13 Surface Areas and Volumes

Lesson at a Glance

1. Cube, cuboid, cylinder, cone, sphere etc. are three dimensional solids.
2. If the length of each edge of a cube is a, then
   (*i*) surface area of each surface of the cube = a^2.
   (*ii*) surface area or total surface area of the cube = $6a^2$.
   (*iii*) lateral surface area of the cube = $4a^2$.
   (*iv*) volume of the cube = a^3.
   (*v*) length of the diagonal of the cube = $a\sqrt{3}$.
3. If length, breadth and height of a cuboid are respectively l, b and h, then
   (*i*) total surface area of the cuboid = $2(lb + bh + hl)$.
   (*ii*) area of the four walls of the room = $2(l + b) \times h$.
   (*iii*) volume of the cuboid = $l \times b \times h$.
   (*iv*) diagonal of the cuboid = $\sqrt{l^2 + b^2 + h^2}$.
4. If the radius is r and the height is h of a right circular cylinder, then
   (*i*) lateral (curved) surface area = $2\pi rh$.
   (*ii*) surface area of either base = πr^2.
   (*iii*) total surface area = $2\pi r(r + h)$.
   (*iv*) volume = $\pi r^2 h$.
5. If the external and internal radii of a hollow cylinder of height h are respectively R and r, then
   (*i*) lateral (curved) surface area = $2\pi(r + R)h$.
   (*ii*) surface area of either base = $\pi(R^2 - r^2)$.
   (*iii*) total surface area = $2\pi(R + r)(h + R - r)$.
   (*iv*) volume = $\pi(R^2 - r^2)h$.

6. If r, h and l denote respectively radius of base, height and slant height of a cone, then

   (i) curved surface area = πrl.

   (ii) surface area of the base = πr^2.

   (iii) total surface area = $\pi r(r + l)$

   (iv) volume = $\frac{1}{3}\pi r^2 h$.

   (v) $l = \sqrt{r^2 + h^2}$.

7. If r is the radius of a sphere, then its

   (i) curved or total surface area = $4\pi r^2$.

   (ii) volume = $\frac{4}{3}\pi r^3$.

8. If r is the radius of a hemisphere, then its

   (i) curved surface area of the hemisphere = $2\pi r^2$.

   (ii) total surface area of the hemisphere = $3\pi r^2$.

   (iii) volume of the hemisphere = $\frac{2}{3}\pi r^3$.

9. If h is the height, l the slant height and r_1, r_2 the base radii of the circular bases of a frustum of a cone, then its

   (i) lateral (curved) surface area = $\pi(r_1 + r_2)\, l$.

   (ii) total surface area = $\pi[r_1^2 + r_2^2 + (r_1 + r_2)\, l]$

   (iii) volume = $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$.

   (iv) volume = $\frac{1}{3}h(A_1 + A_2 + \sqrt{A_1 A_2})$, where A_1 and A_2 are the area of the circular bases.

   (v) slant height $l = \sqrt{h^2 + (r_2 - r_1)^2}$.

10. When a solid is recast into another solid, the volume remains unchanged.

11. 1 m^3 = 1000 litres; 1 litre = 1000 cm^3.

TEXTBOOK QUESTIONS SOLVED

Exercise 13.1 *(Page – 244-245)*

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. *2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.*

Sol. Side of a cube = $\sqrt[3]{64}$ = 4 cm.

When 2 cubes are joined, then $l = (4 + 4)$ cm = 8 cm,

b = 4 cm, h = 4 cm.

Surface area $= 2(lb + bh + lh) = 2(32 + 16 + 32)$ cm^2

$= 160$ cm^2.

2. *A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.*

Sol. Radius of hemisphere = height of hemisphere = 7 cm.

Height of cylinder = (13 – 7) cm

= 6 cm.

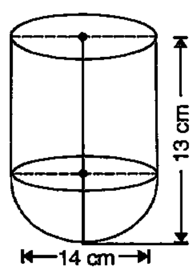
Inner surface area $= 2\pi r^2 + 2\pi rh$

$= 2\pi r(r + h)$

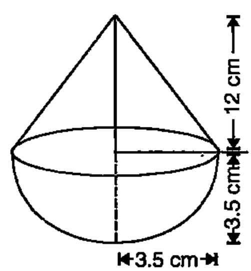
$= 2 \times \frac{22}{7} \times 7(7 + 6)$

$= 44 \times 13$ cm^2

$= 572$ cm^2.



3. *A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.*



Sol. Radius of hemisphere = radius of base of cone = 3.5 cm

Height of cone = (15.5 – 3.5) cm
= 12 cm.

$\therefore$ Slant height (l) = $\sqrt{(12)^2 + (3.5)^2}$ cm

= $\sqrt{144 + 12.25}$ cm

= $\sqrt{156.25}$ cm = 12.5 cm.

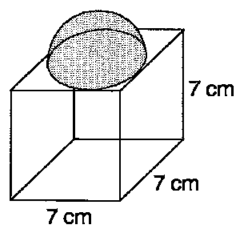
Total surface area of toy = surface area of hemisphere + curved area of cone

$= 2\pi r^2 + \pi rl = \pi r(2r + l)$

$= \frac{22}{7} \times \frac{7}{2}(7 + 12.5)$

$= 11 \times 19.5 = 214.5 \text{ cm}^2.$

4. *A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.*



Sol. Greatest diameter of the hemisphere

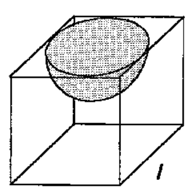
= side of the cube = 7 cm

Surface area of the solid = Surface area of the cube + surface area of the hemisphere – area of the base of the circular portion

$$= \left[6(7)^2 + 2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2} - \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right] \text{ cm}^2$$

$= [294 + 77 - 38.5] \text{ cm}^2 = 332.5 \text{ cm}^2.$

5. *A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.*



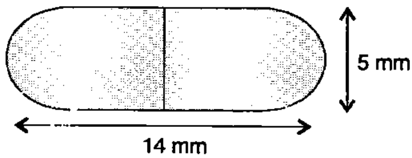
Sol. Surface area of the remaining solid

$$= 6l^2 + 2\pi\left(\frac{l}{2}\right)^2 - \pi\left(\frac{l}{2}\right)^2$$

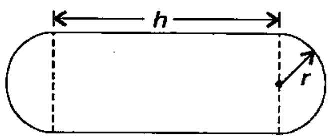
$$= 6l^2 + \frac{\pi l^2}{4}$$

$$= \frac{l^2}{4}(24 + \pi).$$

6. *A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.*



Sol. $r = \frac{5}{2}$ mm, $h = (14 - 2 \times 2.5)$ mm $= 9$ mm



Surface area $= 2 \times 2\pi r^2 + 2\pi rh = 2\pi r(2r + h)$

$$= 2 \times \frac{22}{7} \times \frac{5}{2}(5 + 9) = \frac{110}{7} \times 14 \text{ mm}^2$$

$$= 220 \text{ mm}^2.$$

7. *A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m². (Note that the base of the tent will not be covered with canvas).*

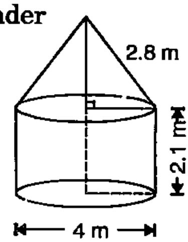
Sol. Radius of base of cylinder, r = radius of base of cone = 2 m.

Area of canvas used = curved area of cylinder

+ lateral area of cone

$$= 2\pi rh + \pi rl = \pi r(2h + l)$$

$$= \frac{22}{7} \times 2(2 \times 2.1 + 2.8)$$



$$= \frac{22}{7} \times 2 \times 7 = 44 \text{ m}^2$$

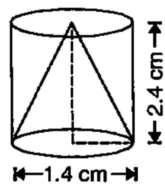
Cost = ₹ (44 × 500) = ₹ 22,000.

8. *From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².*

Sol. Radius of base of cylinder = radius of base of cone

$$= r = \frac{1.4}{2} \text{ cm} = 0.7 \text{ cm}$$

Height of cone = height of cylinder = h = 2.4 cm



Slant height of cone = $l = \sqrt{(2.4)^2 + (0.7)^2}$

$= \sqrt{5.76 + 0.49}$ cm

$= \sqrt{6.25}$ cm = 2.5 cm.

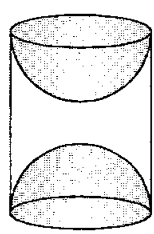
Total surface area of remaining solid = lateral area of cone+ curved area of cylinder + area of top base

$$= \pi rl + 2\pi rh + \pi r^2 = \pi r(l + 2h + r)$$

$$= \frac{22}{7} \times 0.7(2.5 + 2 \times 2.4 + 0.7) \text{ cm}^2$$

$$= 17.6 \text{ cm}^2 \sim 18 \text{ cm}^2.$$

9. *A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.*



Sol. Total surface area = 2 × surface area of a hemisphere + curved area of cylinder

$$= 2 \times 2\pi r^2 + 2\pi rh = 2\pi r[2r + h]$$

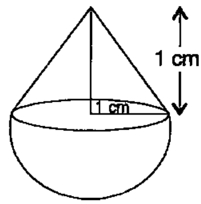
$$= 2 \times \frac{22}{7} \times 3.5 \, [7 + 10] \text{ cm}^2$$

$$= 374 \text{ cm}^2.$$

Exercise 13.2 *(Page – 247-248)*

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. *A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of* π.



Sol. Volume of the solid

= volume of the hemisphere

+ volume of the cone

$$= \frac{2}{3}\pi(1)^3 + \frac{1}{3}\pi(1)^2(1) = \pi \text{ cm}^3.$$

2. *Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)*

Sol. Radius of base of cylinder = radius of base of cone

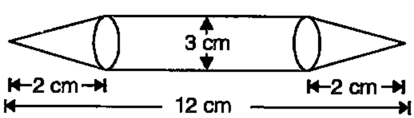
$$= r = \frac{3}{2} \text{ cm}.$$

Height of cone (h_1) = 2 cm

Height of cylinder (h) = (12 – 4) cm = 8 cm

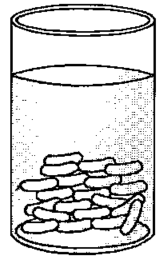
$\therefore$ Volume of air $= 2 \times \frac{1}{3}\pi r^2 h_1 + \pi r^2 h$

$$= \pi r^2 \left[\frac{2}{3}h_1 + h\right]$$



$$= \frac{22}{7} \times \left(\frac{3}{2}\right)^2 \times \left[\frac{2}{3}\times 2 + 8\right] = \frac{22}{7} \times \frac{9}{4} \times \frac{28}{3} = 66 \text{ cm}^3.$$

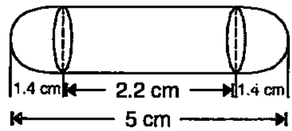
3. *A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see figure).*



Sol. Length of *gulab jamun* = 5 cm

Radius of hemisphere = $r = \frac{2.8}{2}$

$= 1.4$ cm.



Radius of cylinder = radius of hemisphere = r

$= 1.4$ cm.

Height of cylindrical portion = h

$= (5 - 2.8)$ cm $= 2.2$ cm

$\therefore$ Volume of 1 *gulab jamun*

$$= 2 \times \frac{2}{3}\pi r^3 + \pi r^2 h = \pi r^2\left[\frac{4}{3}r + h\right]$$

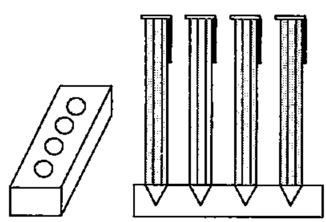
$$= \frac{22}{7} \times (1.4)^2 \left[\frac{4}{3} \times 1.4 + 2.2\right] \text{cm}^3$$

$$= \frac{22}{7} \times (1.4)^2 \times 4.07 = 25.07 \text{ cm}^3.$$

Sugar syrup in 45 *gulab jamuns*

$$= \frac{30}{100} \times 45 \times 25.07 \text{ cm}^3 = 338.445 \text{ cm}^3 \sim 338 \text{ cm}^3.$$

4. *A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of*



the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see figure).

Sol. Volume of cuboid $= 15 \times 10 \times 3.5 \text{ cm}^3$

$= 525 \text{ cm}^3$.

Radius of each depression = 0.5 cm

Depth of each depression = 1.4 cm

$\therefore$ Volume of 4 conical depressions $= 4 \times \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times 1.4 \text{ cm}^3$

$= 1.47 \text{ cm}^3$.

$\therefore$ Volume of wood in the stand $= (525 - 1.47) \text{ cm}^3$

$= 523.53 \text{ cm}^3$.

5. *A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.*

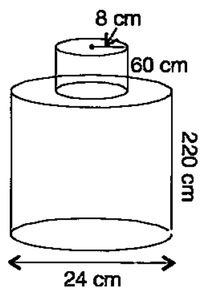
Sol. Volume of x led shots = volume of water flows out

$$\Rightarrow \quad x \times \frac{4}{3} \times \pi \times 0.5 \times 0.5 \times 0.5 = \frac{1}{4}\left(\frac{1}{3} \times \pi \times 5 \times 5 \times 8\right)$$

$$\Rightarrow \quad x = \frac{\pi \times 5 \times 5 \times 8 \times 3}{4 \times \pi \times 0.5 \times 0.5 \times 0.5 \times 4 \times 3} = \frac{10 \times 10 \times 2}{0.5 \times 4} = 100$$

Hence, 100 lead shots should be dropped in the vessel.

6. *A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)*



Sol. Volume of the pole = volume of the cylinder with base diameter 24 cm and height 220 cm + volume of the cylinder with base radius 8 cm and height 60 cm.

$= [\pi(12)^2 \times 220 + \pi \times (8)^2 \times 60)]$ cm^3

$= [31680\pi + 3840\pi]$ cm^3 = 35520π cm^3

Mass of the pole = $\left(35520 \times 3.14 \times \frac{8}{1000}\right)$ kg = 892.26 kg.

7. *A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.*

Sol. Volume of water left = volume of cylinder – volume of solid.

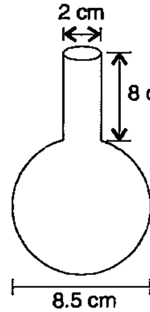
$$= \pi \times (60)^2 \times 180 - \left[\frac{2}{3} \times \pi \times (60)^3 + \frac{1}{3} \times \pi \times (60)^2 \times 120\right] \text{cm}^3$$

$$= \left[648000\pi - \frac{4}{3} \times \pi \times 216000\right] \text{cm}^3$$

$= 360000\pi$ cm^3 = 1131428.57 cm^3

= 1.131 m^3 (approx.).

8. *A spherical glass vessel has a cylinderical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.*



Sol. Volume of glass vessel

$$= \frac{4}{3}\pi r^3 + \pi r_1^2 h$$

$$= \pi\left[\frac{4}{3}r^3 + r_1^2 h\right]$$

$$= 3.14\left[\frac{4}{3} \times \left(\frac{8.5}{2}\right)^3 + (1)^2 \times 8\right]\text{cm}^3$$

$= 3.14[110.35]\text{cm}^3 = 346.51 \text{ cm}^3$

And the child measured $= 345 \text{ cm}^3$

Hence, the child is not correct.

Exercise 13.3 *(Page – 251-252)*

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. *A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.*

Sol. Radius of the sphere $(r_1) = 4.2$ cm

$\therefore$ Volume of the sphere $\left(\frac{4}{3}\pi r_1^3\right)$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} \text{cm}^3$$

Radius of the cylinder $(r_2) = 6$ cm

Let 'h' be the height of the cylinder

$\therefore$ Volume of the cylinder $= \pi r^2 h = \frac{22}{7} \times 6 \times 6 \times h \text{ cm}^3$

Since, Volume of the metallic sphere = Volume of the cylinder.

$$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} = \frac{22}{7} \times 6 \times 6 \times h$$

$$\Rightarrow h = \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} \times \frac{7}{22} \times \frac{1}{6} \times \frac{1}{6} \text{cm}$$

$$= \frac{4 \times 7 \times 7 \times 4}{10 \times 10 \times 10} \text{cm} = \frac{2744}{1000} \text{cm} = \mathbf{2.744 \text{ cm.}}$$

2. *Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.*

Sol. $$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3 + \frac{4}{3}\pi(10)^3$$

$$\Rightarrow \quad R^3 = 1728 \quad \Rightarrow \quad R = 12 \text{ cm}.$$

3. *A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.*

Sol. Volume of earth taken out $= \pi\left(\frac{7}{2}\right)^2 \times 20$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 \text{ m}^3$$

$$= 770 \text{ m}^3.$$

$\therefore$ Height of platform $= \dfrac{\text{volume of earth taken out}}{\text{area of platform}}$

$$= \frac{770}{22 \times 14} = 2.5 \text{ m}.$$

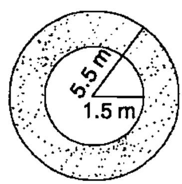
4. *A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it is in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.*

Sol. Volume of the earth taken out $= \pi\left(\frac{3}{2}\right)^2 \times 14 \text{ m}^3$

Area over which earth is spread

$= \pi[(5.5)^2 - (1.5)^2] \text{ m}^2$

$= \pi \times 7 \times 4 \text{ m}^2$



$\therefore$ Height of the embankment

$$= \frac{\pi \times 9 \times 14}{4 \times \pi \times 7 \times 4} \text{ m} = 1.125 \text{ m}.$$

5. *A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top.*

Find the number of such cones which can be filled with ice cream.

Sol. Volume of the circular cylinder = $\pi(6)^2 \times 15$ cm^3

Volume of one ice cream cone

= volume of the hemisphere + volume of the cone

$$= \left[\frac{2}{3}\pi(3)^3 + \frac{1}{3}\pi(3)^2 \times 12\right] \text{ cm}^3$$

$$= 54\,\pi \text{ cm}^3$$

$\therefore$ Number of ice cream cones

$$= \frac{\text{volume of circular cylinder}}{\text{volume of one ice cream cone}}$$

$$= \frac{\pi \times 36 \times 15}{\pi \times 54} = 10.$$

6. *How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?*

Sol. For a circular coin:

Diameter = 1.75 cm

$\Rightarrow$ Radius (r) = $\frac{175}{200}$ cm

Thickness (h) = 2 mm = $\frac{2}{10}$ cm

$\therefore$ Volume = $\pi r^2 h$

$$= \frac{22}{7} \times \left(\frac{175}{200}\right)^2 \times \frac{2}{10} \text{ cm}^3$$

| $\because$ *A coin is like a cylinder*

For a cuboid:

Length (l) = 10 cm, Breadth (b) = 5.5 cm

and Height (h) = 3.5 cm

$\therefore$ Volume = $10 \times \frac{55}{10} \times \frac{35}{10}$ cm^3

Number of coins

Let the number of coins need to melt be 'n'

$$\therefore \quad n = \left[10 \times \frac{55}{10} \times \frac{35}{10}\right] \div \left[\frac{22}{7} \times \frac{175}{200} \times \frac{175}{200} \times \frac{2}{10}\right]$$

$$= 10 \times \frac{55}{10} \times \frac{35}{10} \times \frac{7}{22} \times \frac{200}{175} \times \frac{200}{175} \times \frac{10}{2} = 16 \times 25 = 400$$

Thus, the required number of coins = **400.**

7. *A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.*

Sol. $\pi (18)^2 \times 32 = \frac{1}{3}\pi (r)^2 \times 24 \Rightarrow r = 36$ cm

Slant height = $\sqrt{(36)^2 + (24)^2}$ cm = $12\sqrt{13}$ cm.

8. *Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?*

Sol. Volume of water flows out of canal in 30 minutes

$$= \frac{1.5 \times 6 \times 10000}{2} \text{ m}^3 = 45000 \text{ m}^3.$$

Area irrigated = $45000 \div \frac{8}{100} = 562500 \text{ m}^2$.

9. *A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?*

Sol. Volume of cylindrical tank = volume of water in the pipe of length l.

$$\Rightarrow \quad \pi(5)^2 \times 2 = \pi\left(\frac{1}{10}\right)^2 \times l \Rightarrow l = 5000 \text{ m} = 5 \text{ km}$$

$\therefore$ Time taken = $\frac{5}{3}$ hr = $\frac{5\times 60}{3}$ minutes = 100 minutes.

Exercise 13.4 *(Page – 257)*

Use $\pi = \frac{22}{7}$ unless stated otherwise.

1. *A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameter of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.*

Sol. Capacity $= \frac{\pi h}{3}[R^2 + r^2 + R.r]$

$$= \frac{22}{7} \times \frac{14}{3}[(2)^2 + (1)^2 + 2 \times 1] \text{ cm}^3$$

$$= \frac{44}{3}[4 + 1 + 2] \text{ cm}^3 = \frac{308}{3} \text{ cm}^3 = 102\frac{2}{3} \text{ cm}^3.$$

2. *The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.*

Sol. Perimeters of circular ends are 18 cm and 6 cm.

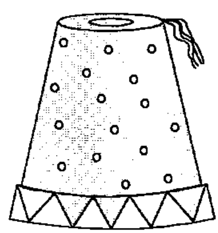
$\therefore$ $2\pi R = 18$ and $2\pi r = 6$

$\Rightarrow$ $R = \frac{9}{\pi}$ cm and $r = \frac{3}{\pi}$ cm.

Curved surface area $= \pi l(R + r)$

$$= \pi \times 4\left(\frac{9}{\pi} + \frac{3}{\pi}\right) \text{ cm}^2 = 48 \text{ cm}^2.$$

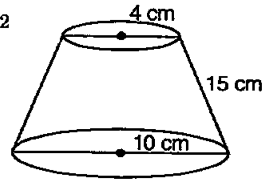
3. *A fez, the cap used by the Turks, is shaped like the frustum of a cone (see figure). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.*



Sol. $l = 15$ cm, $R = 10$ cm, $r = 4$ cm.

Area of material used $= \pi l(R + r) + \pi r^2$

$$= \left[\frac{22}{7} \times 15(10+4) + \frac{22}{7}(4)^2\right] \text{ cm}^2$$



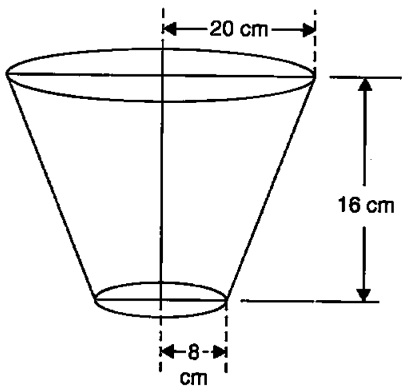
$$= \left[660 + \frac{352}{7}\right] \text{ cm}^2$$

$$= \left[660 + 50\frac{2}{7}\right] \text{ cm}^2 = 710\frac{2}{7} \text{ cm}^2.$$

4. *A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of ₹ 20 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹ 8 per 100 cm². (Take π = 3.14)*

Sol. We have: $r_1 = 20$ cm, $r_2 = 8$ cm

and $h = 16$ cm



∴ Volume of the frustum

$$= \frac{1}{3}\pi h\ [r_1^2 + r_2^2 + r_1 r_2]$$

$$= \frac{1}{3} \times \frac{314}{100} \times 16\ [20^2 + 8^2 + 20 \times 8] \text{ cm}^2$$

$$= \frac{1}{3} \times \frac{314}{100} \times 16\,[400 + 64 + 160] \text{ cm}^2$$

$$= \frac{1}{3} \times \frac{314}{100} \times 16 \times 624 \text{ cm}^3$$

$$= \left[\frac{314}{100} \times 16 \times 208\right] \text{cm}^3$$

$$= \left[\frac{314}{100} \times 16 \times 208\right] \div 1000 \text{ litres}$$

$$= \frac{314 \times 16 \times 208}{100000} \text{ litres}$$

$\therefore$ Cost of milk $= ₹\ 20 \times \dfrac{314 \times 16 \times 208}{100000}$

$$= ₹\ \frac{628 \times 16 \times 208}{10000} = ₹\ \frac{2089984}{10000}$$

$= ₹\ 208.998 \approx$ **₹ 209.**

Now, slant height of the given frustum

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$= \sqrt{16^2 + (20-8)^2} = \sqrt{16^2 + 12^2}$$

$$= \sqrt{256 + 144} = \sqrt{400} = \mathbf{20\ cm}$$

$\therefore$ Curved surface area

$$= \pi (r_1 + r_2)\, l = \frac{314}{100} (20 + 8) \times 20 \text{ cm}^2$$

$$= \frac{314}{100} \times 28 \times 20 \text{ cm}^2 = \frac{314}{5} \times 28 \text{ cm}^2$$

$$= \frac{8792}{5} \text{cm}^2 = \mathbf{1758.4\ cm^2}$$

Area of the bottom

$$= \pi r^2 = \frac{314}{100} \times 8 \times 8 \text{ cm}^2 = \frac{20096}{100} \text{cm}^2$$

$= \mathbf{200.96\ cm^2}$

$\therefore$ Total area of metal required $= 1758.4 \text{ cm}^2 + 200.96 \text{ cm}^2$

$= 1959.36 \text{ cm}^2$

Cost of metal required $= ₹\ \dfrac{8}{100} \times 1959.36 =$ **₹ 156.75.**

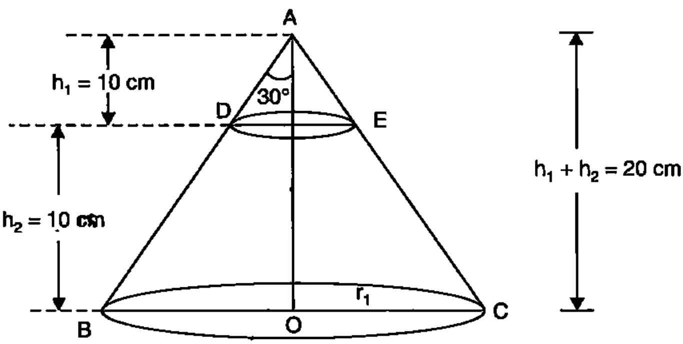
5. *A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained* *be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.*

Sol. Let us consider the frustum DECB of the metallic cone ABC

Here, $r_1 = BO$ and $r_2 = DO$

In Δ AOB, $\frac{r_1}{(h_1 + h_2)} = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad r_1 = (h_1 + h_2) \times \frac{1}{\sqrt{3}} = 20 \times \frac{1}{\sqrt{3}}$



In Δ ADO, $\frac{r_2}{h_1} = \tan 30°$

$\Rightarrow \quad r_2 = h_1 \times \frac{1}{\sqrt{3}} = 10 \times \frac{1}{\sqrt{3}}$

Now, the volume of the frustum DBCE

$$= \frac{1}{3}\pi h_2 \; [r_1^2 + r_2^2 + r_1 r_2]$$

$$= \frac{1}{3} \times \pi \times 10 \left[\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \frac{20}{\sqrt{3}} \times \frac{10}{\sqrt{3}}\right]$$

$$= \frac{\pi}{3} \times 10 \left[\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right] = \frac{\pi}{3} \times 10 \left[\frac{700}{3}\right]$$

Let l be the length and D be diameter of the wire drawn from the frustum. Since the wire is in the form of a cylinder,

$\therefore$ Volume of the wire = $\pi r^2 l$

$$= \pi\left(\frac{D}{2}\right)^2 \times l = \frac{\pi D^2 l}{4} = \frac{\pi l}{4\times16\times16} \qquad \left|\because D = \frac{1}{16}\right.$$

$\because$ [Volume of the frustum] = [Volume of the wire]

$$\therefore \qquad \left[\frac{\pi}{3}\times10\times\frac{700}{3}\right] = \frac{\pi l}{4\times16\times16}$$

$$\Rightarrow \qquad \frac{l}{4\times16\times16} = \frac{10\times700}{3\times3}$$

$$\Rightarrow \qquad l = \frac{10\times700}{3\times3}\times4\times16\times16$$

$$= \frac{7168000}{9\times100} = 7964.44 \text{ m}$$

Thus, the required length of the wire = **7964.44 m.**

Exercise 13.5 (OPTIONAL) *(Page – 258)*

1. *A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm³.*

Sol. Length of wire used in one round = $2\pi \times 5 = 10\pi$ cm.

$$\text{Total rounds required} = 12 \div \frac{3}{10} = \frac{12\times10}{3} = 40$$

$\therefore$ Length of wire required = $40 \times 10\pi = 400\pi$

$= 400 \times 3.14 = 1256$ cm

$$\text{Volume of wire} = \pi\left(\frac{3}{20}\right)^2 \times 400\pi \text{ cm}^3 = 9\pi^2 \text{ cm}^3$$

Mass of wire $= 8.88 \times 9\pi^2 = 8.88 \times 9 \times 3.14 \times 3.14$

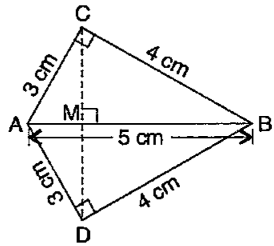
$= 787.98 \text{ g} \simeq 788 \text{ g}$ (approx.).

2. *A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate).*

Sol. Right-angled triangle ABC is made to revolve about its hypotenuse AB.

$$AB = \sqrt{AC^2 + BC^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ cm}$$

Now, $\Delta ABC \sim \Delta CAM$



$$\therefore \quad \frac{AB}{AC} = \frac{BC}{CM} \Rightarrow \frac{5}{3} = \frac{4}{CM}$$

$$\Rightarrow \quad CM = \frac{12}{5} = 2.4 \text{ cm}$$

CM is the radius of cone DACMD as well as cone DBCMD,

i.e., $r = 2.4$ cm

Further $AM = \sqrt{3^2 - \left(\frac{12}{5}\right)^2} = \sqrt{(3)^2 - (2.4)^2} = 1.8$ cm

Also, $BM = 5 - 1.8 = 3.2$ cm

Now, the required volume = volume of cone DACMD + volume of cone DBCMD

$$= \frac{1}{3}\pi r^2(AM) + \frac{1}{3}\pi r^2(BM)$$

$$= \frac{1}{3}\pi r^2(AB)$$

$$= \frac{1}{3} \times 3.14 \times 2.4 \times 2.4 \times 5$$

$$= 30.144 \text{ cm}^3 \sim 30.14 \text{ cm}^2$$

Required surface area $= \pi r(AC) + \pi r(BC)$

$= 3.14 \times 2.4 \times (3 + 4)$

$= 3.14 \times 2.4 \times 7$

$= 52.752 \text{ cm}^2 \sim 52.75 \text{ cm}^2.$

3. *A cistern, internally measuring 150 cm × 120 cm × 110 cm, has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm × 7.5 cm × 6.5 cm?*

Sol. Let number of bricks be n.

Total volume of cistern $= 150 \times 120 \times 110 = 1980000 \text{ cm}^3$

Volume of water in cistern

$= 129600 \text{ cm}^3$

Volume of one brick $= 22.5 \times 7.5 \times 6.5$

$= 1096.875 \text{ cm}^3$

Volume of water absorbed by one brick

$$= \frac{1}{17} \times 1096.875 \text{ cm}^3$$

Volume of water absorbed by n bricks

$$= \frac{n}{17} \times 1096.875 \text{ cm}^3$$

Volume of portion of empty cistern

= Total volume of cistern

– (Volume of water given in it

– volume of water absorbed by n bricks)

$$= 1980000 - \left(129600 - \frac{n}{17} \times 1096.875\right)$$

$$= \left(1850400 + \frac{n}{17} \times 1096.875\right) \text{cm}^3$$

Now, volume of n bricks = volume of portion of empty cistern

$$\Rightarrow \quad n \times 1096.875 = 1850400 + \frac{n}{17} \times 1096.875$$

$$\Rightarrow \quad n \times 1096.875 \left(1 - \frac{1}{17}\right) = 1850400$$

$$\Rightarrow \quad n = \frac{1850400 \times 17}{16 \times 1096.875}$$

$$\Rightarrow \quad n = 1792.41$$

Hence, 1792 bricks can be put in without overflowing the water.

4. *In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 97280 km², show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.*

Sol. Rainfall is 10 cm over an area 97280 km^2

$$\therefore \text{ Total volume of water} = 97280 \times \frac{10}{100 \times 1000}$$

$$= 9.728 \text{ km}^3.$$

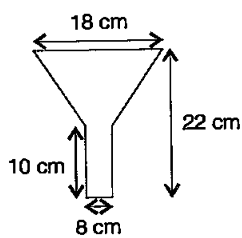
$$\text{Volume of water of 3 rivers} = 3 \times 1072 \times \frac{75}{1000} \times \frac{3}{1000}$$

$$= 0.7236 \text{ km}^3.$$

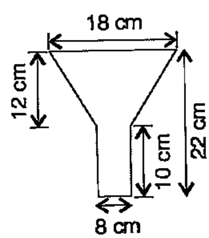
On comparing we notice, these are approximately same.

Hence proved.

5. *An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see figure).*



Sol. Slant height of frustum



$$= \sqrt{(12)^2 + (9-4)^2} \text{ cm} = 13 \text{ cm}.$$

$\therefore$ Area of tin required

$$= [2\pi \times 4 \times 10 + \pi \times 13(9 + 4)]\text{cm}^2$$

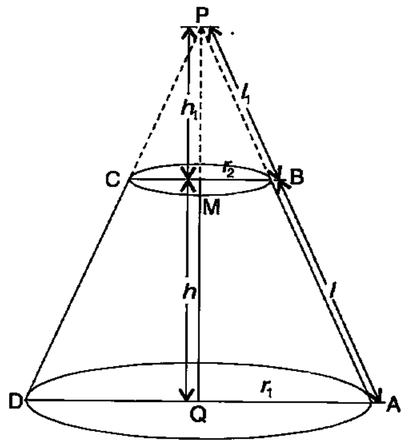
$$= (80\pi + 169\pi) \text{ cm}^2 = 249\,\pi \text{ cm}^2$$

$$= 249 \times \frac{22}{7} \text{ cm}^2$$

$$= \frac{5478}{7} \text{ cm}^2 = 782\frac{4}{7} \text{ cm}^2.$$

6. *Derive the formula for the curved surface area and total surface area of the frustum of a cone, using the symbols which are usually used.*

Sol. In figure, ABCD is a frustum of a cone APD. h and l be respectively the vertical and slant height of the frustum. r_1 and r_2 be the radii of the ends of the frustum.



From the figure,

$$\Delta APQ \sim \Delta BPM$$

$$\therefore \quad \frac{AP}{BP} = \frac{AQ}{BM}$$

$$\Rightarrow \quad \frac{l_1 + l}{l_1} = \frac{r_1}{r_2}$$

or $\quad \dfrac{l_1 + l}{l_1} - 1 = \dfrac{r_1}{r_2} - 1 \quad \Rightarrow \quad \dfrac{l}{l_1} = \dfrac{r_1 - r_2}{r_2}$ ...(*i*)

Now, curved surface area of the frustum (CSA)

= curved surface area of cone APD

− curved surface area of cone BPC

$= \pi r_1(l + l_1) - \pi r_2 l_1$

$= \pi r_1 l + \pi r_1 l_1 - \pi r_2 l_1$

$= \pi r_1 l + \pi(r_1 - r_2) l_1$

$= \pi r_1 l + \pi(r_1 - r_2) \dfrac{r_2 l}{r_1 - r_2}$ [From (*i*)]

$= \pi r_1 l + \pi r_2 l$

$\Rightarrow$ **CSA = $\pi(r_1 + r_2)l$**

Total surface area of the frustum (TSA)

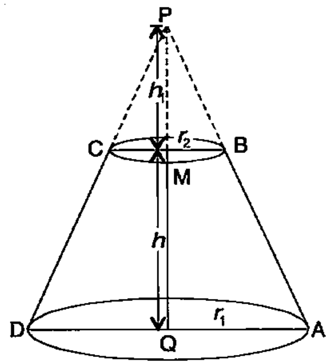
= CSA + surface area of two circular ends

$$= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$$

$\Rightarrow$ $$\boxed{\textbf{TSA} = \pi(r_1 + r_2)l + \pi(r_1^2 + r_2^2)}$$

7. *Derive the formula for the volume of the frustum of a cone, using the symbols which are usually used.*

Sol. In figure, ABCD is a frustum of a cone APD. h be the height of the frustum. r_1 and r_2 be the radii of the frustum.



From the figure,

$$\Delta APQ \sim \Delta BPM$$

$\therefore$ $$\frac{PQ}{PM} = \frac{AQ}{BM}$$

$\Rightarrow$ $$\frac{h_1 + h}{h_1} = \frac{r_1}{r_2}$$

or $$\frac{h_1 + h}{h_1} - 1 = \frac{r_1}{r_2} - 1$$

$\Rightarrow$ $$\frac{h}{h_1} = \frac{r_1 - r_2}{r_2}$$

$\Rightarrow$ $$h_1 = \frac{r_2 h}{r_1 - r_2} \qquad ...(i)$$

Now, volume (V) of the frustum

= volume of cone APD – volume of cone BPC

$$= \frac{1}{3}\pi r_1^2(h_1 + h) - \frac{1}{3}\pi r_2^2 h_1$$

$$= \frac{1}{3}\pi(r_1^2 h_1 + r_1^2 h - r_2^2 h_1)$$

$$= \frac{1}{3}\pi[r_1^2 h + (r_1^2 - r_2^2)h_1]$$

$$= \frac{1}{3}\pi[r_1^2 h + (r_1 - r_2)(r_1 + r_2)h_1]$$

$$= \frac{1}{3}\pi\left[r_1^2 h + (r_1 - r_2)(r_1 + r_2) \times \frac{r_2 h}{r_1 - r_2}\right] \qquad \text{[From (i)]}$$

$$= \frac{1}{3}\pi(r_1^2 h + r_1 r_2 h + r_2^2 h)$$

$\Rightarrow$ $$\mathbf{V} = \frac{\pi h}{3}(r_1^2 + r_1 r_2 + r_2^2).$$

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