16 Probability

Lesson at a Glance

1. Combination and Permutations

1. A **combination** is a selection of some or all of a number of given objects.

The number of combination of n distinct objects taken r at a time ($r \leq n$) is denoted by ${}^nC_r$ and is defined as

$$ {}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\ldots \text{to } r \text{ factors}}{1.2.3\ldots r} $$

For example, ${}^8C_3 = \dfrac{8.7.6}{1.2.3} = 56$, ${}^nC_0 = 1$, ${}^nC_n = 1$.

If $r > \dfrac{n}{2}$, then it is better to simplify ${}^nC_r$ as ${}^nC_{n-r}$.

For example, ${}^{52}C_{50} = {}^{52}C_{52-50} = {}^{52}C_2 = \dfrac{52 \cdot 51}{2 \cdot 1}$

$= 26 \cdot 51 = 1326$

2. The symbol ${}^nP_r$ denotes the number of permutations of n distinct elements taken r at a time and is defined as

$$ {}^nP_r = \frac{n!}{(n-r)!} = n(n-1)(n-2) \ldots\ldots \text{ to } r \text{ factors.} $$

For example, ${}^8P_3 = 8 \cdot 7 \cdot 6 = 336$

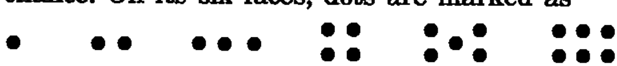
${}^nP_0$ is defined as 1 and ${}^nP_n = n!$

3. The number of **circular** permutations of n different objects is $(n-1)!$

2. Introduction to probability. The science that measures uncertainty is termed as probability.

Probability is a quantitative measure of certainty.

Note: 1. Die. A die is a small cube used in games of chance. On its six faces, dots are marked as



Note: 2. Cards. A pack (or deck) of playing cards has 52 cards, divided into four **SUITS:**

(*i*) Spades (*ii*) Clubs (*iii*) Hearts (*iv*) Diamonds

Each suit has 13 cards, nine cards numbered 2 to 10, an Ace, a King, a Queen and a Jack or Knave. Spades and Clubs are **black-faced cards** while Heart and Diamonds are **red-faced cards.** The Kings, Queens and Jacks are called **face cards.** The Aces, Kings, Queens and Jacks are called **honour cards.** Cards which are not face cards *i.e.*, cards from 1 to 10 are called number cards.

3. **Random experiment.** An experiment having more than one outcomes which cannot be predicted in advance is called a 'random experiment'.
4. **Sample space.** The set of all possible outcomes of a random experiment is called the sample space associated with the random experiment.

   For example, if a coin is tossed n times or n coins are tossed once, the sample space has 2^n points.

   If a coin is tossed once, sample space is {H, T}.

   If a coin is tossed twice, sample space has $2^2 = 4$ points namely {HH, HT, TH, TT}.

   If a coin is tossed thrice, sample space has $2^3 = 8$ points namely {HHH, THH, HTH, HHT, HTT, THT, TTH, TTT}.

   If a die is tossed once, sample space is {1, 2, 3, 4, 5, 6}.

   If a die is tossed n times, then sample space has 6^n points.

   If a dice is tossed twice, sample space has $6^2 = 36$ points namely

   $$\{(1, 1), (1, 2), ..., (1, 6);$$
   $$(2, 1), (2, 2), ..., (2, 6);$$
   $$\vdots$$
   $$(6, 1), (6, 2), ..., (6, 6)\}.$$

5. **Event.** Every subset E of the sample space S of a random experiment is called an event associated with the random experiment.

   If we throw a die, then the sample space is S = {1, 2, 3, 4, 5, 6} . $E_1 = \{1, 3, 5\}$, $E_2 = \{2, 4, 6\}$, $E_3 = \{1, 4\}$, $E_4 = \{2, 3\}$, $E_5 = \{2\}$ are all subsets of S and hence they are events.

   Again $E_1 \cup E_2$, $E_2 \cap E_5$, are also subsets of S and hence, they are also events.

6. **Occurrence of an event.** An event E is said to have occurred if the outcome w of the experiment is such that $w \in E$. If the outcome w is such that $w \notin E$, then we say that the event E has not occurred.

7. Some Special Events

(*a*) **Simple event.** If an event has only one sample point of the sample space, then it is called a simple (or elementary) event.

Consider the experiment of rolling a die. Sample space of this experiment is S = {1, 2, 3, 4, 5, 6}. The events {1}, {4}, {5} are simple events. Every simple event is a singleton set.

(*b*) **Compound event.** If an event has more than one sample points, then it is called a compound event. In the above experiment of rolling a die, the events {2, 5}, {1, 3, 5}, {3, 4, 5, 6} are compound events.

(*c*) **Sure event.** The sample space S is itself a subset of S. For this reason, the event represented by S is called the sure or (certain) event.

For example, let a die be thrown, then S = {1, 2, 3, 4, 5, 6}.

Let E = {w : w is odd or even} = {1, 2, 3, 4, 5, 6} = S.

(*d*) **Impossible event.** The empty set ϕ is also a subset of S, therefore, ϕ represents an event.

Since no outcome of the experiment can belong to ϕ, the event represented by ϕ is called an impossible event. For example, let a die be thrown, then

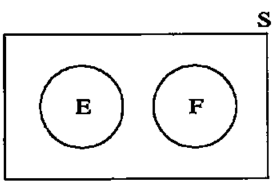
$$S = \{1, 2, 3, 4, 5, 6\}.$$

Let E be the event of getting a number 7 on a die. Then, E is an impossible event.

(*e*) **Equally likely events.** Events are said to be equally likely when there is no reason to expect any one in preference to another. We regard all the elementary events as equally likely to occur when the experiment is performed.

In tossing a coin, head and tail are equally likely to appear. When a card is drawn from a well-shuffled pack, any card may appear in the draw so that the 52 different cases are equally likely.

(*f*) **Mutually exclusive events.** Two events are said to be mutually exclusive (or imcompatible) if the occurrence of one excludes the occurrence of the other. In the



adjoining figure, events E and F have no common element and hence are disjoint.

E and F are mutually exclusive events $\Leftrightarrow$ $E \cap F = \phi$.

(*g*) **Complement (or negation) of an event.** Given an event E of the sample spaces S, the event which occurs when and only when E does not occur is called the complement of E or the negation of E or the event 'not-E' and is denoted by E' or E^c or $\bar{E}$.

Thus, $E' = S - E = \{w : w \in S \text{ and } w \notin E\}$.

(*h*) **Exhaustive events.** Let S be the sample space of a random experiment, then events $E_1, E_2, ..., E_n$ are said to be exhaustive events if $E_1 \cup E_2 \cup ... \cup E_n = S$.

8. Algebra of events.

Let E and F be events of a sample space S.

(*i*) $E \cup F$ *i.e.*, (E or F) is the event "Either E or F or both".

i.e., **$E \cup F$ is the event of happening of at least one of the two events E and F.**

(*ii*) $E \cap F$ *i.e.*, (E and F) is the event "both E and F".

(*iii*) $E - F$ is the event "E but not F".

$$E - F = E - E \cap F$$

(*iv*) **De-Morgan's Laws:**

(*a*) $(E \cup F)' = E' \cap F'$

(*b*) $(E \cap F)' = E' \cup F'$

9. Probability of an event.

$$P(E) = \frac{\text{Number of outcomes favourable to event E}}{\text{Total number of equally likely outcomes}} = \frac{n(E)}{n(S)}$$

For example, if a coin is tossed, there are two equally likely results (outcomes) a head or tail, hence the probability of getting a head

$$= \frac{n(E)}{n(S)} = \frac{1}{2}.$$

$$0 \le P(E) \le 1.$$

If $E = \phi$, *i.e.*, **impossible event,** then $P(E) = 0$

If $E = S$, *i.e.*, **sure event,** then $P(E) = 1$.

10. Odds in favour of an event and odds against an event.

$n(E') = n(S) - n(E)$ will denote the number of cases unfavourable (non-occurrence) to event E.

$$\therefore\ n(E) + n(E') = n(S).$$

Odds in favour of the event E are $\dfrac{n(E)}{n(E')}$ $(= n(E) : n(E'))$

and **odds against** the event E are $\dfrac{n(E')}{n(E)}$ $(= n(E') : n(E))$

For example, if odds in favour of an event E are 3 : 5 $(= n(E) : n(E'))$;

then $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{3}{3+5} = \dfrac{3}{8}$

$$[\because\ n(S) = n(E) + n(E') = 3 + 5 = 8]$$

Remark. $P(E') = 1 - P(E)$ or $P(E) + P(E') = 1$.

11. Theorems on probability:

1. If the events E and F are mutually exclusive,
   i.e., $E \cap F = \phi$,
   then $P(E \cup F) = P(E) + P(F)$
2. If E and F are two events associated with a random experiment,
   then $P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$
   or **$P(E \cup F)$ *i.e.*, probability of at least one of the two events E and F = $P(E) + P(F) - P(E \cap F)$.**
3. For every event E associated with a random experiment, $P(E')$ or $P(\text{not } E) = 1 - P(E)$.
4. For any two events A and B, $P(A \cap B')$
   $= P(A) - P(A \cap B)$

12. Independent events. Two events E and F defined on the sample space S of a random experiment are said to be independent if $P(E \cap F) = P(E) \cdot P(F)$.

This is called the **multiplication rule of probabilities.**

If E and F are independent events then

1. E and F′ are independent.
2. E′ and F are independent.
3. E′ and F′ are independent.

13. Probability of occurrence of at least one of the r independent events.

P(at least one of the events $E_1, E_2, ..., E_r$ occurs)

$$= P(E_1 \cup E_2 \cup ......, \cup E_r)$$

$$= 1 - P(\text{None of } E_1, E_2, ..., E_r \text{ occurs}).$$

$\therefore$ **Probability of at least one = 1 – P(None).**

TEXTBOOK QUESTIONS SOLVED

EXERCISE 16.1 *(Page No.: 386–387)*

In each of the following Exercises 1 to 7, describe the sample space for the indicated experiment.

1. A coin is tossed three times.

Sol. When a coin is tossed once, it can turn up Head (H) or Tail (T). When a coin is tossed 3 times, there are $2 \times 2 \times 2 = 8$ outcomes.

The possible outcomes are:

All heads = (H, H, H) = HHH

Two heads and one tail = (H, H, T), (H, T, H), (T, H, H) = HHT, HTH, THH

One head and two tails = (H, T, T), (T, H, T), (T, T, H) = HTT, THT, TTH

All tails = (T, T, T) = TTT

Thus, the sample space is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

2. A die is thrown two times.

Sol. Let x denote the outcome of first throw and y denote the outcome of second throw, then $x, y \in \{1, 2, 3, 4, 5, 6\}$. The number of elements in the sample space is $6 \times 6 = 36$.

Thus, the sample space is

$$S = \{(x, y) : x, y \in \{1, 2, 3, 4, 5, 6\}\}$$

= {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

3. A coin is tossed four times.

Sol. When a coin is tossed once, it can turn up Head (H) or Tail (T).

When a coin is tossed 4 times, there are $2 \times 2 \times 2 \times 2 = 2^4 = 16$ outcomes. The possible outcomes are:

All heads = HHHH

Three heads and one tail
= HHHT, HHTH, HTHH, THHH

Two heads and two tails
= HHTT, HTHT, HTTH, THHT, THTH, TTHH

One head and three tails = HTTT, THTT, TTHT, TTTH

All tails = TTTT

Thus, the sample space is

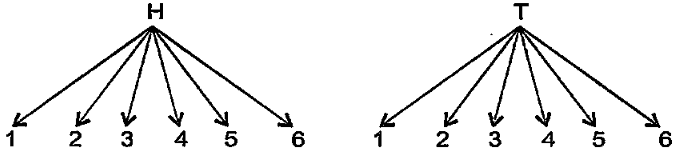
S = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT}

4. A coin is tossed and a die is thrown.

Sol. When a coin is tossed, it can turn up Head (H) or Tail (T).

When a die is thrown, there are six possible outcomes: 1, 2, 3, 4, 5, 6.

When a coin is tossed and a die is thrown, there are $2 \times 6 = 12$ possible outcomes.



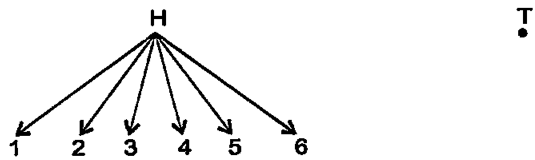
Thus, the sample space is

S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

5. A coin is tossed and then a die is rolled only in case a head is shown on the coin.

Sol. When a coin is tossed, it can turn up Head (H) or Tail (T).

When a die is rolled, there are six possible outcomes: 1, 2, 3, 4, 5, 6.



Thus, the sample space is

S = {H1, H2, H3, H4, H5, H6, T}

6. 2 boys and 2 girls are in room X, and 1 boy and 3 girls in Room Y. Specify the sample space for the experiment in which a room is selected and then a person.

Sol. Let B_1, B_2 be the two boys and G_1, G_2 be the two girls in room X.

Let B_3 be the boy and G_3, G_4, G_5 be the three girls in room Y.

When a room is selected, it is either X or Y.

In room X, when a person (a boy or a girl) is selected, it is B_1 or B_2 or G_1 or G_2. Similarly for room Y.

The sample space S for the experiment is

$$S = \{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}.$$

7. One die of red colour, one of white colour and one of blue colour are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted. Describe the sample space.

Sol. The selected die can be red (R), white (W) or blue (B).

The number on the uppermost face can be 1, 2, 3, 4, 5 or 6.

Thus, the sample space is

S = {R1, R2, R3, R4, R5, R6, W1, W2, W3, W4, W5, W6, B1, B2, B3, B4, B5, B6}

8. An experiment consists of recording boy-girl composition of families with 2 children.

(*i*) **What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?**

(*ii*) **What is the sample space if we are interested in the number of girls in the family?**

Sol. (*i*) The first child can be a boy or a girl followed by a boy or a girl.

∴ Required sample space is S = {BB, BG, GB, GG}

(*ii*) A family with 2 children may have no girl, one girl or two girls.

∴ Required sample space for **the number of girls** in a family is

$$S = \{0, 1, 2\}.$$

9. A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.

Sol. When the first ball drawn is red, the second is white.

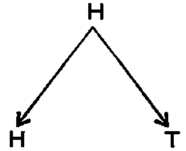
When the first ball drawn is white, the second is either red or white. Denoting red ball by R and white ball by W, the sample space for this experiment is

$$S = \{RW, WR, WW\}.$$

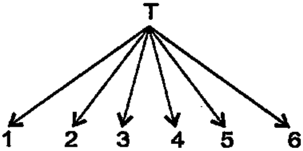
10. An experiment consists of tossing a coin and then throwing it second time if a head occurs. If a tail occurs on the first toss, then a die is rolled once. Find the sample space.

Sol. When a coin is tossed, it can turns up Head (H) or Tail (T).

When the coin turns up Head (H), it is thrown again resulting into Head (H) or Tail (T).



When the coin turns up Tail (T), a die is rolled which can show 1, 2, 3, 4, 5 or 6.



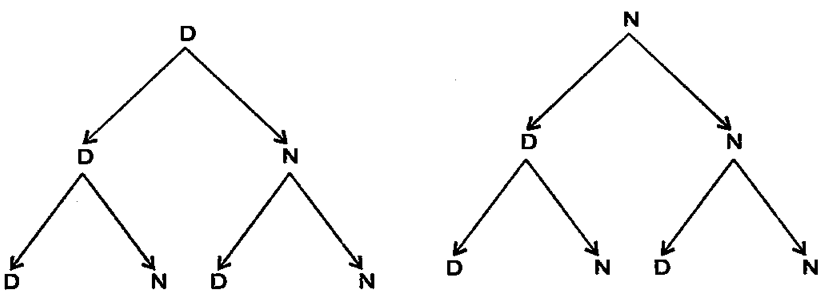
Thus the sample space is

S = {HH, HT, T1, T2, T3, T4, T5, T6}

11. Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non-defective (N). Write the sample space of this experiment.

Sol. The number of bulbs is 3 and each bulb is classified as D or N *i.e.*, in two ways, therefore, total number of possible outcomes is

$$2 \times 2 \times 2 = 8$$

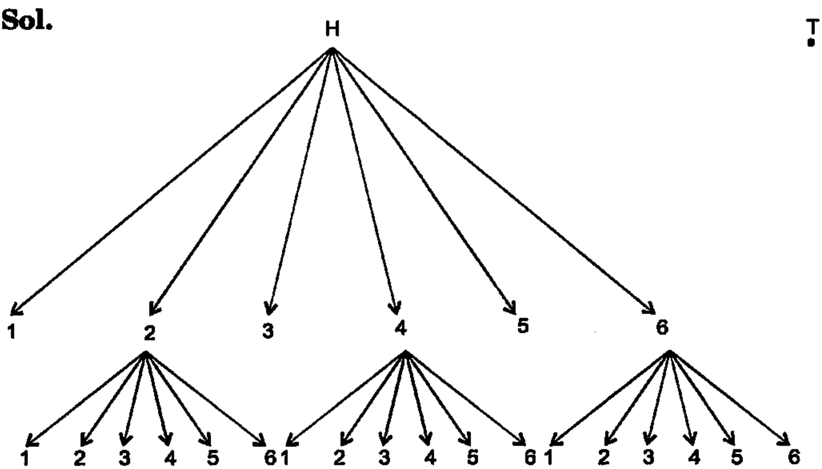


Thus, the sample space is

S = {DDD, DDN, DND, DNN, NDD, NDN, NND, NNN}

12. A coin is tossed. If the outcome is a head, a die is thrown. If the die shows up an even number, the die is thrown again. What is the sample space for the experiment?

Sol.

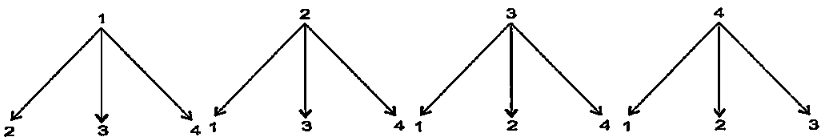


The sample space for the experiment is

S = {T, H1, H3, H5, H21, H22, H23, H24, H25, H26, H41, H42, H43, H44, H45, H46, H61, H62, H63, H64, H65, H66}

13. The numbers, 1, 2, 3 and 4 are written separately on four slips of paper. The slips are put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space for the experiment.

Sol. The first draw can give any one of the four members 1, 2, 3, 4. In each case, the second draw can give any one of the remaining three numbers.



There are 4 × 3 = 12 possible outcomes. The sample space for the experiment is

S = {(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)}.

14. An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.

Sol. If the outcome of rolling a die is even (it can be 2 or 4 or 6), then a coin is tossed once (with possible outcomes H or T).

So we have 2H, 2T, 4H, 4T, 6H, 6T.

If the outcome of rolling a die is odd (it can be 1 or 3 or 5), then a coin is tossed twice (with possible outcomes HH, HT, TH, TT).

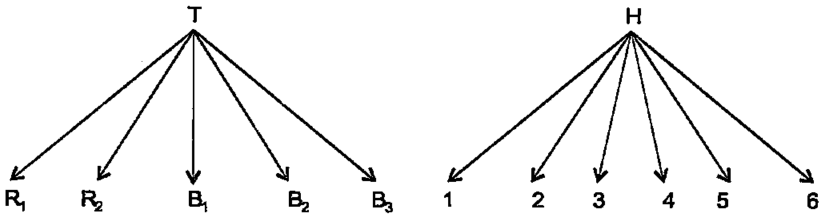
So we have 1HH, 1HT, 1TH, 1TT, 3HH, 3HT, 3TH, 3TT, 5HH, 5HT, 5TH, 5TT.

∴ The required sample space is

S = {1HH, 1HT, 1TH, 1TT, 2H, 2T, 3HH, 3HT, 3TH, 3TT, 4H, 4T, 5HH, 5HT, 5TH, 5TT, 6H, 6T}.

15. A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 red and 3 black balls. If it shows head, we throw a die. Find the sample space for this experiment.

Sol. Let the two red balls be denoted by R_1, R_2 and the three black balls be denoted by B_1, B_2, B_3.



The sample space for the experiment is

$S = \{TR_1, TR_2, TB_1, TB_2, TB_3, H1, H2, H3, H4, H5, H6\}$.

16. A die is thrown repeatedly until a six comes up. What is the sample space for this experiment?

Sol. Let A denote the outcome 1 or 2 or 3 or 4 or 5, *i.e.*, not 6 and B denote the outcome 6. Since 6 can turn up in any of the first, second, third or ... throw, the sample space is

$S = \{B, AB, AAB, AAAB, ...\}$.

EXERCISE 16.2 (*Page No.: 393–394*)

1. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

Sol. We know that, the sample space on rolling a dice is $S = \{1, 2, 3, 4, 5, 6\}$

E: die shows 4 $\therefore$ $E = \{4\}$

F: die shows even number $\therefore$ $F = \{2, 4, 6\}$

Since $E \cap F = \{4\} \cap \{2, 4, 6\} = \{4\} \neq \phi$, therefore, E and F are not mutually exclusive.

2. A die is thrown. Describe the following events:

(*i*) A: a number less than 7

(*ii*) B: a number greater than 7

(*iii*) C: a multiple of 3

(*iv*) D: a number less than 4

(*v*) E: an even number greater than 4

(*vi*) F: a number not less than 3.

Also find $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $A - C$, $D - E$, $E \cap F'$, F'.

Sol. Here, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$

(*i*) A: a number less than 7

$\therefore \quad A = \{1, 2, 3, 4, 5, 6\} = S$

(*ii*) B: a number greater than 7

There is no such number in S. $\quad \therefore \quad B = \phi$

(*iii*) C: a multiple of 3

$\therefore \quad C = \{3, 6\}$

(*iv*) D: a number less than 4

$\therefore \quad D = \{1, 2, 3\}$

(*v*) E: an even number greater than 4 $\quad \therefore \quad E = \{6\}$

(*vi*) F: a number not less than 3 i.e., $\geq 3 \; \therefore \quad F = \{3, 4, 5, 6\}$

Now $\quad A \cup B = \{1, 2, 3, 4, 5, 6\} \cup \phi = \{1, 2, 3, 4, 5, 6\}$

$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \phi = \phi$

$B \cup C = \phi \cup \{3, 6\} = \{3, 6\}$

$E \cap F = \{6\} \cap \{3, 4, 5, 6\} = \{6\}$

$D \cap E = \{1, 2, 3\} \cap \{6\} = \phi$

$A - C = \{1, 2, 3, 4, 5, 6\} - \{3, 6\} = \{1, 2, 4, 5\}$

$D - E = \{1, 2, 3\} - \{6\} = \{1, 2, 3\}$

$E \cap F' = \{6\} \cap \{3, 4, 5, 6\}' = \{6\} \cap \{1, 2\} = \phi$

$F' = \{3, 4, 5, 6\}' = S - \{3, 4, 5, 6\} = \{1, 2\}.$

3. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

A: the sum is greater than 8.

B: 2 occurs on either die

C: the sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

Sol. There are $6 \times 6 = 36$ elements in the sample space S.

$$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),$$
$$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),$$
$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),$$
$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),$$
$$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),$$
$$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

$A = \{(x, y) : x + y > 8\}$

$= \{(x, y) : x + y = 9 \text{ or } 10 \text{ or } 11 \text{ or } 12\}$

($\because$ sum of the numbers on the two dice can't be more than 12)

$= \{(3, 6), (6, 3), (4, 5), (5, 4), (4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)\}$

$B = \{(x, y) : \text{either } x = 2 \text{ or } y = 2 \text{ or } x = y = 2\}$

$= \{(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (1, 2), (3, 2), (4, 2), (5, 2), (6, 2)\}$

$C = \{(x, y) : x + y \geq 7 \text{ and } x + y \text{ is a multiple of } 3\}$

$= \{(x, y) : x + y = 9 \text{ or } 12\}$

$= \{(3, 6), (6, 3), (4, 5), (5, 4), (6, 6)\}$

$A \cap B = \phi, \quad B \cap C = \phi,$

$A \cap C = \{(3, 6), (6, 3), (4, 5), (5, 4), (6, 6)\} \neq \phi.$

$\because$ A and B, B and C are mutually exclusive.

4. Three coins are tossed once. Let A denote the event "three heads show", B denote the event "two heads and one tail show", C denote the event "three tails show" and D denote the event "a head shows on the first coin". Which events are

(*i*) mutually exclusive? (*ii*) simple? (*iii*) compound?

Sol. Here the sample space is (on tossing three coins once),

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

A: three heads show $\therefore$ A = {HHH}

B: two heads and one tail show

$\therefore$ B = {HHT, HTH, THH}

C: three tails show $\therefore$ C = {TTT}

D: a head shows on the first coin

$\therefore$ D = {HHH, HHT, HTH, HTT}

(*i*) $A \cap B$ = {HHH} $\cap$ {HHT, HTH, THH} = ϕ

$\Rightarrow$ A and B are mutually exclusive.

$A \cap C$ = {HHH} $\cap$ {TTT} = ϕ

$\Rightarrow$ A and C are mutually exclusive.

$A \cap D$ = {HHH} $\cap$ {HHH, HHT, HTH, HTT}

= {HHH} $\neq \phi$

$\Rightarrow$ A and D are not mutually exclusive.

$B \cap C$ = {HHT, HTH, THH} $\cap$ {TTT} = ϕ

$\Rightarrow$ B and C are mutually exclusive.

$B \cap D$ = {HHT, HTH, THH} $\cap$ {HHH, HHT, HTH, HTT}

= {HHT, HTH} $\neq \phi$

$\Rightarrow$ B and D are not mutually exclusive.

$C \cap D$ = {TTT} $\cap$ {HHH, HHT, HTH, HTT} = ϕ

$\Rightarrow$ C and D are mutually exclusive.

Hence A and B; A and C; B and C; C and D are mutually exclusive pairs of events.

(*ii*) The events A and C have only one sample point.

$\therefore$ A and C are simple events.

(*iii*) The events B and D have more than one sample points.

$\therefore$ B and D are compound events.

5. Three coins are tossed. Describe

(*i*) Two events which are mutually exclusive.

(*ii*) Three events which are mutually exclusive and exhaustive.

(*iii*) Two events which are not mutually exclusive.

(*iv*) Two events which are mutually exclusive but not exhaustive.

(*v*) Three events which are mutually exclusive but not exhaustive.

Sol. We know that the sample space of the experiment of tossing three coins once is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

[**Note.** Many examples are possible for each part. Here, we give one example for each part and advise the student to frame a few more.]

(*i*) Let A: getting exactly one head

and B: getting exactly one tail

then A = {HTT, THT, TTH},

B = {THH, HTH, HHT}

Since $A \cap B = \phi$, the events A and B are mutually exclusive.

(*ii*) Let A: getting at most ($\Rightarrow$ **Maximum**) one head *i.e.*, one head or no head.

B: getting exactly two heads

C: getting exactly three heads

then A = {TTT, HTT, THT, TTH}

$B = \{HHT, HTH, THH\}$, $C = \{HHH\}$

Since $A \cap B = \phi$, $A \cap C = \phi$, $B \cap C = \phi$,

and $A \cup B \cup C = S$, the events A, B and C are mutually exclusive and exhaustive.

(*iii*) Let A: getting at most one head i.e., one head or no head.

B: getting at least (⇒ **Minimum**) one head i.e., one head or two heads or three heads

then $A = \{TTT, HTT, THT, TTH\}$

$B = \{HTT, THT, TTH, HHT, HTH, THH, HHH\}$

Since $A \cap B = \{HTT, THT, TTH\} \neq \phi$, the events A and B are not mutually exclusive.

(*iv*) Let A: getting exactly one head,

B: getting exactly two heads

then $A = \{HTT, THT, TTH\}$,

$B = \{HHT, HTH, THH\}$

Since $A \cap B = \phi$, the events A and B are mutually exclusive.

But $A \cup B \neq S$, therefore, A and B are not exhaustive events.

(*v*) Let A: getting exactly one head,

B: getting exactly two heads

C: getting exactly three heads

then $A = \{HTT, THT, TTH\}$,

$B = \{HHT, HTH, THH\}$,

$C = \{HHH\}$

Since $A \cap B = \phi$, $A \cap C = \phi$, $B \cap C = \phi$, the events A, B and C are mutually exclusive.

But $A \cup B \cup C \neq S$, therefore, A, B and C are not exhaustive events.

6. Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5.

Describe the events:

(*i*) A′ **(*ii*) not B** **(*iii*) A or B**
(*iv*) A and B **(*v*) A but not C** **(*vi*) B or C**
(*vii*) B and C **(*viii*) $A \cap B' \cap C'$**

Sol. Here the sample space S is the same as in solution 3 of this exercise.

$$A = \{(x, y): x \text{ is even}\}$$
$$= \{(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$
$$B = \{(x, y): x \text{ is odd}\}$$
$$= \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$$
$$C = \{(x, y): x + y \le 5\}$$
$$= \{(x, y): x + y = 2 \text{ or } 3 \text{ or } 4 \text{ or } 5\}$$
$$= \{(1, 1), (1, 2), (2, 1), (1, 3), (3, 1), (2, 2), (1, 4), (4, 1), (2, 3), (3, 2)\}$$

(*i*) A′ = not A: getting an odd number on the first die
$\Rightarrow$ $A' = B$ [**OR** $A' = S - A = B$]

(*ii*) not B: getting an even number on the first die
$\Rightarrow$ $B' = A$ [**OR** $B' = S - B = A$]

(*iii*) A or B $= A \cup B = S$

(*iv*) A and B $= A \cap B = \phi$

(*v*) A but not C $= A - C = \{(2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

(*vi*) B or C $= B \cup C$
$$= \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (2, 1), (2, 2), (2, 3), (4, 1)\}$$

(*vii*) B and C $= B \cap C = \{(1, 1), (1, 2), (1, 3), (1, 4), (3, 1), (3, 2)\}$

(*viii*) $A \cap B' \cap C' = A \cap A \cap C'$ [By (*ii*)]
$$= A \cap C' = A - C$$
$$= \{(2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

7. Refer to question 6 above, state true or false: (give reason for your answer)

(*i*) A and B are mutually exclusive.

(*ii*) A and B are mutually exclusive and exhaustive.

(*iii*) A = B′.

(*iv*) A and C are mutually exclusive.

(*v*) A and B′ are mutually exclusive.

(*vi*) A′, B′ and C are mutually exclusive and exhaustive.

Sol. (*i*) **True**, since $A \cap B = \phi$

(*ii*) **True**, since $A \cap B = \phi$ and $A \cup B = S$

(*iii*) **True** [By Question 6 (*ii*)]

(*iv*) **False**, since $A \cap C = \{(2, 1), (2, 2), (2, 3), (4, 1)\} \neq \phi$

(*v*) **False** ($\because$ By Question 6 (*ii*), $A \cap B' = A \cap A = A \neq \phi$)

(*vi*) **False** [$\because A' \cap B' = B \cap A$ (By (*i*) and (*ii*) $= \phi$ (by (*iv*)].
But $A' \cap C = B \cap C$ (By (*i*) $\neq \phi$ by (*vi*).
Of course, $A' \cup B' \cup C = B \cup A \cup C = S \cup C$ (By (*iii*) $= S$].

EXERCISE 16.3 (*Page No.: 403–406*)

1. Which of the following cannot be valid assignment of probabilities for outcomes of sample space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(*a*)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(*b*)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
(*c*)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(*d*)	– 0.1	0.2	0.3	0.4	– 0.2	0.1	0.3
(*e*)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Sol. (*a*) Each $P(\omega_i)$ is positive and less than one

$\Rightarrow$ Condition (*i*) is satisfied.

$$\sum_{i=1}^{7} P(\omega_i) = 0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 = 1$$

$\Rightarrow$ Condition (*ii*) is satisfied

Therefore, the assignment is valid.

(*b*) Each $P(\omega_i)$ is positive and less than one.

$\Rightarrow$ Condition (*i*) is satisfied.

$$\sum_{i=1}^{7} P(\omega_i) = P(\omega_1) + P(\omega_2) + P(\omega_3) + P(\omega_4) + P(\omega_5) + P(\omega_6) + P(\omega_7)$$

$$= \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{7}{7} = 1$$

$\Rightarrow$ Condition (*ii*) is satisfied.

Therefore, the assignment is valid.

(*c*) Each $P(\omega_i)$ is positive and less than one.

$\Rightarrow$ Condition (*i*) is satisfied.

$$\sum_{i=1}^{7} P(\omega_i) = 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7 = 2.8 \neq 1$$

$\Rightarrow$ Condition (*ii*) is not satisfied.

Therefore, the assignment is not valid.

(*d*) $P(\omega_1) = -0.1$ is negative.

$\Rightarrow$ Condition (*i*) is not satisfied

Therefore, the assignment is not valid.

(*e*) Each $P(\omega_i)$ is positive. Since $P(\omega_7) = \frac{15}{14} > 1$, condition (*i*) is not satisfied.

Therefore, the assignment is not valid.

2. A coin is tossed twice, what is the probability that at least one tail occurs?

Sol. When a coin is tossed twice, there are $2 \times 2 = 4$ possible outcomes:

HH, HT, TH, TT

$\therefore$ Sample space S = {HH, HT, TH, TT}

Let E: at least one tail occurs.

then E = {HT, TH, TT}. Here $n(E) = 3$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}.$$

3. A die is thrown, find the probability of following events:

(*i*) A prime number will appear,

(ii) A number greater than or equal to 3 will appear,

(iii) A number less than or equal to one will appear,

(iv) A number more than 6 will appear,

(v) A number less than 6 will appear.

Sol. When a die is thrown, possible outcomes are 1, 2, 3, 4, 5, 6.

$\therefore$ Sample space S = {1, 2, 3, 4, 5, 6} $\quad \therefore n(5) = 6$

(*i*) Let E_1: a prime number will appear

then $E_1 = \{2, 3, 5\}$ $\quad \therefore n(E_1) = 3$

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{6} = \frac{1}{2}.$$

(*ii*) Let E_2: a number greater than or equal to 3 will appear

$\therefore n(E_2) = 4$

then $E_2 = \{3, 4, 5, 6\}$

$$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

(*iii*) Let E_3: a number less than or equal to one will appear

$\therefore n(E_3) = 1$

then $E_3 = \{1\}$

$$\therefore \quad P(E_3) = \frac{n(E_3)}{n(S)} = \frac{1}{6}.$$

(*iv* Let E_4: a number more than 6 will appear.

Since there is no such number in S, $E_4 = \phi \because n(E_4) = 0$

$\Rightarrow$ E_4 is an impossible event and hence $P(E_4) = 0$

(*v*) Let E_5: a number less than 6 will appear, then

$E_5 = \{1, 2, 3, 4, 5\}$ $\quad \because n(E_5) = 5$

$$\therefore \quad P(E_5) = \frac{n(E_5)}{n(S)} = \frac{5}{6}.$$

4. A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the card is (i) an ace (ii) black card.

Sol. When a card is selected from a pack of 52 cards, there are 52 equally likely possible outcomes.

(*a*) $n(S) = 52$

(*b*) Let E: card is an ace of spades

Since there is only one ace of spades, $n(E) = 1$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{52}$$

(*c*) (*i*) Let F: card is an ace

Since there are 4 aces in the pack, $n(F) = 4$

$$\therefore \quad P(F) = \frac{n(F)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(*ii*) Let G: card is black.

Since there are 26 black cards, 13 clubs and 13 spades,

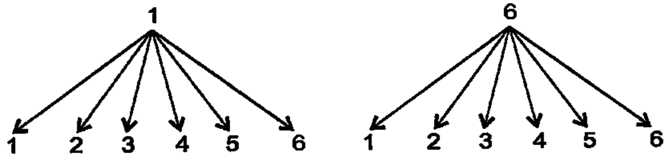
$n(G) = 26$

$$\therefore \quad P(G) = \frac{n(G)}{n(S)} = \frac{26}{52} = \frac{1}{2}.$$

5. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed, find the probability that the sum of numbers that turn up is (*i*) 3 (*ii*) 12.

Sol. The possible outcomes are

Coin:



The sample space is

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$n(S) = 2 \times 6 = 12$

(*i*) Let E: sum of numbers is 3

then E = {(1, 2)}, $n(E) = 1$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{12}$$

(*ii*) Let F: sum of numbers is 12

then F = {(6, 6)}, $n(F) = 1$

$$\therefore \quad P(F) = \frac{n(F)}{n(S)} = \frac{1}{12}.$$

6. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Sol. Number of men = 4, Number of women = 6

Total number of council members = 4 + 6 = 10

Let E: a woman member is selected

then $$P(E) = \frac{^6C_1}{^{10}C_1} = \frac{6}{10} = \frac{3}{5}.$$

7. A fair coin is tossed four times, and a person wins ₹ 1 for each head and loses ₹ 1.50 for each tail that turns up.

From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Sol. The sample space of the experiment is

S = {HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}

[Write all the possible outcomes when a coin is tossed three times. Prefix H with each outcome, then prefix T with each outcome.]

$$n(S) = 2^4 = 16$$

(*i*) When the outcome is **all heads,** the person wins ₹ (1 × 4) = ₹ 4.

(*ii*) When the outcome is **three heads and one tail,** the person wins ₹ (1 × 3) = ₹ 3 and loses ₹ 1.50.

Therefore, he gets ₹ 3 – ₹ 1.50 = ₹ 1.5.

(*iii*) When the outcome is **two heads and two tails,** the person wins ₹ (1 × 2) = ₹ 2 and loses ₹ (1.50 × 2) = ₹ 3. Therefore, he gets ₹ 2 – ₹ 3 = – ₹ 1 *i.e.,* on the whole, he loses ₹ 1.

(*iv*) When the outcome is **one head and three tails**, the person wins ₹ 1 and loses ₹ (1.50 × 3) = ₹ 4.50. Therefore, he gets ₹ 1 – ₹ 4.50 = – ₹ 3.50, *i.e.*, on the whole, he loses ₹ 3.50.

(*v*) When the outcome is **all tails**, the person loses ₹ (1.50 × 4) = ₹ 6. Therefore, he gets – ₹ 6.

Thus, the amounts of money the person can have after four tosses is as follows:

₹ 4, ₹ 1.5, – ₹ 1, – ₹ 3.50, – ₹ 6 (negative sign shows loss)

P(person wins ₹ 4) $= P(HHHH) = \frac{1}{16}$

P(person wins ₹ 1.5) $= P(HHHT, HHTH, HTHH, THHH)$

$= \frac{4}{16} = \frac{1}{4}$

P(person loses ₹ 1) $= P(HHTT, HTHT, HTTH, THHT, THTH, TTHH)$

$= \frac{6}{16} = \frac{3}{8}$

P(person loses ₹ 3.50) $= P(HTTT, THTT, TTHT, TTTH)$

$= \frac{4}{16} = \frac{1}{4}$

P(person loses ₹ 6) $= P(TTTT) = \frac{1}{16}$.

8. Three coins are tossed once. Find the probability of getting

(*i*) 3 heads **(*ii*) 2 heads**
(*iii*) at least 2 heads **(*iv*) atmost 2 heads**
(*v*) no head **(*vi*) 3 tails**
(*vii*) exactly two tails **(*viii*) no tail**
(*ix*) atmost two tails.

Sol. When three coins are tossed, there are 2 × 2 × 2 = 8 equally likely possible outcomes. The sample space is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

(*i*) Let E_1: 3 heads

then E_1 = {HHH}

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{8}$$

(*ii*) Let E_2: 2 heads

then E_2 = {HHT, HTH, THH}

$$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{8}$$

(*iii*) Let E_3: at least 2 heads *i.e.,* 2 or 3 heads

then E_3 = {HHT, HTH, THH, HHH}

$$\therefore \quad P(E_3) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(*iv*) Let E_4: atmost 2 heads *i.e.,* no head or 1 head or 2 heads then E_4 = {TTT, HTT, THT, TTH, HHT, HTH, THH}

$$\therefore \quad P(E_4) = \frac{n(E_4)}{n(S)} = \frac{7}{8}$$

(*v*) Let E_5: no head *i.e.,* all tails

then E_5 = {HHH}

$$\therefore \quad P(E_5) = \frac{n(E_5)}{n(S)} = \frac{1}{8}$$

(*vi*) Let E_6: 3 tails

then E_6 = {TTT}

$$\therefore \quad P(E_6) = \frac{n(E_6)}{n(S)} = \frac{1}{8}$$

(*vii*) Let E_7: exactly two tails

then E_7 = {TTH, THT, HTT}

$$\therefore \quad P(E_7) = \frac{n(E_7)}{n(S)} = \frac{3}{8}$$

(*viii*) Let E_8: no tails *i.e.,* all heads

then E_8 = {HHH}

$$\therefore \quad P(E_8) = \frac{n(E_8)}{n(S)} = \frac{1}{8}$$

(*ix*) Let E_9: atmost two tails *i.e.,* no tail or one tail or two tails; then E_9 = {HHH, THH, HTH, HHT, TTH, THT, HTT}

$$\therefore \quad P(E_9) = \frac{n(E_9)}{n(S)} = \frac{7}{8}.$$

9. If $\frac{2}{11}$ is the probability of an event A, what is the probability of the event 'not A'.

Sol. Given $P(A) = \frac{2}{11}$

$$\therefore \quad P(\text{not } A) = 1 - P(A) = 1 - \frac{2}{11} = \frac{9}{11}.$$

10. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is (*i*) a vowel (*ii*) a consonant.

Sol. The word ASSASSINATION has 13 letters of which 6 are vowels (A, A, A, I, I, O) and 7 are consonants (S, S, S, S, N, N, T).

(*i*) $P(\text{a vowel}) = \frac{6}{13}$

(*ii*) $P(\text{consonant}) = \frac{7}{13}$.

11. In a lottery, a person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game?

Sol. Six natural numbers can be chosen randomly out of first twenty natural numbers in ${}^{20}C_6$ ways.

$$\therefore \quad n(S) = {}^{20}C_6.$$

Let E: the person wins the prize.

Since there is only one choice which will match the numbers already chosen by the lottery committee.

$$n(E) = 1$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{{}^{20}C_6} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15}$$

$$= \frac{1}{38760}.$$

12. Check whether the following probabilities P(A) and P(B) are consistently defined

(*i*) $P(A) = 0.5$, $P(B) = 0.7$, $P(A \cap B) = 0.6$

(*ii*) $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.8$.

Sol. (*i*) We know that $A \cap B \subset A$ and $A \cap B \subset B$

$\therefore$ We must have $P(A \cap B) \le P(A)$ and $P(A \cap B) \le P(B)$

Here $P(A) = 0.5$ and $P(A \cap B) = 0.6$ so that

$P(A \cap B) > P(A)$, which is not true *i.e.*, not consistent.

(*ii*) We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ... (*i*)

Here $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.8$

Putting these values in (*i*),

$0.8 = 0.5 + 0.4 - P(A \cap B)$

$\Rightarrow 0.8 = 0.9 - P(A \cap B) \Rightarrow P(A \cap B) = 0.9 - 0.8 = 0.1$

which is possible (*i.e.*, true) ($\because 0 \le P(E) \le 1$ must always hold)

$\because$ The probabilities P(A) and P(B) are consistently defined.

13. Fill in the blanks in the following table:

	P(A)	P(B)	$P(A \cap B)$	$P(A \cup B)$
(*i*)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	...
(*ii*)	0.35	...	0.25	0.6
(*iii*)	0.5	0.35	...	0.7

Sol. Using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, we have

(*i*) $P(A \cup B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{15} = \frac{5+3-1}{15} = \frac{7}{15}$

(*ii*) $\quad 0.6 = 0.35 + P(B) - 0.25$

$\Rightarrow \quad 0.6 = 0.10 + P(B)$

$\Rightarrow \quad P(B) = 0.6 - 0.1 = 0.5$

(*iii*) $\quad 0.7 = 0.5 + 0.35 - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = 0.85 - 0.70 = 0.15.$

14. Given $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find P(A or B), if A and B are mutually exclusive events.

Sol. Since A and B are mutually exclusive events, $A \cap B = \phi$ so that $P(A \cap B) = 0$

$\therefore \quad P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{3}{5} + \frac{1}{5} - 0 = \frac{4}{5}.$$

15. If E and F are events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$, find (*i*) P(E or F) (*ii*) P(not E and not F).

Sol. (*i*) $P(E \text{ or } F) = P(E \cup F)$

$$= P(E) + P(F) - P(E \cap F)$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}.$$

(*ii*) $P(\text{not } E \text{ and not } F) = P(E' \cap F')$

$= P(E \cup F)'$ [De Morgan's Law]

$= 1 - P(E \cup F)$

$= 1 - \frac{5}{8}$ [See part (*i*)]

$= \frac{3}{8}.$

16. Events E and F are such that P(not E or not F) = 0.25. State whether E and F are mutually exclusive.

Sol. $P(\text{not } E \text{ or not } F) = 0.25$

$\Rightarrow \quad P(E' \cup F') = 0.25$

$\Rightarrow \quad P(E \cap F)' = 0.25$ [De Morgan's Law]

$\Rightarrow \quad 1 - P(E \cap F) = 0.25$

$\Rightarrow \quad P(E \cap F) = 1 - 0.25 = 0.75 \neq 0$

$\Rightarrow$ E and F are not mutually exclusive.

17. A and B are events such that P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16. Determine (*i*) P(not A) (*ii*) P(not B) and (*iii*) P(A or B).

Sol. (*i*) P(not A) = 1 – P(A) = 1 – 0.42 = 0.58

(*ii*) P(not B) = 1 – P(B) = 1 – 0.48 = 0.52

(*iii*) P(A or B) = $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.42 + 0.48 - 0.16 = 0.74.$

18. In class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Sol. Let M: a student studies Mathematics

B: a student studies Biology

Given: $P(M) = \frac{40}{100}$, $P(B) = \frac{30}{100}$, $P(M \cap B) = \frac{10}{100}$

$\therefore$ P(M or B) = $P(M \cup B) = P(M) + P(B) - P(M \cap B)$

$$= \frac{40}{100} + \frac{30}{100} - \frac{10}{100} = \frac{60}{100} = 0.6.$$

19. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Sol. Let E_1: a student passes the first examination

E_2: a student passes the second examination

Given: $P(E_1) = 0.8$, $P(E_2) = 0.7$

P(a student passes at least one examination)

$= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2) = 0.95$

$\Rightarrow P(E_1) + P(E_2) - P(E_1 \cap E_2) = 0.95$

$\Rightarrow 0.8 + 0.7 - P(E_1 \cap E_2) = 0.95$

$\Rightarrow P(E_1 \cap E_2) = 1.50 - 0.95 = 0.55$

$\Rightarrow$ P(a student passes both examinations) = 0.55.

20. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Sol. Let E: a student passes in English

H: a student passes in Hindi

Given: P(passing both English and Hindi) = $P(E \cap H) = 0.5$

P(passing neither English nor Hindi) = $P(E' \text{ and } H') = P(E' \cap H')$ $= 0.1$

$$P(E) = 0.75$$

Now $P(E' \cap H') = 0.1$

$\Rightarrow$ $P(E \cup H)' = 0.1$ [De Morgan's Law]

$\Rightarrow$ $1 - P(E \cup H) = 0.1$

$\Rightarrow$ $P(E \cup H) = 1 - 0.1 = 0.9$

$\Rightarrow$ $P(E) + P(H) - P(E \cap H) = 0.9$

$\Rightarrow$ $0.75 + P(H) - 0.5 = 0.9$

$\Rightarrow$ $0.25 + P(H) = 0.9$

$\Rightarrow$ $P(H) = 0.90 - 0.25 = 0.65.$

21. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(*i*) The student opted for NCC or NSS,

(*ii*) The student has opted neither NCC nor NSS,

(*iii*) The student has opted NSS but not NCC.

Sol. Let E : a student opted for NCC

F : a student opted for NSS

Given: $P(E) = \frac{30}{60}$, $P(F) = \frac{32}{60}$,

and $P(E \cap F) = \frac{24}{60}$

(*i*) P(a student opted for NCC or NSS)

$= P(E \text{ or } F) = P(E \cup F)$

$= P(E) + P(F) - P(E \cap F)$

$$= \frac{30}{60} + \frac{32}{60} - \frac{24}{60} = \frac{38}{60} = \frac{19}{30}$$

(*ii*) P(a student has opted neither NCC nor NSS)

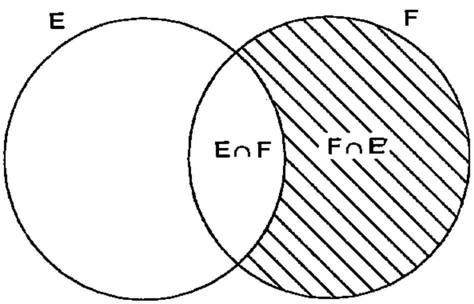
$= P(\text{neither E nor F}) = P(E' \cap F')$

$= P(E \cup F)'$ [De Morgan's Law]

$= 1 - P(E \cup F)$

$= 1 - \frac{19}{30}$ [See part (*i*)]

$= \frac{11}{30}$.



(*iii*) P(a student has opted NSS but not NCC)

$= P(\text{F and E}')$

$= P(F \cap E')$

$= P(F) - P(E \cap F)$

$= \frac{32}{60} - \frac{24}{60}$

$= \frac{8}{60} = \frac{2}{15}$.

MISCELLANEOUS EXERCISE ON CHAPTER 16

(*Page No.: 408–409*)

1. A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that

(*i*) all will be blue? (*ii*) at least one will be green?

Sol. Total number of marbles = 10 + 20 + 30 = 60

5 marbles out of 60 can be drawn in $^{60}C_5$ ways

(*i*) P(all 5 marbles are blue) $= \frac{^{20}C_5}{^{60}C_5}$

(*ii*) **P(at least one marble is green)**

= 1 – P(no marble is green)

$$= 1 - \frac{^{30}C_5}{^{60}C_5}$$

[∵ there are 10 + 20 = 30 non-green marbles]

2. 4 cards are drawn from a well-shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

Sol. 4 cards can be drawn out of 52 in $^{52}C_4$ ways.

3 diamonds out of 13 can be drawn in $^{13}C_3$ ways.

1 spade out of 13 can be drawn in $^{13}C_1$ ways.

∵ number of ways of getting 3 diamond cards and 1 spade card out of 4 cards drawn

$= 13C_3 \times 13C_1$

[By F.P.C.(multiplication)]

$$\therefore \quad \text{P(3 diamonds and one spade)} = \frac{^{13}C_3 \times {}^{13}C_1}{^{52}C_4}.$$

3. A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine

(*i*) P(2) (*ii*) P(1 or 3) (*iii*) P(not 3).

Sol. The six faces of the die are marked 1, 1, 2, 2, 2, 3.

(*i*) $P(2) = \frac{3}{6}$ (∵ 3 faces out of 6 are marked 2)

$= \frac{1}{2}$

(*ii*) $P(1 \text{ or } 3) = P(1) + P(3) = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

(*iii*) $P(\text{not } 3) = 1 - P(3) = 1 - \frac{1}{6} = \frac{5}{6}$.

4. In a certain lottery 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy (*a*) one ticket (*b*) two tickets (*c*) 10 tickets.

Sol. Total number of tickets = 10,000

Number of prize winning tickets = 10

Number of tickets without prize = 10,000 – 10 = 9990

(*a*) One ticket out of 10,000 can be drawn in

$^{10000}C_1 = 10000$ ways

One ticket without prize can be drawn out of 9990 tickets in $^{9990}C_1 = 9990$ ways.

$\therefore$ P(not getting a prize with one ticket) = $\dfrac{9990}{10000} = \dfrac{999}{1000}$

(*b*) Two tickets out of 10,000 can be drawn in $^{10000}C_2$ ways. Two tickets out of 9990 can be drawn in $^{9990}C_2$ ways.

$\therefore$ P(not getting a prize with two tickets) = $\dfrac{^{9990}C_2}{^{10000}C_2}$

(*c*) 10 tickets out of 10,000 can be drawn in $^{10000}C_{10}$ ways. 10 tickets out of 9990 can be drawn in $^{9990}C_{10}$ ways.

$\therefore$ P(not getting a prize with 10 tickets) = $\dfrac{^{9990}C_{10}}{^{10000}C_{10}}$.

5. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that

(*a*) you both enter the same section?

(*b*) you both enter the different sections?

Sol. [☞ Note that formation of one section simultaneously results into the formation of the other.]

Two sections of 40 and 60 can be formed out of 100 in $^{100}C_{40}$ (or $^{100}C_{60}$) ways. Let us call them section A and section B respectively.

(*a*) When both enter section A, 38 more students out of remaining 98 are to be selected for section A. This can be done in $^{98}C_{38}$ ways. When both enter section B, 40 out of remaining 98 are to be selected for Section A. This can be done in $^{98}C_{40}$ ways.

$\therefore$ P(both enter the same section)

$$= \frac{^{98}C_{38} + {}^{98}C_{40}}{^{100}C_{40}} = \frac{\dfrac{98!}{38!\,60!} + \dfrac{98!}{40!\,58!}}{\dfrac{100!}{40!\,60!}}$$

$$= \frac{40!\,60!}{100!} \,.\, 98!\left[\frac{1}{38!\,60!} + \frac{1}{40!\,58!}\right]$$

$$= \frac{98!}{100!}\left[\frac{40!}{38!} + \frac{60!}{58!}\right]$$

$$= \frac{98!}{100 \times 99 \times 98!}\left[\frac{40 \times 39 \times 38!}{38!} + \frac{60 \times 59 \times 58!}{58!}\right]$$

$$= \frac{1}{9900}(1560 + 3540)$$

$$= \frac{5100}{9900} = \frac{17}{33}.$$

(*b*) P(both enter different sections)

$$= 1 - \text{P(both enter the same section)}$$

$$= 1 - \frac{17}{33} = \frac{16}{33}$$

OR (Independently)

For Section A, one out of you two can be selected in ${}^2C_1$ ways and 39 more students out of remaining 98 can be selected in ${}^{98}C_{39}$ ways.

$$\therefore \quad \text{P(both enter different sections)} = \frac{{}^2C_1 \times {}^{98}C_{39}}{{}^{100}C_{40}}$$

$$= 2 \times \frac{98!}{39!\,59!} \times \frac{40!\,60!}{100!}$$

$$= 2 \times \frac{98!}{100!} \times \frac{40!}{39!} \times \frac{60!}{59!}$$

$$= 2 \times \frac{1}{100 \times 99} \times 40 \times 60$$

$$= \frac{48}{99} = \frac{16}{33}.$$

6. Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Sol. Let the three letters be L_1, L_2, L_3 and the corresponding directed envelopes be E_1, E_2, E_3 respectively.

	E_1	E_2	E_3
(*i*)	L_1	L_2	L_3
(*ii*)	L_1	L_3	L_2
(*iii*)	L_2	L_1	L_3
(*iv*)	L_2	L_3	L_1
(*v*)	L_3	L_1	L_2
(*vi*)	L_3	L_2	L_1

Three letters can be put in the three envelopes in

$${}^3P_3 = 3! = 6 \text{ ways.}$$

The number of ways in which no letter is put in its proper envelope is 2, *viz.* (*iv*) and (*v*).

$\therefore$ P(at least one letter is in its proper envelope)

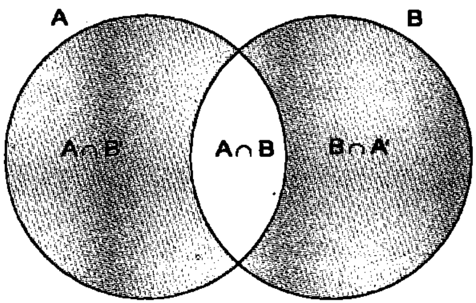
$= 1 - $ P(no letter is in its proper envelope)

$= 1 - \frac{2}{6} = 1 - \frac{1}{3} = \frac{2}{3}.$

7. A and B are two events such that P(A) = 0.54, P(B) = 0.69 and P(A ∩ B) = 0.35. Find
(i) P(A ∪ B) (ii) P(A′ ∩ B′) (iii) P(A ∩ B′) (iv) P(B ∩ A′).

Sol. (*i*) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.54 + 0.69 - 0.35$
$= 1.23 - 0.35 = 0.88$

(*ii*) $P(A' \cap B') = P(A \cup B)'$ [De Morgan's Law]
$= 1 - P(A \cup B)$
$= 1 - 0.88$ [part (*i*)]
$= 0.12$



(*iii*) $P(A \cap B') = P(A) - P(A \cap B)$ (See the above figure)
$= 0.54 - 0.35 = 0.19$

(*iv*) $P(B \cap A') = P(B) - P(A \cap B)$ (See the above figure)
$= 0.69 - 0.35 = 0.34.$

8. From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S. No.	Name	Sex	Age in years
1.	Harish	M	30
2.	Rohan	M	33
3.	Sheetal	F	46
4.	Alis	F	28
5.	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

Sol. Let E: spokesperson is a male

F: spokesperson is over 35 years

then $E \cap F$: spokesperson is a male over 35 yeras

We can observe from the given information that

$n(S) = {}^5C_1 = 5, n(E) = {}^3C_1 = 3, n(F) = {}^2C_1 = 2, n(E \cap F) = 1$

$$\therefore \ P(E) = \frac{3}{5}, \quad P(F) = \frac{2}{5}, \qquad P(E \cap F) = \frac{1}{5}$$

$$\therefore \quad P(E \text{ or } F) = P(E) + P(F) - P(E \cap F)$$

$$= \frac{3}{5} + \frac{2}{5} - \frac{1}{5} = \frac{4}{5}.$$

9. If 4-digit numbers greater than 5000 are randomly formed from the digits 0, 1, 3, 5 and 7, what is the probability of forming a number divisible by 5 when,

(*i*) the digits are repeated?

(*ii*) the repetition of digits is not allowed?

Sol. (*i*) **When digits are repeated**

Since the required numbers are greater than 5000, the thousand's place can be filled by 5 or 7 only *i.e.*, in 2 ways. Each of the remaining three places can be filled in 5 ways.

Th	H	T	U
5	0	0	0
7	1	1	1
	3	3	3
	5	5	5
	7	7	7

$\therefore$ Total numbers formed

$= 2 \times 5 \times 5 \times 5 - 1$

$= 250 - 1 = 249$

($\because$ one of these 250 numbers formed, one will be 5000 also and we are to form numbers > 5000)

If the number is divisible by 5 then unit's place can be filled by 0 or 5 only *i.e.*, in 2 ways.

Th	H	T	U
5	0	0	0
7	1	1	5
	3	3	
	5	5	
	7	7	

∴ Total numbers greater than 5000 and divisible by 5
$= 2 \times 5 \times 5 \times 2 - 1 = 100 - 1 = 99$ (reason for subtracting 1 is given above)

Required probability $= \frac{99}{249} = \frac{33}{83}$.

(*ii*) **When repetition of digits is not allowed**

The thousand's place can be filled by 5 or 7 only, *i.e.,* in 2 ways. The remaining three places can be filled by the remaining 4 digits in $^4P_3$ ways.

∴ Total numbers formed $= 2 \times {}^4P_3 = 2 \times 4 \times 3 \times 2$
$= 48$

The numbers divisible by 5 have either 0 or 5 in the unit's place.

When 0 is in unit's place, thousand's place can be filled by 5 or 7 *i.e.,* in 2 ways. The remaining two places can be filled by the remaining three digits in $^3P_2$ ways.

Th	H	T	U
5 7			0

∴ Total numbers divisible by 5
$= 2 \times {}^3P_2 = 2 \times 3 \times 2 = 12$

When 5 is in unit's place, thousand's place can be filled by 7 only *i.e.,* in 1 way. The remaining two places can be filled by the remaining three digits in $^3P_2$ ways.

Th	H	T	U
7			5

∴ Total numbers divisible by 5 $= {}^3P_2 = 3 \times 2 = 6$

Combining the two cases, total numbers divisible by 5
$= 12 + 6 = 18$

∴ Required probability $= \frac{18}{48} = \frac{3}{8}$.

Remark: In (*ii*) part, 1 has neither been subtracted from $n(5) = 48$ nor from $n(E) = 18$ because none of the numbers formed will be 5000 as repetition of digits is not allowed.

10. The number lock of a suitcase has 4 wheels, each labelled with ten digits *i.e.*, from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Sol. Each of the four wheels is labelled with 10 digits.

Number of sequences with distinct digits = $^{10}P_4$

$$= 10 \times 9 \times 8 \times 7$$

$$= 5040$$

There is only one right sequence

$\therefore$ Required probability = $\frac{1}{5040}$.