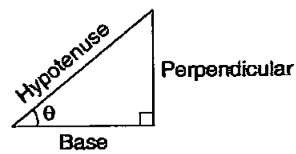
8 Introduction to Trigonometry

Lesson at a Glance

1. Trigonometry is the study of relationships between the sides and angles of a triangle.

2. $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$,



$\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}}$,

$\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$, $\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}}$,

$\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}}$, $\text{cosec}\,\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$.

3. $\cot\theta = \dfrac{1}{\tan\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$, $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$,

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

4. $\sin(90° - \theta) = \cos\theta$, $\cos(90° - \theta) = \sin\theta$,

$\tan(90° - \theta) = \cot\theta$, $\cot(90° - \theta) = \tan\theta$,

$\sec(90° - \theta) = \text{cosec}\,\theta$, $\text{cosec}(90° - \theta) = \sec\theta$.

5. $\sin^2\theta + \cos^2\theta = 1$, $\sec^2\theta - \tan^2\theta = 1$, $\text{cosec}^2\theta - \cot^2\theta = 1$.

6. The value of $\sin\theta$ or $\cos\theta$, is 0 or more but never exceeds 1.

7. The value of $\sec\theta$ or $\text{cosec}\,\theta$ is always greater than or equal to 1.

TEXTBOOK QUESTIONS SOLVED

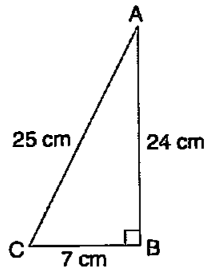
Exercise 8.1 *(Page – 181)*

1. *In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine:*

   (i) sin A, cos A

   (ii) sin C, cos C

Sol. AB = 24 cm, BC = 7 cm, using Pythagoras Theorem,



$$AC = \sqrt{24^2 + 7^2} = \sqrt{576 + 49}$$

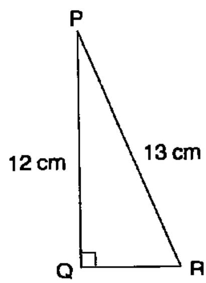
$$= \sqrt{625} = 25 \text{ cm.}$$

(i) $\sin A = \dfrac{BC}{AC} = \dfrac{7}{25}$,

$\cos A = \dfrac{AB}{AC} = \dfrac{24}{25}$.

(ii) $\sin C = \dfrac{AB}{AC} = \dfrac{24}{25}$, $\cos C = \dfrac{BC}{AC} = \dfrac{7}{25}$.

2. *In figure, find tan P – cot R.*



Sol. PQ = 12 cm, PR = 13 cm.
Using Pythagoras Theorem,

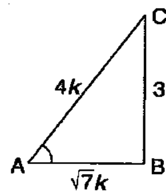
$$RQ = \sqrt{PR^2 - PQ^2} = \sqrt{13^2 - 12^2}$$

$$= \sqrt{169 - 144} = \sqrt{25} = 5 \text{ cm}$$

$\tan P = \dfrac{RQ}{PQ} = \dfrac{5}{12}$, $\cot R = \dfrac{RQ}{PQ} = \dfrac{5}{12}$.

$\therefore \tan P = \cot R \Rightarrow \tan P - \cot R = 0.$

3. *If* $\sin A = \frac{3}{4}$, *calculate* $\cos A$ *and* $\tan A$.



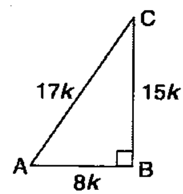
Sol. $\sin A = \frac{3}{4} = \frac{3k}{4k}$

$$AB = \sqrt{16k^2 - 9k^2} = \sqrt{7}\,k$$

$$\therefore \quad \cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}.$$

4. *Given* $15 \cot A = 8$, *find* $\sin A$ *and* $\sec A$.



Sol. $15 \cot A = 8 \quad \Rightarrow \quad \cot A = \frac{8}{15}$

Let $AB = 8k$, $BC = 15k$, then

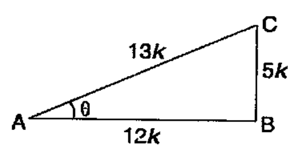
$$AC = \sqrt{AB^2 + BC^2} = \sqrt{64k^2 + 225k^2}$$

$$= \sqrt{289k^2} = 17k$$

$$\therefore \quad \sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17},$$

$$\sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}.$$

5. *Given* $\sec \theta = \frac{13}{12}$, *calculate all other trigonometric ratios.*



Sol. $\sec \theta = \frac{13}{12} = \frac{13k}{12k}$, *i.e.,* $\frac{AC}{AB}$

$$\therefore \quad BC = \sqrt{169k^2 - 144k^2} = 5k$$

$$\sin \theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}, \quad \cos \theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

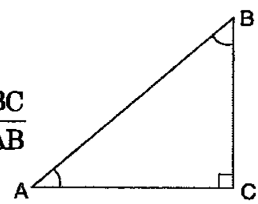
$$\tan\theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12},\ \cot\theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$

$$\text{and } \operatorname{cosec}\theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}.$$

6. *If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.*

Sol. ∠A and ∠B are two acute angles either in same right-angled triangle or in different.

Case I: Let ∠A and ∠B are acute angles in right ΔABC.



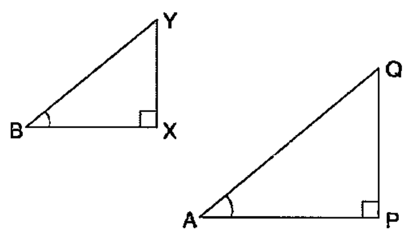
$$\cos A = \frac{AC}{AB} \text{ and } \cos B = \frac{BC}{AB}$$

As given cos A = cos B $\Rightarrow \dfrac{AC}{AB} = \dfrac{BC}{AB}$

$\Rightarrow$ AC = BC

$\therefore$ ∠B = ∠A. [Angles opposite to equal sides are equal]

Case II: Let ∠A belong to rt ΔAPQ and ∠B belong to rt ΔBXY.



In ΔAPQ,

$$\cos A = \frac{AP}{AQ} \qquad ...(i)$$

In ΔBXY,

$$\cos B = \frac{BX}{BY} \qquad ...(ii)$$

As given cos A = cos B $\Rightarrow \dfrac{AP}{AQ} = \dfrac{BX}{BY}$

[From (*i*) and (*ii*)]

$$\Rightarrow \quad \frac{AP}{BX} = \frac{AQ}{BY} = k \text{ (say)}$$

$\therefore$ AP = k BX and AQ = k BY

$$\text{Now,} \quad \frac{PQ}{XY} = \frac{\sqrt{AQ^2 - AP^2}}{\sqrt{BY^2 - BX^2}} = \frac{\sqrt{k^2BY^2 - k^2BX^2}}{\sqrt{BY^2 - Bk^2}}$$

$$\therefore \qquad \frac{PQ}{XY} = \frac{k\sqrt{BY^2 - BX^2}}{\sqrt{BY^2 - Bk^2}} = k$$

Therefore, $\dfrac{AP}{BX} = \dfrac{AQ}{BY} = \dfrac{PQ}{XY}$

$\therefore$ $\Delta APQ \sim \Delta BXY$ [By SSS similarity theorem]

$\therefore$ $\angle A = \angle B.$

7. *If cot $\theta = \dfrac{7}{8}$, evaluate:*

(i) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$, *(ii)* $\cot^2\theta$

Sol. (*i*) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \dfrac{1-\sin^2\theta}{1-\cos^2\theta} = \dfrac{\cos^2\theta}{\sin^2\theta}$

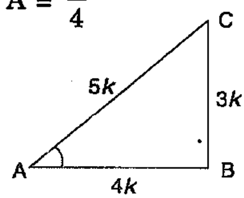
$$= \cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}.$$

(*ii*) $\cot^2\theta = \left(\dfrac{7}{8}\right)^2 = \dfrac{49}{64}.$

8. *If 3 cot A = 4, check whether* $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ *or not.*

Sol. $3\cot A = 4 \Rightarrow \cot A = \dfrac{4}{3}$ or $\tan A = \dfrac{3}{4}$

If AB = $4k$ and BC = $3k$, then



$$AC = \sqrt{AB^2 + BC^2} = \sqrt{(4k)^2 + (3k)^2}$$

$$= \sqrt{25k^2} = 5k$$

$\therefore$ $\cos A = \dfrac{AB}{AC} = \dfrac{4}{5}$ and $\sin A = \dfrac{BC}{AC} = \dfrac{3}{5}$

$$\text{LHS} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{16-9}{16+9} = \frac{7}{25}$$

$$\text{RHS} = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16-9}{25} = \frac{7}{25}$$

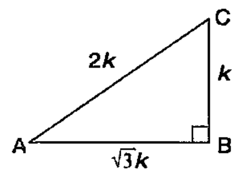
Hence, $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A.$

9. *In triangle ABC, right-angled at B, if* $\tan A = \frac{1}{\sqrt{3}}$, *find the value of:*

(*i*) *sin A cos C + cos A sin C*

(*ii*) *cos A cos C – sin A sin C*

Sol. $\tan A = \dfrac{1}{\sqrt{3}} \Rightarrow AB = \sqrt{3}k$ and $BC = k$, then



$$AC = \sqrt{AB^2 + BC^2} = \sqrt{(3k)^2 + k^2}$$

$$= \sqrt{3k^2 + k^2} = 2k.$$

$$\sin A = \frac{BC}{AC} = \cos C \text{ and } \sin C = \frac{AB}{AC} = \cos A$$

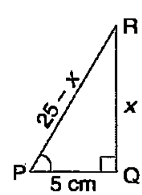
(*i*) $\sin A \cos C + \cos A \sin C = \dfrac{1}{2} \cdot \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{3}}{2}$

$$= \frac{1}{4} + \frac{3}{4} = 1.$$

(*ii*) $\cos A \cos C - \sin A \sin C = \dfrac{\sqrt{3}}{2} \cdot \dfrac{1}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{3}}{2}$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0.$$

10. *In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.*



Sol. PQ = 5 cm and PR + QR = 25 cm

Let RQ = x, then PR = $25 - x$

Using Pythagoras theorem,

$$PR^2 = PQ^2 + RQ^2$$

$$(25 - x)^2 = x^2 + (5)^2 \Rightarrow 625 - 50x + x^2 = x^2 + 25$$

$$\Rightarrow \quad 50x = 600 \Rightarrow x = 12$$

$\therefore$ RQ = 12 cm and PR = 13 cm.

$\therefore$ $\sin P = \frac{12}{13}$, $\cos P = \frac{5}{13}$ and $\tan P = \frac{12}{5}$.

11. *State whether the following are true or false. Justify your answer.*

(*i*) *The value of tan A is always less than 1.*

(*ii*) *sec A* $= \frac{12}{5}$ *for some value of angle A.*

(*iii*) *cos A is the abbreviation used for the cosecant of angle A.*

(*iv*) *cot A is the product of cot and A.*

(*v*) *sin* $\theta = \frac{4}{3}$ *for some angle* θ.

Sol. (*i*) False, As $\tan A = \frac{p}{b}$ where perpendicular is not always less than base in a right-angled triangle.

(*ii*) True, $\sec A = \frac{12}{5}$, true as $\cos A = \frac{5}{12} < 1$, true.

(*iii*) False, cos A is abbreviation used for the cosine of angle A.

(*iv*) False, $\cot A \neq \cot \times A$

(*v*) False, as $\sin\theta$ is always less than or equal to 1.

Exercise 8.2 *(Page – 187)*

1. *Evaluate the following:*

(*i*) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(*ii*) $2 \tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5 \cos^2 60° + 4 \sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Sol. (i) $\dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \cdot \dfrac{1}{2} = \dfrac{3}{4} + \dfrac{1}{4} = 1$

(ii) $2(1)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{\sqrt{3}}{2}\right)^2 = 2 + \dfrac{3}{4} - \dfrac{3}{4} = 2$

(iii) $\dfrac{\dfrac{1}{\sqrt{2}}}{\dfrac{2}{\sqrt{3}} + 2} = \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2 + 2\sqrt{3}}$

$$= \frac{\sqrt{3}}{2\sqrt{2}(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)\sqrt{2}}{(\sqrt{3}-1)\sqrt{2}}$$

$$= \frac{(3-\sqrt{3})\sqrt{2}}{2 \times 2 \times (3-1)} = \frac{3\sqrt{2}-\sqrt{6}}{8}.$$

(iv) $\dfrac{\dfrac{1}{2} + 1 - \dfrac{2}{\sqrt{3}}}{\dfrac{2}{\sqrt{3}} + \dfrac{1}{2} + 1} = \dfrac{\dfrac{3}{2} - \dfrac{2}{\sqrt{3}}}{\dfrac{2}{\sqrt{3}} + \dfrac{3}{2}} = \dfrac{3\sqrt{3}-4}{3\sqrt{3}+4} = \dfrac{(3\sqrt{3}-4)^2}{27-16}$

[Rationalising the denominator]

$$= \frac{27 + 16 - 24\sqrt{3}}{11} = \frac{43 - 24\sqrt{3}}{11}.$$

(v) $\dfrac{5\left(\dfrac{1}{2}\right)^2 + 4\left(\dfrac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2} = \dfrac{\dfrac{5}{4} + \dfrac{16}{3} - 1}{\dfrac{1}{4} + \dfrac{3}{4}}$

$$= \frac{15 + 64 - 12}{12} \times \frac{4}{1+3} = \frac{67}{12}.$$

2. *Choose the correct option and justify your choice:*

(i) $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(A) $\sin 60^\circ$ (B) $\cos 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

(ii) $\dfrac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$

(A) $\tan 90^\circ$ (B) 1 (C) $\sin 45^\circ$ (D) 0

(iii) $\sin 2A = 2\sin A$ is true when $A =$

(A) 0° (B) 30° (C) 45° (D) 60°

(iv) $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ} =$

(A) $\cos 60^\circ$ (B) $\sin 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

Sol. (i) (A). $\dfrac{2\cdot\dfrac{1}{\sqrt{3}}}{1+\left(\dfrac{1}{\sqrt{3}}\right)^2} = \dfrac{\dfrac{2}{\sqrt{3}}}{1+\dfrac{1}{3}} = \dfrac{2}{\sqrt{3}} \times \dfrac{3}{4} = \dfrac{\sqrt{3}}{2} = \sin 60^\circ.$

(ii) (D). $\dfrac{1-(1)^2}{1+(1)^2} = \dfrac{0}{2} = 0.$

(iii) (A). Because $\sin 2A = \sin 0 = 0$
and $2\sin A = 2\sin 0^\circ = 0.$

(iv) (C). $\dfrac{2\times\dfrac{1}{\sqrt{3}}}{1-\left(\dfrac{1}{\sqrt{3}}\right)^2} = \dfrac{\dfrac{2}{\sqrt{3}}}{1-\dfrac{1}{3}} = \dfrac{2}{\sqrt{3}} \times \dfrac{3}{2} = \sqrt{3} = \tan 60^\circ.$

3. *If* $\tan(A+B) = \sqrt{3}$ *and* $\tan(A-B) = \dfrac{1}{\sqrt{3}}$; $0^\circ < A + B \le 90^\circ$; $A > B$, *find A and B.*

Sol. $\tan(A+B) = \sqrt{3} = \tan 60^\circ \Rightarrow A + B = 60^\circ$ ...(i)

$\tan(A-B) = \dfrac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow A - B = 30^\circ$ ...(ii)

Solving (i) and (ii), we get $A = 45^\circ$ and $B = 15^\circ$.

4. *State whether the following are true or false. Justify your answer.*

(i) *sin (A + B) = sin A + sin B.*

(ii) *The value of sin θ increases as θ increases.*

(iii) *The value of cos θ increases as θ increases.*

(iv) *sin θ = cos θ for all values of θ.*

(v) *cot A is not defined for A = 0°.*

Sol. (i) False. Let A = 30°, B = 60°

$$\sin (A + B) = \sin (30° + 60°) = \sin 90° = 1$$

$$\sin A + \sin B = \sin 30° + \sin 60° = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$$

$\therefore \quad \sin (A + B) \neq \sin A + \sin B.$

(ii) True. As $\sin 0° = 0$, $\sin 30° = \frac{1}{2}$ and $\sin 90° = 1$.

(iii) False. As $\cos 0° = 1$, $\cos 60° = \frac{1}{2}$ and $\cos 90° = 0$.

(iv) False. Only for θ = 45°, cos θ = sin θ.

(v) True. $\cot A = \frac{\cos A}{\sin A}$

$$\Rightarrow \quad \cot 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0} \text{ which is not defined.}$$

Exercise 8.3 *(Page – 189-190)*

1. *Evaluate:*

(i) $\frac{\sin 18°}{\cos 72°}$ (ii) $\frac{\tan 26°}{\cot 64°}$

(iii) $\cos 48° - \sin 42°$ (iv) $\text{cosec } 31° - \sec 59°$

Sol. (i) $\frac{\sin 18°}{\cos 72°} = \frac{\sin(90° - 72°)}{\cos 72°} = \frac{\cos 72°}{\cos 72°} = 1.$

[∵ sin (90° – θ) = cos θ]

(*ii*) $\frac{\tan 26°}{\cot 64°} = \frac{\tan 26°}{\cot (90° - 26°)} = \frac{\tan 26°}{\tan 26°} = 1$

[$\because$ $\cot (90° - \theta) = \tan \theta$]

(*iii*) $\cos 48° - \sin 42° = \cos (90° - 42°) - \sin 42°$

$= \sin 42° - \sin 42° = 0.$

[$\because$ $\cos (90° - \theta) = \sin \theta$]

(*iv*) $\text{cosec } 31° - \sec 59° = \text{cosec } 31° - \sec (90° - 31°)$

$= \text{cosec } 31° - \text{cosec } 31° = 0.$

[$\because$ $\sec (90° - \theta) = \text{cosec } \theta$]

2. *Show that:*

(*i*) *tan 48° tan 23° tan 42° tan 67° = 1*

(*ii*) *cos 38° cos 52° – sin 38° sin 52° = 0.*

Sol. (*i*) LHS $= \tan 48° \tan 23° \tan 42° \tan 67°$

$= \tan 48° \tan 23° \tan (90° - 48°) \tan (90° - 23°)$

$= \tan 48° \tan 23° \cot 48° \cot 23°$

$= \tan 48° \tan 23° \times \frac{1}{\tan 48°} \times \frac{1}{\tan 23°}$

$= 1 =$ RHS.

(*ii*) LHS $= \cos 38° \cos 52° - \sin 38° \sin 52°$

$= \cos (90° - 52°) \cos (90° - 38°) - \sin 38° \sin 52°$

$= \sin 52° \sin 38° - \sin 38° \sin 52° = 0 =$ RHS.

3. *If tan 2A = cot(A – 18°), where 2A is an acute angle, find the value of A.*

Sol. $\tan 2A = \cot (A - 18°) = \tan \{90° - (A - 18°)\}$

$\Rightarrow \quad 2A = 90° - A + 18° \quad \Rightarrow \quad 3A = 108°$

$\Rightarrow \quad A = \frac{108°}{3} = 36°.$

4. *If tan A = cot B, prove that A + B = 90°.*

Sol. $\tan A = \cot B = \tan (90° - B)$

$\Rightarrow \quad A = 90° - B \quad \Rightarrow \quad A + B = 90°$. **Hence proved.**

5. *If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A.*

Sol. sec 4A = cosec (A – 20°)

$\Rightarrow$ cosec (90° – 4A) = cosec (A – 20°)

$\Rightarrow$ 90° – 4A = A – 20°

$\Rightarrow$ 5A = 110° $\Rightarrow$ A = 22°.

6. *If A, B and C are interior angles of a triangle ABC, then show that*

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}.$$

Sol. In ΔABC,

A + B + C = 180° $\Rightarrow$ B + C = 180° – A

$$\Rightarrow \frac{B+C}{2} = 90° - \frac{A}{2}$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90° - \frac{A}{2}\right) = \cos\frac{A}{2}.$$

7. *Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.*

Sol. Consider sin 67° + cos 75°

= sin (90° – 23°) + cos (90° – 15°) = cos 23° + sin 15°.

Exercise 8.4 *(Page – 193-194)*

1. *Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.*

Sol. (*i*) $\sin A = \frac{1}{\text{cosec } A} = \frac{1}{\sqrt{1+\cot^2 A}}$

(*ii*) $\sec A = \sqrt{1+\tan^2 A} = \sqrt{1+\frac{1}{\cot^2 A}} = \frac{\sqrt{\cot^2 A+1}}{\cot A}$

(*iii*) $\tan A = \frac{1}{\cot A}$.

2. *Write all the other trigonometric ratios of ∠A in terms of sec A.*

Sol. $\sin A = \tan A \cdot \cos A = \dfrac{\sqrt{\sec^2 A - 1}}{\sec A}$; $\left[\cos A = \dfrac{1}{\sec A}\right]$

$\tan A = \sqrt{\sec^2 A - 1}$; $\cot A = \dfrac{1}{\tan A} = \dfrac{1}{\sqrt{\sec^2 A - 1}}$

$\operatorname{cosec} A = \dfrac{1}{\sin A} = \dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$.

3. *Evaluate:*

(*i*) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(*ii*) $\sin 25° \cos 65° + \cos 25° \sin 65°$.

Sol. (*i*) Consider $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$$= \frac{\sin^2 (90° - 27°) + \sin^2 27°}{\cos^2 17° + \cos^2 (90° - 17°)} = \frac{\cos^2 27° + \sin^2 27°}{\cos^2 17° + \sin^2 17°}$$

$= \dfrac{1}{1} = 1.$ $[\because \sin^2 \theta + \cos^2 \theta = 1]$

(*ii*) Consider $\sin 25° \cos 65° + \cos 25° \sin 65°$

$= \sin 25° \cos (90° - 25°) + \cos 25° \sin (90° - 25°)$

$= \sin 25° \sin 25° + \cos 25° \cos 25°$

$= \sin^2 25° + \cos^2 25° = 1.$ $[\because \sin^2 \theta + \cos^2 \theta = 1]$

4. *Choose the correct option. Justify your choice.*

(*i*) $9 \sec^2 A - 9 \tan^2 A =$

(*A*) 1 (*B*) 9 (*C*) 8 (*D*) 0

(*ii*) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) =$

(*A*) 0 (*B*) 1 (*C*) 2 (*D*) – 1

(*iii*) $(\sec A + \tan A)(1 - \sin A) =$

(*A*) $\sec A$ (*B*) $\sin A$ (*C*) $\operatorname{cosec} A$ (*D*) $\cos A$

(*iv*) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} =$

(*A*) $\sec^2 A$ (*B*) – 1 (*C*) $\cot^2 A$ (*D*) $\tan^2 A$.

Sol. (*i*) (B). Consider $9 \sec^2 A - 9 \tan^2 A$

$$= 9 (\sec^2 A - \tan^2 A)$$

$$= 9 \times 1 = 9.$$

(*ii*) (C). Consider $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$$

$$= \frac{(\cos\theta + \sin\theta)^2 - 1}{\cos\theta\sin\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta + 2\sin\theta.\cos\theta - 1}{\cos\theta.\sin\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\cos\theta\sin\theta} = 2.$$

(*iii*) (D). Consider $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1 + \sin A)}{\cos A}.(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A.$$

(*iv*) (D). Consider $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \dfrac{\sec^2 A}{\text{cosec}^2 A}$

$$= \frac{1}{\cos^2 A}.\sin^2 A$$

$$= \tan^2 A.$$

5. *Prove the following identities, where the angles involved are acute angles for which the expressions are defined.*

(*i*) $(\text{cosec}\,\theta - \cot\theta)^2 = \dfrac{1 - \cos\theta}{1 + \cos\theta}$

(*ii*) $\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A} = 2 \sec A$

(iii) $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\text{cosec}\,\theta$

[**Hint:** *Write the expression in terms of sin θ and cos θ*]

(iv) $\dfrac{1+\sec A}{\sec A}=\dfrac{\sin^2 A}{1-\cos A}$

[**Hint:** *Simplify LHS and RHS separately*]

(v) $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}$

$=\text{cosec}\,A+\cot A$, *using the identity* $\text{cosec}^2 A=1+\cot^2 A$.

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\dfrac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+\text{cosec}\,A)^2+(\cos A+\sec A)^2$

$=7+\tan^2 A+\cot^2 A$

(ix) $(\text{cosec}\,A-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

[**Hint:** *Simplify LHS and RHS separately*]

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A.$

Sol. (i) LHS $=(\text{cosec}\,\theta-\cot\theta)^2$

$$=\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2=\frac{(1-\cos\theta)^2}{\sin^2\theta}=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$=\frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}=\frac{1-\cos\theta}{1+\cos\theta}=\text{RHS}.$$

(ii) $$\text{LHS}=\frac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)\cos A}$$

$$=\frac{\cos^2 A+1+\sin^2 A+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)\cos A} = \frac{2(1+\sin A)}{(1+\sin A)\cos A}$$

$$= \frac{2}{\cos A} = 2\sec A = \text{RHS.}$$

(*iii*) $\text{LHS} = \dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta}$

$$= \frac{\dfrac{\sin\theta}{\cos\theta}}{1-\dfrac{\cos\theta}{\sin\theta}} + \frac{\dfrac{\cos\theta}{\sin\theta}}{1-\dfrac{\sin\theta}{\cos\theta}}$$

$$= \frac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{(\sin\theta-\cos\theta)\cos\theta} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\sec\theta\,.\cos\theta\,(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta\,(\sin\theta-\cos\theta)}$$

$$= \frac{1+\sin\theta\cos\theta}{\sin\theta\,.\cos\theta}$$

$$= \operatorname{cosec}\theta\,.\,\sec\theta + 1 = \text{RHS.}$$

(*iv*) $\text{RHS} = \dfrac{\sin^2 A}{1-\cos A} = \dfrac{1-\cos^2 A}{1-\cos A}$

$$= \frac{(1+\cos A)(1-\cos A)}{(1-\cos A)}$$

$$= 1+\cos A = 1+\frac{1}{\sec A} = \frac{\sec A+1}{\sec A} = \text{LHS.}$$

(*v*) $\text{LHS} = \dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing each term of numerator and denominator by sin A, we have

$$= \frac{\frac{\cos A}{\sin A} - 1 + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + 1 - \frac{1}{\sin A}} = \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{(\cot A + \operatorname{cosec} A)\{1 - (\operatorname{cosec} A - \cot A)\}}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{(\cot A + \operatorname{cosec} A)(1 - \operatorname{cosec} A + \cot A)}{\cot A - \operatorname{cosec} A + 1}$$

$= \cot A + \operatorname{cosec} A =$ RHS.

(*vi*) LHS $= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \frac{1+\sin A}{\cos A}$$

$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A =$ RHS.

(*vii*) LHS $= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$

$$= \frac{\sin\theta(\cos^2\theta - \sin^2\theta)}{\cos\theta(\cos^2\theta - \sin^2\theta)}$$

$= \frac{\sin\theta}{\cos\theta} = \tan\theta =$ RHS.

(*viii*) LHS $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A$
$+ \sec^2 A + 2 \cos A \sec A$

$= 1 + 1 + \cot^2 A + 2 + 1 + \tan^2 A + 2$

[As $\sin\theta \cdot \operatorname{cosec}\theta = 1$ and $\cos\theta \cdot \sec\theta = 1$]

$= 7 + \cot^2 A + \tan^2 A =$ RHS.

(*ix*) $$\text{LHS} = \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \frac{1-\sin^2 A}{\sin A} \times \frac{1-\cos^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A \cos A$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

On dividing numerator and denominator by sin A cos A, we have

$$= \frac{1}{\tan A + \cot A} = \text{RHS.}$$

(*x*) $$\text{LHS} = \frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\text{cosec}^2 A} = \frac{1}{\cos^2 A} \cdot \sin^2 A$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A = \text{RHS.}$$

Now consider, $$\text{LHS} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$= \left\{\frac{\tan A\,(1-\tan A)}{\tan A - 1}\right\}^2 = \{-\tan A\}^2$$

$$= \tan^2 A = \text{RHS.}$$