

12

Areas Related to Circles

Lesson at a Glance

1. Perimeter of a circle = $2\pi r$.
2. Perimeter of a semicircle = $(\pi + 2)r$.
3. Perimeter of a quadrant of a circle = $\left(\frac{\pi}{2} + 2\right)r$.
4. Area of a circle = πr^2 .
5. Area of a semicircle = $\frac{1}{2}\pi r^2$.
6. Area of a quadrant of a circle = $\frac{1}{4}\pi r^2$.
7. Length of an arc of a sector of a circle with radius r and angle $\theta = \frac{\theta}{360^\circ} \times 2\pi r$.
8. Perimeter of the sector = $2r + \frac{\theta}{360^\circ} \times 2\pi r$.
9. Area of the sector = $\frac{\theta}{360^\circ} \times \pi r^2$.
10. Area of a segment of a circle = area of the corresponding sector
– area of the corresponding triangle.

TEXTBOOK QUESTIONS SOLVED

Exercise 12.1 (Page – 225-226)

Unless stated otherwise, use $\pi = \frac{22}{7}$.

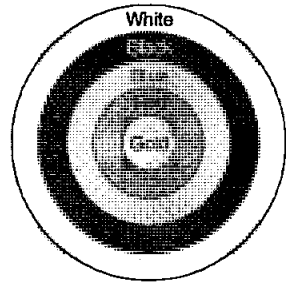
1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Sol. $2\pi r = 2\pi r_1 + 2\pi r_2 \Rightarrow r = r_1 + r_2 = 19 + 9 = 28$ cm.

2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Sol. $\pi r^2 = \pi r_1^2 + \pi r_2^2 \Rightarrow r^2 = (8)^2 + (6)^2 = 64 + 36 = 100$
 $\Rightarrow r = 10$ cm.

3. Adjoining figure depicts an archery target marked with its five scoring regions from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.



Sol. Area representing Gold
 $= \pi (10.5)^2 \text{ cm}^2$

$$r_1 = \frac{21}{2} \text{ cm} = 10.5 \text{ cm}$$

For region representing Red

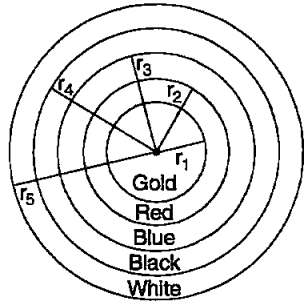
$$= \pi (r_2^2 - r_1^2)$$

$$= \pi \{(21)^2 - (10.5)^2\} \text{ cm}^2$$

For region representing Blue = $\pi (r_3^2 - r_2^2)$

$$= \pi \{(31.5)^2 - (21)^2\} \text{ cm}^2$$

Similarly, we can find other regions.



4. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Sol. Speed of car = 66 km / hr

Distance covered in 10 minutes = $\frac{66000}{60} \times 10$ m
 $= 11000$ m.

Distance covered in one revolution = $2 \times \frac{22}{7} \times 40$ cm
 $= \frac{1760}{7}$ cm = $\frac{176}{70}$ m

$$\therefore \text{Number of revolutions} = 11000 \times \frac{70}{176} = 4375.$$

Thus, number of revolutions are 4375.

5. Tick the correct answer in the following and justify your choice: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(A) 2 units (B) π units (C) 4 units (D) 7 units

Sol. Option (A) is correct.

Exercise 12.2 (Page – 230-231)

Unless stated otherwise, use $\pi = \frac{22}{7}$.

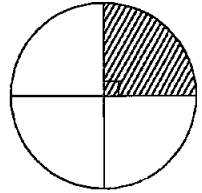
1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60° .

Sol. Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2 = \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2 = \frac{132}{7} \text{ cm}^2$.

2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Area of quadrant = $\frac{90^\circ}{360^\circ} \times \pi r^2 = \frac{1}{4} \pi r^2$

Now, $2\pi r = 22 \Rightarrow r = \frac{11 \times 7}{22} = \frac{7}{2} \text{ cm}$



$$\therefore \text{Area of quadrant} = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \text{ cm}^2 = \frac{77}{8} \text{ cm}^2.$$

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Sol. $r = 14 \text{ cm}$

Angle formed in 5 minutes = $\frac{360^\circ}{60} \times 5 = 30^\circ$

$$\therefore \text{Area covered} = \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times (14)^2 \text{ cm}^2$$

$$= \frac{1}{12} \times 22 \times 2 \times 14 = \frac{154}{3} \text{ cm}^2.$$

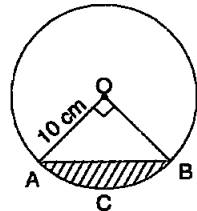
4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:
 (i) minor segment (ii) major sector. (Use $\pi = 3.14$)

Sol. Length of the radius (r) = 10 cm

Sector angle $\theta = 90^\circ$

Area of the sector with $\theta = 90^\circ$ and $r = 10$ cm

$$\begin{aligned} &= \frac{90}{360} \times 10 \times 10 \times \frac{314}{100} \text{ cm}^2 \\ &= \frac{1}{4} \times 314 \text{ cm}^2 = \frac{157}{2} \text{ cm}^2 = 78.5 \text{ cm}^2 \end{aligned}$$



Now,

- (i) Area of the minor segment

$$\begin{aligned} &= [\text{Area of minor sector}] - [\text{Area of rt. } \Delta \text{ AOB}] \\ &= [78.5 \text{ cm}^2] - \left[\frac{1}{2} \times 10 \times 10 \text{ cm}^2 \right] = 78.5 \text{ cm}^2 - 50 \text{ cm}^2 \\ &= \mathbf{28.5 \text{ cm}^2}. \end{aligned}$$

- (ii) Area of major segment

$$\begin{aligned} &= [\text{Area of the circle}] - [\text{Area of the minor segment}] \\ &= \pi r^2 - 78.5 \text{ cm}^2 \\ &= \left[\frac{314}{100} \times 10 \times 10 - 78.5 \right] \text{ cm}^2 = (314 - 78.5) \text{ cm}^2 \\ &= \mathbf{235.5 \text{ cm}^2}. \end{aligned}$$

5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) the length of the arc

(ii) area of the sector formed by the arc

(iii) area of the segment formed by the corresponding chord

Sol. (i) $r = 21$ cm; $\theta = 60^\circ$

$$\begin{aligned} \therefore \text{Length of arc} &= \frac{\theta \pi r}{180^\circ} = \frac{60^\circ \pi \times 21}{180^\circ} \\ &= 7\pi \text{ cm} = 22 \text{ cm}. \end{aligned}$$

$$(ii) \text{ Area of sector} = \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$$

$$= \frac{1}{6} \times 22 \times 3 \times 21 = 231 \text{ cm}^2$$

(iii) Area of segment formed

$$= \text{area of sector} - \frac{1}{2} \cdot (21)^2 \sin 60^\circ$$

$$= \left(231 - \frac{441\sqrt{3}}{4} \right) \text{ cm}^2.$$

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

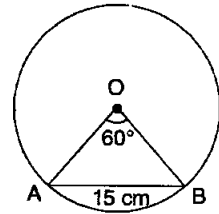
(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Sol. As $\angle AOB = 60^\circ$

$$\therefore \angle OAB = \angle OBA = 60^\circ$$

$\therefore \triangle OAB$ is equilateral.

$$\therefore OA = OB = 15 \text{ cm.}$$



Area of minor segment

$$= \text{area of sector OAB} - \text{area of } \triangle OAB$$

$$= \frac{60^\circ}{360^\circ} \times 3.14 \times (15)^2 - \frac{1}{2} \times (15)^2 \times \sin 60^\circ$$

$$= \frac{1}{6} \times 3.14 \times 225 - \frac{1}{2} \times 225 \times \frac{\sqrt{3}}{2}$$

$$= (117.75 - 97.3125) \text{ cm}^2 = 20.4375 \text{ cm}^2.$$

$$\text{Area of major segment} = [3.14 \times (15)^2 - 20.4375] \text{ cm}^2$$

$$= [706.5 - 20.4375] \text{ cm}^2$$

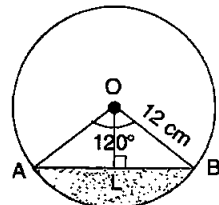
$$= 686.0625 \text{ cm}^2.$$

7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Sol. $\angle LOB = 60^\circ$

$$\frac{OL}{OB} = \cos 60^\circ$$



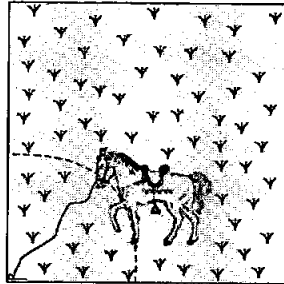
$$\Rightarrow OL = 12 \times \frac{1}{2} = 6 \text{ cm}$$

$$\frac{LB}{OB} = \sin 60^\circ \Rightarrow LB = 12 \times \frac{\sqrt{3}}{2} = 6\sqrt{3} \text{ cm}$$

$$\therefore AB = 2LB = 12\sqrt{3} \text{ cm.}$$

$$\begin{aligned} \therefore \text{Area of segment} &= \left[\frac{120^\circ}{360^\circ} \times 3.14 \times (12)^2 - \frac{1}{2} \times 12\sqrt{3} \times 6 \right] \text{ cm}^2 \\ &= [3.14 \times 48 - 36 \times 1.73] \text{ cm}^2 \\ &= [150.72 - 62.28] \text{ cm}^2 = 88.44 \text{ cm}^2. \end{aligned}$$

8. A horse is tied to a peg at one corner of a square-shaped grass field of side 15 m by means of a 5 m long rope (see figure). Find

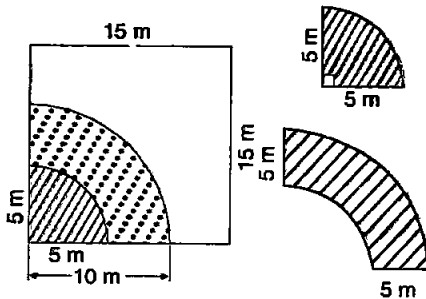


- (i) the area of that part of the field in which the horse can graze.
- (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m.

(Use $\pi = 3.14$)

Sol. (i) Horse can graze the shaded area

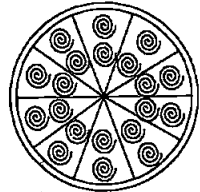
$$\begin{aligned} &= \frac{90^\circ}{360^\circ} \times 3.14 \times (5)^2 \text{ m}^2 \\ &= 19.625 \text{ m}^2 \end{aligned}$$



(ii) Increase in grazing area

$$\begin{aligned}
 &= \frac{90^\circ}{360^\circ} [\pi (10)^2 - \pi (5)^2] \\
 &= \frac{\pi}{4} (100 - 25) = \frac{3.14}{4} \times 75 \text{ m}^2 \\
 &= 58.875 \text{ m}^2.
 \end{aligned}$$

9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find:



(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.

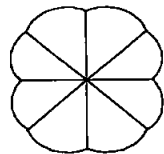
Sol. (i) Total length of the silver wire

$$\begin{aligned}
 &= \text{circumference of the circle} + 5 \times \text{diameter of the circle} \\
 &= (\pi \times 35 + 5 \times 35) \text{ mm} = (110 + 175) \text{ mm} \\
 &= 285 \text{ mm}.
 \end{aligned}$$

$$(ii) \text{ Central angle of a sector} = \frac{360^\circ}{10} = 36^\circ$$

$$\begin{aligned}
 \therefore \text{ Area of each sector} &= \frac{36^\circ}{360^\circ} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \text{ mm}^2 \\
 &= \frac{385}{4} \text{ mm}^2.
 \end{aligned}$$

10. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Sol. Central angle formed by any two consecutive ribs

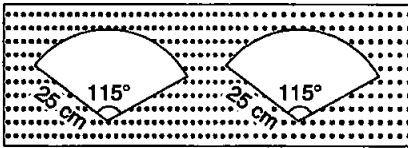
$$= \frac{360^\circ}{8} = 45^\circ$$

Area between two consecutive ribs.

$$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45 \times 45 \text{ cm}^2 = \frac{22275}{28} \text{ cm}^2.$$

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades.

Sol. Total area = $2 \left[\frac{115^\circ}{360^\circ} \times \pi(25)^2 \right] \text{ cm}^2$



$$= \frac{23 \times 11 \times 25 \times 25}{18 \times 7}$$

$$= \frac{158125}{126} \text{ cm}^2$$

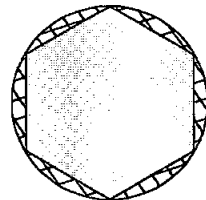
12. To warn ships for underwater rocks, a lighthouse spreads a red- coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)

Sol. $\theta = 80^\circ, r = 16.5 \text{ km}.$

$$\text{Area of the sea} = \frac{80^\circ}{360^\circ} \times 3.14 \times (16.5)^2 \text{ km}^2$$

$$= 189.97 \text{ km}^2.$$

13. A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2 . (Use $\sqrt{3} = 1.7$)



Sol. 6 equal designs = $6 \times$ area of one segment.

Here, central angle $\theta = 60^\circ$.

∴ Area of one segment

$$= \left[\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (28)^2 - \frac{1}{2} \times (28)^2 \times \sin 60^\circ \right]$$

$$\therefore \text{Area of 6 segments} = \left[\frac{22}{7} \times (28)^2 - \frac{3\sqrt{3}}{2} (28)^2 \right] \text{ cm}^2$$

$$= (2464 - 1999.2) \text{ cm}^2$$

$$= 464.8 \text{ cm}^2$$

∴ Cost of making the designs = ₹ (464.8 × 0.35)

$$= ₹ 162.68 \text{ cm}^2.$$

14. Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $\frac{p}{180} \times 2\pi R$

(B) $\frac{p}{180} \times \pi R^2$

(C) $\frac{p}{360} \times 2\pi R$

(D) $\frac{p}{720} \times 2\pi R^2$

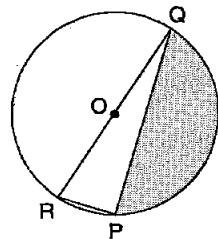
Sol. Area of the sector = $\frac{p}{360} \times \pi R^2 = \frac{p}{720} \times 2\pi R^2$.

Option (D) is correct.

Exercise 12.3 (Page – 234-238)

Unless stated otherwise use $\pi = \frac{22}{7}$

1. Find the area of the shaded region in figure, if $PQ = 24$ cm, $PR = 7$ cm and O is the centre of the circle.



Sol. $RQ = \sqrt{(24)^2 + (7)^2}$ cm
 $= \sqrt{576 + 49}$ cm = 25 cm.

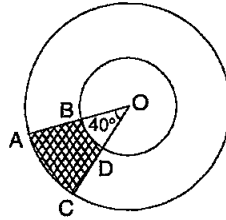
Area of the shaded region

= area of semicircle – area of ΔPQR .

$$= \left[\frac{1}{2} \times \frac{22}{7} \times \left(\frac{25}{2} \right)^2 - \frac{1}{2} \times 24 \times 7 \right] \text{ cm}^2$$

$$= \frac{6875}{28} - 84 = \frac{4523}{28} \text{ cm}^2.$$

2. Find the area of the shaded region in figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$.

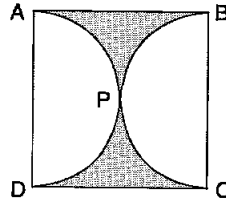


Sol. Area of the shaded region

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times (14^2 - 7^2)$$

$$= \frac{1}{9} \times \frac{22}{7} \times (196 - 49) = \frac{22 \times 147}{9 \times 7} = \frac{154}{3} \text{ cm}^2.$$

3. Find the area of the shaded region in figure, if $ABCD$ is a square of side 14 cm and APD and BPC are semicircles.



Sol. Area of the shaded region

= area of the square – 2

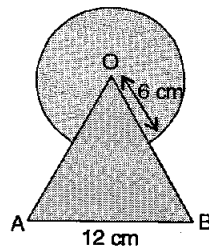
× area of one semicircle with diameter 14 cm

$$= \left[14 \times 14 - 2 \times \frac{1}{2} \times \frac{22}{7} \times (7)^2 \right] \text{ cm}^2$$

$$= (196 - 154) \text{ cm}^2$$

$$= 42 \text{ cm}^2.$$

4. Find the area of the shaded region in figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Sol. Area of the shaded portion

= area of an equilateral triangle of side 12 cm
+ area of a circle of radius 6 cm – area of
a sector of radius 6 cm and central angle 60°

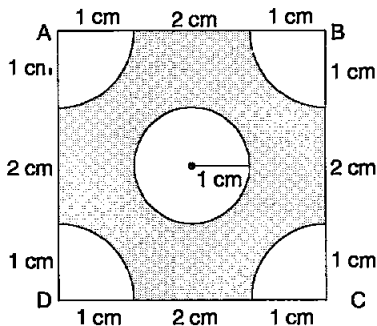
$$= \left[\frac{\sqrt{3}}{4} (12)^2 + \frac{22}{7} \times (6)^2 - \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2 \right] \text{ cm}^2$$

$$= \left[\frac{\sqrt{3}}{4} \times 144 + \frac{5}{6} \times \frac{22}{7} \times 36 \right] \text{ cm}^2 = \left(36\sqrt{3} + \frac{660}{7} \right) \text{ cm}^2.$$

5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in figure. Find the area of the remaining portion of the square.

Sol. Area of the remaining portion

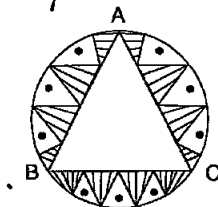
= area of the square of side 4 cm – $4 \times$ area of a quadrant
of radius 1 cm – area of the circle of diameter 2 cm.



$$= \left[4 \times 4 - 4 \times \frac{90^\circ}{360^\circ} \times \pi(1)^2 - \pi(1)^2 \right] \text{ cm}^2$$

$$= \left[16 - 2 \times \frac{22}{7} \right] \text{ cm}^2 = \frac{4}{7} (28 - 11) \text{ cm}^2 = \frac{68}{7} \text{ cm}^2.$$

6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in figure. Find the area of the design.



Sol. Area of the design = 3 × area of minor segment

Here, $\angle BOC = 120^\circ \Rightarrow \angle LOC = 60^\circ$

$$\frac{OL}{OC} = \cos 60^\circ$$

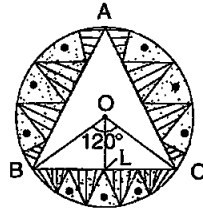
$$\Rightarrow OL = \frac{1}{2} \times 32 = 16 \text{ cm}$$

$$\frac{LC}{OC} = \sin 60^\circ \Rightarrow LC = 32 \times \frac{\sqrt{3}}{2} = 16\sqrt{3} \text{ cm}$$

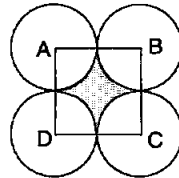
$$\therefore BC = 2LC = 32\sqrt{3} \text{ cm.}$$

\therefore Area of the design

$$\begin{aligned} &= 3 \left[\frac{120^\circ}{360^\circ} \times \frac{22}{7} \times (32)^2 - \frac{1}{2} \times 32\sqrt{3} \times 16 \right] \text{ cm}^2 \\ &= \left[\frac{22}{7} \times (32)^2 - 768\sqrt{3} \right] \text{ cm}^2 \\ &= \left(\frac{25528}{7} - 768\sqrt{3} \right) \text{ cm}^2. \end{aligned}$$



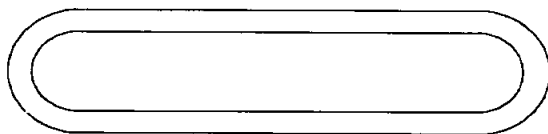
7. In figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded region.



Sol. Area of the shaded region

$$\begin{aligned} &= \text{area of a square with side 14 cm} \\ &\quad - 4 \times \text{area of a quadrant with radius 7 cm} \\ &= \left[14 \times 14 - 4 \times \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7 \right] \text{ cm}^2 \\ &= (196 - 154) \text{ cm}^2 = 42 \text{ cm}^2. \end{aligned}$$

8. Given figure depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:



- (i) the distance around the track along its inner edge
 (ii) the area of the track.

Sol. (i) Distance along the inner track

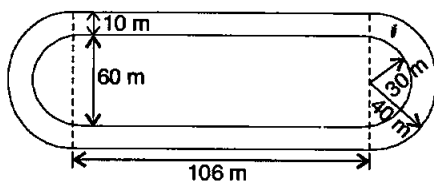
$$= 106 + 106 + 2 \times \text{circumference of semicircle with radius 30 m}$$

$$= \left(212 + 2 \times \frac{22}{7} \times 30 \right) \text{ m} = \frac{1484 + 1320}{7}$$

$$= \frac{2804}{7} \text{ m}$$

- (ii) Area of the track = $2 \times$ area of rectangle of dimensions 106 m and 10 m + $2 \times$ area of a semicircular path of width 10 m

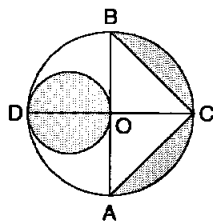
$$= \left[2 \times 106 \times 10 + 2 \times \frac{1}{2} \times \frac{22}{7} \{ (40)^2 - (30)^2 \} \right] \text{ m}^2$$



$$= \left[2120 + \frac{22}{7} \times 700 \right] \text{ m}^2 = (2120 + 2200) \text{ m}^2$$

$$= 4320 \text{ m}^2.$$

9. In figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.



Sol. Area of the shaded region

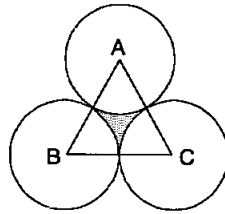
$$= \text{area of circle with diameter 7 cm}$$

+ area of semicircle with radius 7 cm – area of triangle with base 14 cm and height 7 cm

$$= \left[\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} + \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 - \frac{1}{2} \times 14 \times 7 \right] \text{ cm}^2$$

$$= (38.5 + 77 - 49) \text{ cm}^2 = 66.5 \text{ cm}^2.$$

10. The area of an equilateral triangle ABC is 17320.5 cm^2 . With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see figure). Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)



Sol. Area of an equilateral triangle = 17320.5 cm^2 .

Let side of the triangle be $a \text{ cm}$.

$$\therefore \frac{\sqrt{3}}{4} \times a^2 = 17320.5 \Rightarrow a^2 = \frac{17320.5 \times 4}{1.73205} = 40000$$

$$\Rightarrow a = 200 \text{ cm}.$$

$$\therefore \text{Radius of each sector} = 100 \text{ cm and central angle is } 60^\circ.$$

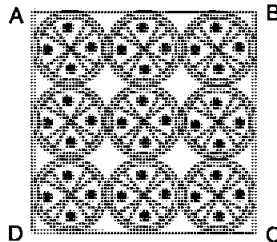
$$\therefore \text{Area of 3 sectors} = 3 \times \frac{60^\circ}{360^\circ} \times 3.14 \times (100)^2 \text{ cm}^2$$

$$= 15700 \text{ cm}^2$$

$$\therefore \text{Area of the shaded portion} = [17320.5 - 15700] \text{ cm}^2$$

$$= 1620.5 \text{ cm}^2.$$

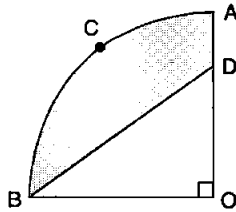
11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see figure). Find the area of the remaining portion of the handkerchief.



Sol. Radius of each circular design = 7 cm.

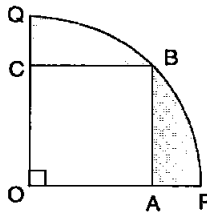
- \therefore Side of the square = $3 \times 14 \text{ cm} = 42 \text{ cm}$
 \therefore Area of the square = $(42)^2 \text{ cm}^2 = 1764 \text{ cm}^2$
 \therefore Area of 9 circular designs = $9 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$
 $= 1386 \text{ cm}^2$.
 \therefore Area of the remaining portion = $(1764 - 1386) \text{ cm}^2$
 $= 378 \text{ cm}^2$.

12. In figure, $OACB$ is a quadrant of a circle with centre O and radius 3.5 cm . If $OD = 2 \text{ cm}$, find the area of the
 (i) quadrant $OACB$, (ii) shaded region.



- Sol.** (i) Area of quadrant = $\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \text{ cm}^2 = \frac{77}{8} \text{ cm}^2$
 (ii) Area of the shaded region = $\left(\frac{77}{8} - \frac{1}{2} \times \frac{7}{2} \times 2\right) \text{ cm}^2$
 $= \left(\frac{77}{8} - \frac{7}{2}\right) \text{ cm}^2 = \frac{49}{8} \text{ cm}^2$.

13. In figure, a square $OABC$ is inscribed in a quadrant $OPBQ$. If $OA = 20 \text{ cm}$, find the area of the shaded region. (Use $\pi = 3.14$)



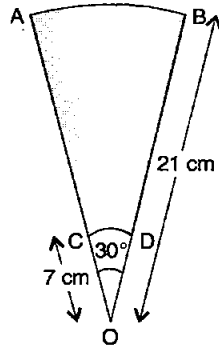
Sol. Radius (OB) of the quadrant = $20\sqrt{2}$ cm.

Area of the shaded region = area of the quadrant
 – area of the square

$$= \left[\frac{90^\circ}{360^\circ} \times 3.14 \times (20\sqrt{2})^2 - (20)^2 \right] \text{ cm}^2$$

$$= (628 - 400) \text{ cm}^2 = 228 \text{ cm}^2.$$

14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see figure). If $\angle AOB = 30^\circ$, find the area of the shaded region.



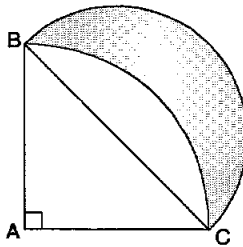
Sol. Area of the shaded portion

$$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} [(21)^2 - (7)^2] \text{ cm}^2$$

$$= \frac{1}{12} \times \frac{22}{7} \times (441 - 49) \text{ cm}^2$$

$$= \frac{1}{6} \times \frac{11}{7} \times 392 \text{ cm}^2 = \frac{308}{3} \text{ cm}^2.$$

15. In figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



Sol. $BC = 14\sqrt{2}$ cm.

\therefore Radius of semicircle = $7\sqrt{2}$ cm

Area of the shaded portion

= area of semicircle of radius $7\sqrt{2}$ cm + area of an isosceles triangle with equal sides of 14 cm each – area of quadrant of radius 14 cm

$$= \left[\frac{\pi}{2} (7\sqrt{2})^2 + \frac{1}{2} \times 14 \times 14 - \frac{90^\circ}{360^\circ} \times \pi \times (14)^2 \right] \text{ cm}^2$$

$$= (49\pi + 98 - 49\pi) \text{ cm}^2 = 98 \text{ cm}^2.$$

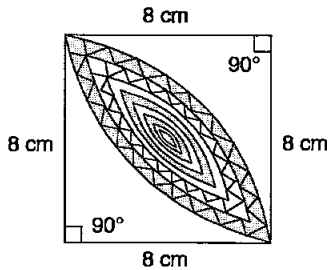
16. Calculate the area of the designed region in figure. common between the two quadrants of circles of radius 8 cm each.

Sol. Area of the shaded portion

$$= 2 \times \text{area of segment}$$

$$= 2 \left[\frac{90^\circ}{360^\circ} \times \pi \times (8)^2 - \frac{1}{2} \times 8 \times 8 \right] \text{ cm}^2$$

$$= 2 \left[16 \times \frac{22}{7} - 32 \right] \text{ cm}^2$$



$$= \frac{2 \times 16}{7} (22 - 14) \text{ cm}^2$$

$$= \frac{256}{7} \text{ cm}^2.$$

