{y}{2} = 30 \quad \Rightarrow \quad 4x - y = 60 \qquad ...(iii)$$

Solving (*i*) and (*iii*), we get $x = 19, y = 16$

And solving (*ii*) and (*iii*), we get $x = 21, y = 24$

Hence, either Ani's age = 19 years and Biju's age = 16 years

Or Ani's age = 21 years and Biju's age = 24 years.

2. *One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II]*

Sol. Let amounts of first and second be ₹ x and ₹ y respectively.

We have $x + 100 = 2(y - 100)$

$\Rightarrow$ $x - 2y = -300$ ...(*i*)

and $6(x - 10) = y + 10$

$\Rightarrow$ $6x - y = 70$ ...(*ii*)

Solving (*i*) and (*ii*), we get $x = 40, y = 170$.

Amount of first = ₹ 40 and amount of second = ₹ 170.

3. *A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.*

Sol. Let distance covered by the train = x km

Let speed of the train = y km / hr.

According to given conditions,

$$(y + 10)\left(\frac{x}{y} - 2\right) = x, \text{ i.e., } y^2 + 10y - 5x = 0 \quad ...(i)$$

$$(y - 10)\left(\frac{x}{y} + 3\right) = x, \text{ i.e., } 3y^2 - 30y - 10x = 0 \quad ...(ii)$$

On solving equations (*i*) and (*ii*), we will obtain x = 600 km.

4. *The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.*

Sol. Let number of students in each row = x

and number of rows = y

$\therefore$ Total number of students = xy ...(*i*)

According to the given conditions,

$$(x + 3)(y - 1) = xy$$

$$\Rightarrow \quad -x + 3y = 3 \quad ...(ii)$$

and $$(x - 3)(y + 2) = xy$$

$$\Rightarrow \quad 2x - 3y = 6 \quad ...(iii)$$

Solving (*ii*) and (*iii*), we get

$$x = 9, y = 4$$

From (*i*), we get

The total number of students are 9 × 4, *i.e.*, 36.

5. *In a Δ ABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.*

Sol. In Δ ABC,

$$\angle A + \angle B + \angle C = 180° \quad ...(i)$$

Also, $$\angle C = 2(\angle A + \angle B) \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$\frac{1}{2}\angle C + \angle C = 180°$$

$$\Rightarrow \qquad \frac{3}{2}\angle C = 180° \quad \Rightarrow \quad \angle C = 120°$$

Also, $3\angle B = \angle C \quad \Rightarrow \quad 3\angle B = 120° \quad \Rightarrow \quad \angle B = 40°$

From (*i*), we get $\qquad \angle A = 20°$

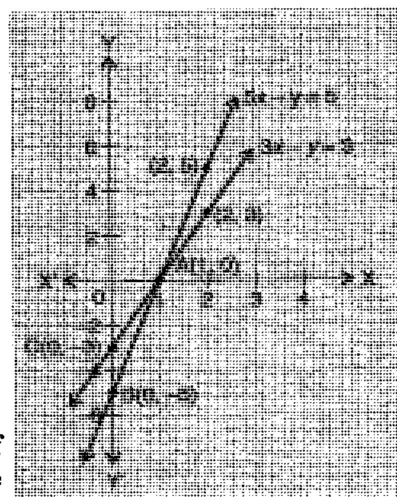
Hence, $\angle A = 20°$, $\angle B = 40°$ and $\angle C = 120°$.

6. *Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the coordinates of the vertices of the triangle formed by these lines and the y axis.*

Sol. Consider equation $\qquad 5x - y = 5$

Some points on the graph are:

x	1	0	2
y	0	– 5	5



Consider equation

$3x - y = 3$

Some points on the graph are:

x	1	0	2
y	0	– 3	3

Plotting these points on the graph the coordinates of the vertices of the triangle formed by these lines and the y-axis are A(1, 0), B(0, – 5) and C(0, – 3).

7. *Solve the following pair of linear equations:*

(*i*) $px + qy = p - q$

$qx - py = p + q$

(*ii*) $ax + by = c$

$bx + ay = 1 + c$

(*iii*) $\dfrac{x}{a} - \dfrac{y}{b} = 0$

$ax + by = a^2 + b^2$

(*iv*) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$

$(a + b)(x + y) = a^2 + b^2$

(*v*) $152x - 378y = -74$

$-378x + 152y = -604$

Sol. (*i*) Consider equations

$$px + qy = p - q \qquad ...(i)$$

and $$qx - py = p + q \qquad ...(ii)$$

Multiplying (*i*) by p and (*ii*) by q and adding the results, we get

$$p^2x + pqy = p(p - q)$$

$$q^2x - pqy = q(p + q)$$

$$(p^2 + q^2)\,x = p^2 + q^2$$

$\Rightarrow$ $$x = 1$$

Substituting the value for x in (*i*), we get

$$p + qy = p - q \quad \Rightarrow \quad qy = -q$$

$\Rightarrow$ $$y = -1$$

Hence, the solution is $x = 1$, $y = -1$.

(*ii*) Consider equations

$$ax + by = c, \text{ i.e., } \quad ax + by - c = 0 \qquad ...(i)$$

$$bx + ay = 1 + c, \text{ i.e., } bx + ay - (1 + c) = 0 \qquad ...(ii)$$

Using method of cross-multiplication, we get

$$\frac{x}{\begin{vmatrix} b & -c \\ a & -(1+c) \end{vmatrix}} = \frac{-y}{\begin{vmatrix} a & -c \\ b & -(1+c) \end{vmatrix}} = \frac{1}{\begin{vmatrix} a & b \\ b & a \end{vmatrix}}$$

$$\Rightarrow \quad \frac{x}{-b - bc + ac} = \frac{-y}{-a - ac + bc} = \frac{1}{a^2 - b^2}$$

$$\Rightarrow \quad x = \frac{c(a-b) - b}{a^2 - b^2} = \frac{c}{a+b} - \frac{b}{a^2 - b^2}$$

and $$y = \frac{c(a-b) + a}{a^2 - b^2} = \frac{a}{a^2 - b^2} + \frac{c}{a+b}.$$

(*iii*) Consider equations

$$\frac{x}{a} - \frac{y}{b} = 0 \qquad ...(i)$$

and $$ax + by = a^2 + b^2 \qquad ...(ii)$$

Substituting the value for x, from (*i*) in (*ii*), we get

$$a\left(\frac{ay}{b}\right) + by = a^2 + b^2$$

$\Rightarrow \quad (a^2 + b^2)\,y = b(a^2 + b^2) \quad \Rightarrow \quad y = b$

Substituting $y = b$ in (*i*), we get $x = a$

Hence, the solution is $x = a,\ y = b$.

(*iv*) Consider equations

$$(a - b)x + (a + b)y = a^2 - 2ab - b^2 \qquad ...(i)$$

$$(a + b)x + (a + b)y = a^2 + b^2 \qquad ...(ii)$$

Subtracting (*ii*) from (*i*), we get

$$(a - b - a - b)x = -\,2ab - 2b^2$$

$\Rightarrow \quad -\,2bx = -\,2b(a + b) \quad \Rightarrow \quad x = a + b$

Substituting the value for x in (*i*), we get

$$a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$$

$$\Rightarrow \quad y = \frac{-2ab}{a+b}$$

Hence, the solution is $x = a + b,\ y = \dfrac{-2ab}{a+b}$.

(*v*) Consider equations

$$152x - 378y = -\,74 \qquad ...(i)$$

and $$-\,378x + 152y = -\,604 \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$-\,226x - 226y = -\,678$$

$$\Rightarrow \quad x + y = 3 \qquad ...(iii)$$

Subtracting (*ii*) from (*i*), we get

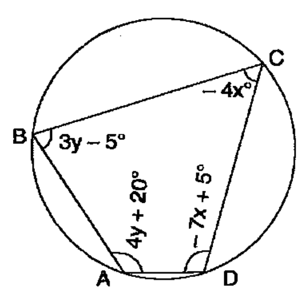
$$530x - 530y = 530$$

$$\Rightarrow \quad x - y = 1 \qquad ...(iv)$$

Solving (*iii*) and (*iv*), we get

$$x = 2,\ y = 1.$$

8. *ABCD is a cyclic quadrilateral (see figure). Find the angles of the cyclic quadrilateral.*



Sol. As ABCD is a cyclic quadrilateral.

$\therefore \quad \angle A + \angle C = 180°$

$\Rightarrow \quad 4y + 20° - 4x = 180°$

$\Rightarrow \quad -4x + 4y = 160°$

$\Rightarrow \quad -x + y = 40°$ ...(*i*)

and $\quad \angle B + \angle D = 180°$

$\Rightarrow \quad -7x + 5 + 3y - 5 = 180°$

$\Rightarrow \quad -7x + 3y = 180°$ ...(*ii*)

From (*i*) and (*ii*), we get

$$\begin{array}{r} -7x + 7y = 280° \\ -7x + 3y = 180° \\ + \quad - \quad\quad - \\ \hline 4y = 100° \end{array}$$

(On subtraction)

$\Rightarrow \quad y = 25°$

Substituting $y = 25°$ in (*i*), we get $x = -15°$

$\therefore \quad \angle A = 4y + 20° = 100° + 20° = 120°$

$\angle B = 3y - 5 = 75° - 5° = 70°$

$\angle C = -4x = 60°$

$\angle D = -7x + 5° = 105° + 5° = 110°.$