15 Probability

Lesson at a Glance

1. $P(E) = \frac{\text{number of outcomes favourable to E}}{\text{number of all possible outcomes}}$.
2. An outcome of a random experiment is called an elementary event.
3. The sum of the probabilities of all the elementary events of an experiment is 1.
4. The probability of an impossible event is 0.
5. The probability of a sure event is 1.
6. For any event E,

   $0 \le P(E) \le 1$.
7. Probability of an event cannot be negative.
8. If E and $\overline{E}$ represent respectively occuring and not occuring an event, then,

   $P(E) + P(\overline{E}) = 1$.
9. E and $\overline{E}$ are called complementary events.

TEXTBOOK QUESTIONS SOLVED

Exercise 15.1 *(Page – 308-311)*

1. *Complete the following statements:*
   (i) *Probability of an event E + Probability of the event 'not E' =* ..... .
   (ii) *The probability of an event that cannot happen is* ..... . *Such an event is called* ..... .
   (iii) *The probability of an event that is certain to happen is* ..... . *Such an event is called* ..... .

(*iv*) *The sum of the probabilities of all the elementary events of an experiment is* ..... .

(*v*) *The probability of an event is greater than or equal to* ..... *and less than or equal to* ..... .

Sol. (*i*) 1 (*ii*) 0, impossible event (*iii*) 1, sure event

(*iv*) 1 (*v*) 0, 1.

2. *Which of the following experiments have equally likely outcomes? Explain.*

(*i*) *A driver attempts to start a car. The car starts or does not start.*

(*ii*) *A player attempts to shoot a basketball. She/he shoots or misses the shot.*

(*iii*) *A trial is made to answer a true-false question. The answer is right or wrong.*

(*iv*) *A baby is born. It is a boy or a girl.*

Sol. In case (*i*) and (*ii*) events can be favourable to particular event, so not equally likely, whereas in (*iii*) and (*iv*) events are equally likely.

3. *Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?*

Sol. Because, both the events 'Head' or 'Tail' are equally likely to occur.

4. *Which of the following cannot be the probability of an event?*

(A) $\frac{2}{3}$ (B) -1.5 (C) 15% (D) 0.7

Sol. (B), as probability of an event cannot be negative.

5. *If P(E) = 0.05, what is the probability of 'not E'?*

Sol. P(not E) = 1 – P(E) = 1 – 0.05 = 0.95.

6. *A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out*

(*i*) *an orange flavoured candy?*

(*ii*) *a lemon flavoured candy?*

Sol. (*i*) As there are only lemon flavoured candies, therefore,

getting an orange flavoured candy is an impossible event. Hence probability is 0.

(*ii*) Lemon flavoured candy is a sure event, hence probability is 1.

7. *It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?*

Sol. P(two students having the same birthday)

$= 1 - P$

(two students not having the same birthday)

$= 1 - 0.992 = 0.008.$

8. *A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red? (ii) not red?*

Sol. Total balls (3 red + 5 black) = 8 balls.

(*i*) Probability of a red ball $= \frac{3}{8}$

(*ii*) Probability of not a red ball = 1 – P(red ball)

$= 1 - \frac{3}{8} = \frac{5}{8}.$

9. *A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red? (ii) white? (iii) not green?*

Sol. Total marbles in a box are 17, *i.e.*, (5 red + 8 white + 4 green).

(*i*) Favourable cases for red marble are 5.

$\therefore$ Probability of a red marble $= \frac{5}{17}.$

(*ii*) Probability of a white marble $= \frac{8}{17}.$

(*iii*) P(not green) = 1 – P(green) $= 1 - \frac{4}{17} = \frac{13}{17}.$

10. *A piggy bank contains hundred 50 p coins, fifty ₹ 1 coins, twenty ₹ 2 coins and ten ₹ 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned*

upside down, what is the probability that the coin (i) will be a 50 p coin? (ii) will not be a ₹5 coin?

Sol.

50 p coins	₹ 1 coins	₹ 2 coins	₹ 5 coins
100	50	20	10

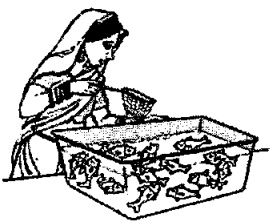
Total coins in the piggy bank = 100 + 50 + 20 + 10 = 180

(*i*) P(50 p coin) = $\frac{100}{180} = \frac{5}{9}$.

(*ii*) P(not a ₹ 5 coin) = 1 – P(₹ 5 coin)

$= 1 - \frac{10}{180} = \frac{17}{18}$.

11. *Gopy buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (see figure). What is the probability that the fish taken out is a male fish?*

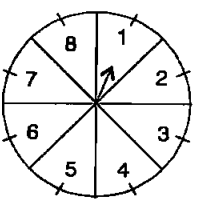


Sol. Total number of fish in the tank

= (5 male + 8 female) fish = 13 fish

∴ Probability of male fish = $\frac{5}{13}$.

12. *A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see figure), and these are equally likely outcomes. What is the probability that it will point at*



(*i*) *8?*

(*ii*) *an odd number?*

(*iii*) *a number greater than 2?*

(*iv*) *a number less than 9?*

Sol. Total numbers on the disc are 8, *i.e.,* 1, 2, 3, 4, 5, 6, 7, 8.

(*i*) Probability of pointing at 8 = $\frac{1}{8}$.

(*ii*) Favourable odd numbers are 4, *i.e.,* 1, 3, 5, 7.

$\therefore$ Probability of pointing towards odd numbers

$$= \frac{4}{8} = \frac{1}{2}.$$

(*iii*) Favourable numbers greater than 2 are 6, *i.e.,* 3, 4, 5, 6, 7, 8.

$\therefore$ Probability of pointing towards number greater than 2 = $\frac{6}{8} = \frac{3}{4}$.

(*iv*) Favourable numbers less than 9 are 8, *i.e.,* 1, 2, 3, 4, 5, 6, 7, 8.

$\therefore$ Probability of pointing towards a number less than 9 is 1.

13. *A die is thrown once. Find the probability of getting*

(*i*) *a prime number;*

(*ii*) *a number lying between 2 and 6;*

(*iii*) *an odd number*

Sol. Total outcomes: 1, 2, 3, 4, 5, 6

(*i*) Favourable outcomes for a prime number are 3, *i.e.,* 2, 3, 5.

$\therefore$ Probability of getting a prime number = $\frac{3}{6} = \frac{1}{2}$.

(*ii*) Favourable cases for number between 2 and 6 are 3, *i.e.,* 3, 4, 5.

$\therefore$ Probability of getting a number between 2 and 6

$$= \frac{3}{6} = \frac{1}{2}.$$

(*iii*) Probability of getting an odd number (*i.e.,* 1, 3, 5)

$$= \frac{3}{6} = \frac{1}{2}.$$

14. *One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting*

(*i*) *a king of red colour* (*ii*) *a face card*

(*iii*) *a red face card* (*iv*) *the jack of hearts*

(*v*) *a spade* (*vi*) *the queen of diamonds.*

Sol. (*i*) 2 cases favourable, *i.e.*, 1 king of heart and 1 king of diamond, therefore, probability $= \frac{2}{52} = \frac{1}{26}$

(*ii*) Face card, *i.e.*, there are 12 face cards (4 jacks + 4 queens + 4 kings), therefore, probability

$$= \frac{12}{52} = \frac{3}{13}.$$

(*iii*) There are 6 red face cards.

$$\therefore \text{ Probability of a red face card} = \frac{6}{52} = \frac{3}{26}.$$

(*iv*) There is one jack of hearts.

$$\therefore \quad \text{P(jack of heart)} = \frac{1}{52}.$$

(*v*) There are 13 spades.

$$\therefore \quad \text{P(getting a spade)} = \frac{13}{52} = \frac{1}{4}.$$

(*vi*) There is one queen of diamonds.

$$\therefore \text{ P(getting of a queen of diamonds)} = \frac{1}{52}.$$

15. *Five cards—the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.*

(*i*) *What is the probability that the card is the queen?*

(*ii*) *If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?*

Sol. (*i*) Total cards = 5

Favourable for queen = 1. Therefore, probability is $\frac{1}{5}$.

(*ii*) (*a*) Four possible cases and one is favourable, so, probability is $\frac{1}{4}$.

(*b*) None is favourable. Therefore, probability is 0.

16. *12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.*

Sol. Total pens are 144, (12 defective + 132 good ones).

Favourable cases for good pen are 132.

$\therefore$ Probability of drawing a good pen $= \frac{132}{144} = \frac{11}{12}$.

17. (*i*) *A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?*

(*ii*) *Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?*

Sol. (*i*) Total bulbs = 20, defective bulbs = 4

$\therefore$ Probability of a defective bulb $= \frac{4}{20} = \frac{1}{5}$.

(*ii*) As bulb is non-defective and non-replaced. Therefore, total bulbs now are 19 (4 defective + 15 non-defective).

$\therefore$ Probability of a non-defective bulb $= \frac{15}{19}$.

18. *A box containing 90 discs are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.*

Sol. Total discs are 90, numbered 1 to 90.

(*i*) Favourable two-digit numbered discs are 81, *i.e.*, (10, 11, ..., 90)

$\therefore$ Probability of getting a two-digit numbered disc $= \frac{81}{90} = \frac{9}{10}$.

(*ii*) Favourable cases for a perfect square number are 1, 4, 9, 16, 25, 36, 49, 64, 81, *i.e.*, 9.

$\therefore$ Probability of getting a perfect square numbered disc $= \frac{9}{90} = \frac{1}{10}$.

(*iii*) Favourable cases for a number divisible by 5 are 18, *i.e.*, (5, 10, 15, ..., 90)

$\therefore$ Probability of getting a disc numbered divisible by 5 $= \frac{18}{90} = \frac{1}{5}$.

19. *A child has a die whose six faces show the letters as given below:*

A	B	C	D	E	A

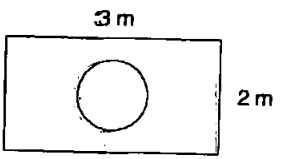
The die is thrown once. What is the probability of getting (i) A? (ii) D?

Sol. (*i*) Here, A is shown on two faces so favourable cases = 2

$\therefore$ Probability of getting A $= \frac{2}{6} = \frac{1}{3}$.

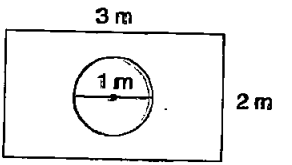
(*ii*) Favourable case is only 1. So, probability of getting D $= \frac{1}{6}$.

20. *Suppose you drop a die at random on the rectangular region shown in Fig. What is the probability that it will land inside the circle with diameter 1 m?*



Sol. Total area of the region

$= 3 \times 2 \text{ m}^2 = 6 \text{ m}^2$



Area of the circular region

$$= \pi\left(\frac{1}{2}\right)^2 \text{ m}^2 = \frac{11}{14} \text{ m}^2$$

$\therefore$ Probability that the die will land inside the circular region $= \frac{11}{14} \div 6 = \frac{11}{84}$.

21. *A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that*

(i) She will buy it?

(ii) She will not buy it?

Sol. Total ball pens are 144, defective ball pens are 20, non-defective ball pens are 124.

(i) Probability that Nuri buys these pens

$$= \text{P(non-defective pen)} = \frac{124}{144} = \frac{31}{36}.$$

(ii) Probability that Nuri does not buy it

$$= \text{P(defective pen)} = \frac{20}{144} = \frac{5}{36}.$$

22. *Refer to Example 13. (i) Complete the following table:*

Event: 'Sum on 2 dice'	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$. *Do you agree with this argument? Justify your answer.*

Sol. When two dice are thrown, then total favourable cases are 36, *i.e.,* (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6).

(i) Favourable cases for "sum on 2 dice as 3" are 2, *i.e.,* (1, 2), (2, 1).

$\therefore$ Probability of getting sum on two dice 3 = $\frac{2}{36}$.

Similarly, favourable cases for sum on 2 dice

— as 4 are: (1, 3), (2, 2), (3, 1)

$\Rightarrow$ P (sum as 4) = $\frac{3}{36} = \frac{1}{12}$

— as 5 are: (1, 4), (2, 3), (3, 2), (4, 1)

$\Rightarrow$ P (sum as 5) = $\frac{4}{36} = \frac{1}{9}$

— as 6 are: (1, 5), (2, 4), (3, 3), (4, 2), (5, 1)

$\Rightarrow$ P (sum as 6) = $\frac{5}{36}$

— as 7 are: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

$\Rightarrow$ P (sum as 7) = $\frac{6}{36} = \frac{1}{6}$

— as 9 are: (3, 6), (4, 5), (5, 4), (6, 3)

$\Rightarrow$ P (sum as 9) = $\frac{4}{36} = \frac{1}{9}$

— as 10 are: (4, 6), (5, 5), (6, 4)

$\Rightarrow$ P (sum as 10) = $\frac{3}{36} = \frac{1}{12}$

— as 11 are: (6, 5), (5, 6)

$\Rightarrow$ P (sum as 11) = $\frac{2}{36} = \frac{1}{18}$

(*ii*) Argument is wrong, as cases are not equally likely to occur.

23. *A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.*

Sol. Total outcomes are HHH, HHT, HTH, THH, HTT, THT, TTH, TTT.

Hanif loses if he does not get HHH or TTT.

$\therefore$ Probability of losing the game = $\frac{6}{8} = \frac{3}{4}$.

24. *A die is thrown twice. What is the probability that*

(i) 5 will not come up either time?

(ii) 5 will come up at least once?

(**Hint.** *Throwing a die twice and throwing two dice simultaneously are treated as the same experiment*)

Sol. Total outcomes when a die is thrown twice are 36

(*i*) 5 will not come either time

$\Rightarrow$ 5 is not favourable on any of the dice.

Favourable outcomes are 25, *i.e.*, (1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 6).

$\therefore$ Probability of not getting 5 either time = $\frac{25}{36}$.

(*ii*) Favourable cases that 5 comes at least once are 11, *i.e.*, (1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6).

$\therefore$ Probability of getting 5 at least once = $\frac{11}{36}$.

25. *Which of the following arguments are correct and which are not correct? Give reasons for your answer.*

(i) If two coins are tossed simultaneously, there are three possible outcomes—two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.

(ii) If a die is thrown, there are two possible outcomes—an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Sol. (*i*) Argument is not correct as there are four possible outcomes.

One on each $\Rightarrow$ HT or TH, *i.e.*, two outcomes.

On both are same $\Rightarrow$ HH and TT, *i.e.*, two outcomes.

$\therefore$ P(HH) = $\frac{1}{4}$ = P(TT)

and P(a head or a tail) = $\frac{2}{4} = \frac{1}{2}$

(*ii*) Correct, as both the outcomes are equally likely to occur.

Exercise 15.2 (OPTIONAL) *(Page – 311-312)*

1. *Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on (i) the same day? (ii) consecutive days? (iii) different days?*

Sol. Total cases of the days pairs are 25; *i.e.,* (Tuesday, Tuesday), (Tuesday, Wednesday), ..., (Saturday, Saturday).

(*i*) Favourable cases for same day are 5, *i.e.,* Tuesday to Saturday

$\therefore$ Probability = $\frac{5}{25} = \frac{1}{5}$.

(*ii*) Favourable cases for consecutive days are:

(Tuesday, Wednesday), (Wednesday, Tuesday), (Wednesday, Thursday), (Thursday, Wednesday), (Thursday, Friday), (Friday, Thursday), (Friday, Saturday), (Saturday, Friday)

$\therefore$ Probability = $\frac{8}{25}$.

(*iii*) Favourable cases for different days = 25 – 5 = 20.

(Tuesday, Wednesday), ..., (Tuesday, Saturday), (Wednesday, Thursday), ... (Wednesday, Tuesday), ... (Saturday, Friday).

$\therefore$ Probability = $\frac{20}{25} = \frac{4}{5}$.

2. *A die is numbered in such a way that its faces show the numbers 1, 2, 2, 3, 3, 6. It is thrown two times and the total score in two throws is noted. Complete the following table which gives a few values of the total score on the two throws:*

Number in first throw

Number in second throw						
+	*1*	*2*	*2*	*3*	*3*	*6*
1	*2*	*3*	*3*	*4*	*4*	*7*
2	*3*	*4*	*4*	*5*	*5*	*8*
2					*5*	
3						
3			*5*			*9*
6	*7*	*8*	*8*	*9*	*9*	*12*

What is the probability that the total score is

(i) even? *(ii) 6?* *(iii) at least 6?*

Sol.

+	1	2	2	3	3	6
1	2	3	3	4	4	7
2	3	4	4	5	5	8
2	3	4	4	5	5	8
3	4	5	5	6	6	9
3	4	5	5	6	6	9
6	7	8	8	9	9	12

(*i*) Favourable outcomes for even total are 18.

$\therefore$ Probability of an even total $= \frac{18}{36} = \frac{1}{2}$.

(*ii*) Favourable outcomes for a total of 6 are 4.

$\therefore$ Probability for total score to be 6 $= \frac{4}{36} = \frac{1}{9}$.

(*iii*) Favourable outcomes for a total score 'at least 6' are 15.

$\therefore$ Probability $= \frac{15}{36} = \frac{5}{12}$.

3. *A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.*

Sol. Let number of blue balls be x. Red balls are 5.

Total balls = $5 + x$.

Probability of blue ball = 2 × probability of red ball

$$\frac{x}{5+x} = 2 \times \frac{5}{5+x} \quad \Rightarrow \quad x = 10.$$

$\therefore$ Total blue balls are 10.

4. *A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball?*

If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.

Sol. (*i*) Probability of black ball = $\frac{x}{12}$.

(*ii*) Total black balls are $(x + 6)$ and now total balls in the box are

$$12 + 6 = 18$$

According to the condition,

$$\frac{x+6}{18} = 2 \times \frac{x}{12} \quad \Rightarrow \quad x + 6 = 3x$$

$$\Rightarrow \qquad 2x = 6 \quad \Rightarrow \quad x = 3.$$

5. *A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.*

Sol. Let number of blue marbles = x.

$\therefore$ Number of green marbles = $24 - x$

According to given condition,

$$\frac{24-x}{24} = \frac{2}{3}$$

$$\Rightarrow \qquad 24 - x = 16 \quad \Rightarrow \quad x = 8$$

$\therefore$ Number of blue marbles = 8.