9 Mechanical Properties of Solids

Lesson at a Glance

• Intermolecular Force

In a solid, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighbouring molecules. These forces are known as intermolecular forces.

• Elasticity

The property of the body to regain its original configuration (length, volume or shape) when the deforming forces are removed, is called elasticity.

• The change in the shape or size of a body when external forces act on it is determined by the forces between its atoms or molecules. These short range atomic forces are called elastic forces.

• Perfectly elastic body

A body which regains its original configuration immediately and completely after the removal of deforming force from it, is called perfectly elastic body. Quartz and phospher bronze are the examples of nearly perfectly elastic bodies.

• Plasticity

The inability of a body to return to its original size and shape even on removal of the deforming force is called plasticity and such a body is called a plastic body.

• Stress

Stress is defined as the ratio of the internal force F, produced when the substance is deformed, to the area A over which this force acts. In equilibrium, this force is equal in magnitude to the externally applied force. In other words,

$$\text{Stress} = \frac{F}{A}$$

The SI unit of stress is newton per square metre (Nm^{-2}). In CGS units, stress is measured in dyne cm^{-2}. Dimensional formula of stress is $[ML^{-1}T^{-2}]$.

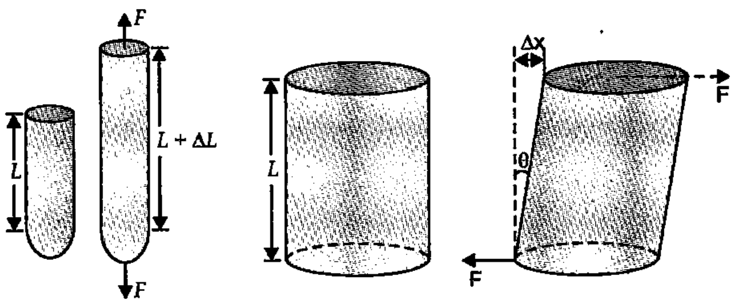
• Strain

It is defined as the ratio of the change in size or shape to the original size or shape. It has no dimensions, it is just a number.

Strain is of three types:

(*i*) **Longitudinal strain:** If the deforming force produces a change in length alone, the strain produced in the body is called longitudinal strain or tensile strain. It is given as:

$$\text{Longitudinal strain} = \frac{\text{Change in length } (\Delta l)}{\text{Original length } (l)}$$



(a) (b)

(*ii*) **Volumetric strain:** If the deforming force produces a change in volume alone, the strain produced in the body is called volumetric strain. It is given as:

$$\text{Volumetric strain} = \frac{\text{Change in volume } (\Delta V)}{\text{Original volume } (V)}$$

(*iii*) **Shear strain:** The angle tilt caused in the body due to tangential stress expressed is called shear strain. It is given as:

$$\text{Shear strain} = \theta = \frac{\Delta L}{L}.$$

• The maximum stress to which the body can regain its original status on the removal of the deforming force is called elastic limit.

• Hooke's Law

Hooke's law states that, within elastic limits, the ratio of stress to the corresponding strain produced is a constant. This constant is called the modulus of elasticity. Thus

$$\text{Modulus of elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

Since strain is a pure number, the units of this constant are the same as those of stress, *i.e.*, Nm^{-2}.

• Young's Modulus

For a solid, in the form of a wire or a thin rod, Young's modulus of elasticity within elastic limit is defined as the ratio of longitudinal stress to longitudinal strain. It is given as:

$$\text{Young's modulus, } Y = \frac{F/A}{\Delta l/l} = \frac{Fl}{A.\Delta l} = \frac{mgl}{\pi r^2.\Delta l}$$

It has the unit of longitudinal stress and dimensions of $[ML^{-1}T^{-2}]$. Its unit is Pascal or N/m^2.

• Bulk Modulus

Within elastic limit the bulk modulus is defined as the ratio of longitudinal stress and volumetric strain. It is given as:

$$\text{Bulk modulus, } B = \frac{F/A}{\Delta V/V} = -\frac{P}{\Delta V/V}$$

– ve indicates that the volume variation and pressure variation always negate each other.

• Reciprocal of bulk modulus is commonly referred to as the "compressibility". It is defined as the fractional change in volume per unit change in pressure.

• Shear Modulus or Modulus of Rigidity

It is defined as the ratio of the tangential stress to the shear strain. Modulus of rigidity is given by

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}} = \frac{F/A}{\theta}.$$

• Poisson's Ratio

The ratio of change in diameter (ΔD) to the original diameter (D) is called lateral strain.

The ratio of change in length (Δl) to the original length (l) is called longitudinal strain.

The ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = -\frac{\Delta D/D}{\Delta l/l}$$

TEXTBOOK QUESTIONS SOLVED

9.1. *A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m² stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0×10^{-5} m² under a given load. What is the ratio of the Young's modulus of steel to that of copper?*

Sol. For steel $l_1 = 4.7$m, $A_1 = 3.0 \times 10^{-5}$ m²

If F newton is the stretching force and Δl metre the extension in each case, then

$$Y_1 = \frac{Fl_1}{A_1 \Delta l} \Rightarrow Y_1 = \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta l} \quad ...(i)$$

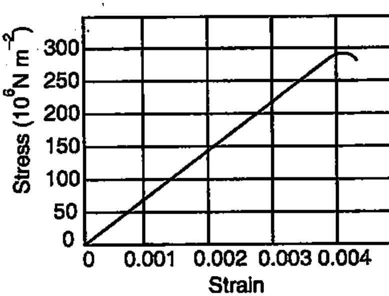
For copper $l_2 = 3.5$m, $A_2 = 4.0 \times 10^{-5}$ m²

Now, $$Y_2 = \frac{F \times 3.5}{4.0 \times 10^{-5} \times \Delta l} \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{Y_1}{Y_2} = \frac{4.7}{3.0 \times 10^{-5}} \times \frac{4.0 \times 10^{-5}}{3.5} = \frac{4.7 \times 4.0}{3.0 \times 3.5} = 1.79.$$

9.2. *Figure shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?*



Sol. (*a*) Young's modulus of the material (Y) is given by

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{150 \times 10^6}{0.002}$$

$$= \frac{150 \times 10^6}{2 \times 10^{-3}} = 75 \times 10^9 \text{ Nm}^{-2} = 7.5 \times 10^{10} \text{ Nm}^{-2}$$

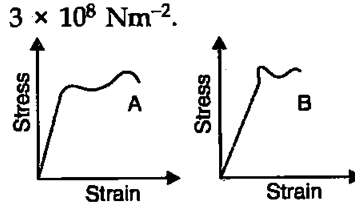
(*b*) Yield strength of a material is defined as the maximum stress it can sustain.

From graph, the approximate yield strength of the given material

$= 300 \times 10^6$ Nm⁻² $= 3 \times 10^8$ Nm⁻².

9.3. *The stress-strain graphs for materials A and B are shown in figure.*

The graphs are drawn to the same scale.



(a) Which of the materials has the greater Young's modulus?

(b) Which of the two is the stronger material?

Sol. (*a*) From the two graphs we note that for a given strain, stress for A is more than that of B. Hence Young's modulus $\left(=\frac{\text{Stress}}{\text{Strain}}\right)$ is greater for A than that of B.

(*b*) Strength of a material is determined by the amount of stress required to cause fracture. This stress corresponds to the point of fracture. The stress corresponding to the point of fracture in A is more than for B. So, material A is stronger than material B.

9.4. *Read the following two statements below carefully and state, with reasons, if it is true or false.*

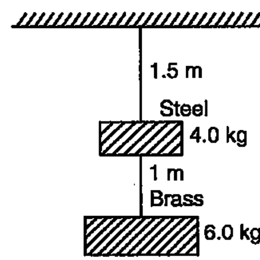
(a) The Young's modulus of rubber is greater than that of steel;

(b) The stretching of a coil is determined by its shear modulus.

Sol. (*a*) **False.** The Young's modulus is defined as the ratio of stress to the strain within elastic limit. For a given stretching force elongation is more in rubber and quite less in steel. Hence, rubber is less elastic than steel.

(*b*) **True.** Stretching of a coil is determined by its shear modulus. When equal and opposite forces are applied at opposite ends of a coil, the distance as well as shape of helicals of the coil change and it involves shear modulus.

9.5. *Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is 2.0×10^{11} Pa. Compute the elongations of steel and brass wires. ($1\ Pa = 1\ Nm^{-2}$).*



Sol. For steel wire; total force on steel wire;

$$F_1 = 4 + 6 = 10 \text{ kg } f = 10 \times 9.8 \text{ N};$$

$$l_1 = 1.5 \text{ m}, \quad \Delta l_1 = ?; \quad 2r_1 = 0.25 \text{ cm}$$

or $$r_1 = \left(\frac{0.25}{2}\right) \text{cm} = 0.125 \times 10^{-2} \text{ m}$$

$$Y_1 = 2.0 \times 10^{11} \text{ Pa}$$

For brass wire,

$$F_2 = 6.0 \text{ kg } f = 6 \times 9.8 \text{ N};$$

$$2r_2 = 0.25 \text{ cm}$$

or $r_2 = \left(\frac{0.25}{2}\right) = 0.125 \times 10^{-2}$ m;

$Y_2 = 0.91 \times 10^{11}$ Pa, $l_2 = 1.0$ m, $\Delta l_2 = ?$

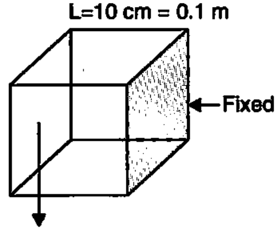
Since, $Y_1 = \frac{F_1 \times l_1}{A_1 \times \Delta l_1} = \frac{F_1 \times l_1}{\pi r_1^2 \times \Delta l_1} \Rightarrow \Delta l_1 = \frac{F_1 \times l_1}{\pi r_1^2 \times \Delta l_1}$

or $\Delta l_1 = \frac{(10 \times 9.8) \times 1.5 \times 7}{22 \times (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}} = 1.49 \times 10^{-4}$ m.

And $\Delta l_2 = \frac{F_2 \times l_2}{\pi r_2^2 \times Y_2} = \frac{(6 \times 9.8) \times 1 \times 7}{22 \times (0.125 \times 10^{-2})^2 \times (0.91 \times 10^{11})}$

$= 1.3 \times 10^{-4}$ m.

9.6. *The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?*



Sol. Here, side of cube, $L = 10 \text{ cm} = \frac{10}{100} = 0.1$ m

$\therefore$ Area of each face, $A = (0.1)^2 = 0.01 \text{ m}^2$

Tangential force acting on the face,

$F = 100 \text{ kg} = 100 \times 9.8 = 980$ N

Shear modulus, $\eta = 25 \text{ GPa} = 25 \times 10^9 \text{ Nm}^{-2}$

Since shear modulus is given as:

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

$$\therefore \text{ Shearing strain} = \frac{\text{Tangential stress}}{\text{Shear modulus}}$$

$$= \frac{F}{A\eta} = \frac{980}{0.01 \times 25 \times 10^9} = 3.92 \times 10^{-6}$$

Now, $\frac{\text{Lateral Strain}}{\text{Side of cube}} = \text{Shearing strain}$

$\therefore$ Lateral Strain $=$ Shearing strain $\times$ Side of the cube

$= 3.92 \times 10^{-6} \times 0.1$

$= 3.92 \times 10^{-7} \text{ m} \approx 4 \times 10^{-7}$ m.

9.7. *Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. Young's modulus, Y = 2.0 × 10^{11} Pa.*

Sol. Here total mass to be supported, M = 50,000 kg

∴ Total weight of the structure to be supported = Mg

$$= 50{,}000 \times 9.8 \text{ N}$$

Since this weight is to be supported by 4 columns,

∴ Compressional force on each column (F) is given by

$$F = \frac{Mg}{4} = \frac{50{,}000 \times 9.8}{4} \text{ N}$$

Inner radius of a column, r_1 = 30 cm = 0.3 m

Outer radius of a column, r_2 = 60 cm = 0.6 m.

∴ Area of cross-section of each column is given by

$$A = \pi(r_2^2 - r_1^2)$$

$$= \pi[(0.6)^2 - (0.3)^2] = 0.27\,\pi \text{ m}^2$$

Young's modulus, $Y = 2 \times 10^{11}$ Pa

Compressional strain of each column = ?

$$\therefore \quad Y = \frac{\text{Compressional force / area}}{\text{Compressional Strain}}$$

$$= \frac{F/A}{\text{Compressional Strain}}$$

or Compressional strain of each column

$$= \frac{F}{AY} = \frac{50{,}000 \times 9.8 \times 7}{4 \times 0.27 \times 22 \times 2 \times 10^{11}}$$

$$= 0.722 \times 10^{-6}$$

∴ Compressional strain of all columns is given by

$$= 0.722 \times 10^{-6} \times 4 = 2.88 \times 10^{-6}$$

$$= 2.88 \times 10^{-6} \ .$$

9.8. *A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain? Shear modulus of elasticity of copper is 42 × 10^9 N/m^2.*

Sol. Here, $A = 15.2 \times 19.2 \times 10^{-6}$ m^2;

$F = 44500$ N; $\eta = 42 \times 10^9$ Nm^{-2}

$$\text{Strain} = \frac{\text{Stress}}{\text{modulus of elasticity}} = \frac{F/A}{\eta}$$

$$= \frac{F}{A\eta} = \frac{44500}{(15.2\times19.2\times10^{-6})\times42\times10^{9}} = 3.65 \times 10^{-3}.$$

9.9. *A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 Nm^{-2}, what is the maximum load the cable can support?*

Sol. Maximum load = Maximum stress × Cross-sectional area

$$= 10^8 \text{ Nm}^{-2} \times \frac{22}{7} \times (1.5 \times 10^{-2} \text{ m})^2$$
$$= 7.07 \times 10^4 \text{ N.}$$

9.10. *A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.*

Sol. Since each wire is to have same tension therefore, each wire has same extension. Moreover, each wire has the same initial length. So, strain is same for each wire.

Now, $$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/(\pi D^2/4)}{\text{Strain}}$$

or $$Y \propto \frac{1}{D^2} \Rightarrow D \propto \frac{1}{\sqrt{Y}}$$

$$\frac{D_{copper}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{copper}}} = \sqrt{\frac{190\times10^9}{110\times10^9}} = \sqrt{\frac{19}{11}} = 1.314$$

9.11. *A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1 m, is whirled in a vertical circle with an angular velocity of 2 rev./s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path. Y_{steel} = 2 × 10^{11} Nm^{-2}.*

Sol. Here, m = 14.5 kg; $l = r$ = 1 m; v = 2 rps; A = 0.065 × 10^{-4} m^2

Total pulling force on mass, when it is at the lowest position of the vertical circle is

$$F = mg + mr\,\omega^2 = mg + mr\,4\,\pi^2\,v^2$$
$$= 14.5 \times 9.8 + 14.5 \times 1 \times 4 \times (22/7)^2 \times 2^2$$
$$= 142.1 + 2291.6 = 2433.9 \text{ N}$$

$$Y = \frac{F}{A}\times\frac{l}{\Delta l}$$

or $$\Delta l = \frac{Fl}{AY} = \frac{2433.7\times1}{(0.065\times10^{-4})\times(2\times10^{11})}$$
$$= 1.87 \times 10^{-3} \text{ m} = 1.87 \text{ m.}$$

9.12. *Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013 × 10^5 Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.*

Sol. Here P = 100 atmosphere

$= 100 \times 1.013 \times 10^5$ Pa ($\because$ 1 atm = 1.013×10^5 Pa)

Initial volume,

$V_1 = 100$ litre $= 100 \times 10^{-3}$ m^3

Final volume,

$V_2 = 100.5$ litre $= 100.5 \times 10^{-3}$ m^3

$\therefore$ Change in volume

$= \Delta V = V_2 - V_1$

$= (100.5 - 100) \times 10^{-3}$ m^3

$= 0.5 \times 10^{-3}$ m^3

Using formula of bulk modulus,

$$B = \frac{P}{\frac{\Delta V}{V}} = \frac{PV}{\Delta V} = \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}}$$

$$B = 2.026 \times 10^9 \text{ Pa}$$

Also we know that the bulk modulus of air = 1.0×10^5 Pa

Now,
$$\frac{\text{Bulk modulus of water}}{\text{Bulk modulus of air}} = \frac{2.026 \times 10^9}{1.0 \times 10^5}$$

$$= 2.026 \times 10^4 = 20260$$

The ratio is too large. This is due to the fact that the strain for air is much larger than for water at the same temperature. In other words, the intermolecular distances in case of liquids are very small as compared to the corresponding distances in the case of gases. Hence there are larger interatomic forces in liquids than in gases.

9.13. *What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03 × 10^3 kg m^{-3}?*

Sol. Compressibility of water,

$$k = \frac{1}{B} = 45.8 \times 10^{-11} \text{ Pa}^{-1}$$

Change in pressure,

Δp = 80 atm − 1 atm

= 79 atm = $79 \times 1.013 \times 10^5$ Pa

Density of water at the surface,

$$\rho = 1.03 \times 10^3 \text{ kg m}^{-3}$$

As $B = \frac{\Delta p.V}{\Delta V}$ or $\frac{\Delta V}{V} = \frac{\Delta p}{B} = \Delta p \times \frac{1}{B} = \Delta p \times k$

or $\frac{\Delta V}{V} = 79 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 3.665 \times 10^{-5}$

Now $\frac{\Delta V}{V} = \frac{(M/\rho)-(M/\rho')}{(M/\rho)} = 1 - \frac{\rho}{\rho'}$

or $\frac{\rho}{\rho'} = 1 - \frac{\Delta V}{V}$ or $\rho' = \frac{\rho}{1-(\Delta V/V)}$

or $\rho' = \frac{1.03\times10^3}{1-3.665\times10^{-3}} = \frac{1.03\times10^3}{0.996}$

$= 1.034\times10^3 \text{ kg/m}^3.$

9.14. *Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.*

Sol. Here, $P = 10 \text{ atm} = 10 \times 1.013 \times 10^5$ Pa; $k = 37 \times 10^9 \text{ Nm}^{-2}$

Volumetric strain $= \frac{\Delta V}{V} = \frac{P}{K} = \frac{10\times1.013\times10^5}{37\times10^9} = 2.74 \times 10^{-5}$

$\therefore$ Fractional change in volume $= \frac{\Delta V}{V} = 2.74 \times 10^{-5}$.

9.15. *Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0 × 10⁶ Pa.*

Sol. Here a side of copper cube a = 10 cm, hence volume $V = a^3 = 10^{-3}$ m^3, hydraulic pressure applied $p = 7.0 \times 10^6$ Pa and from table we find that bulk modulus of copper

$$B = 140 \text{ G Pa} = 140 \times 10^9 \text{ Pa.}$$

Using the relation

$B = -\frac{P}{\frac{\Delta V}{V}}$, we have decrease in

volume $\Delta V = \frac{PV}{B}$

$\therefore \Delta V = \frac{7.0\times10^6\times10^{-3}}{140\times10^9} = 5 \times 10^{-8} \text{ m}^3 = 5 \times 10^{-2} \text{ cm}^3.$

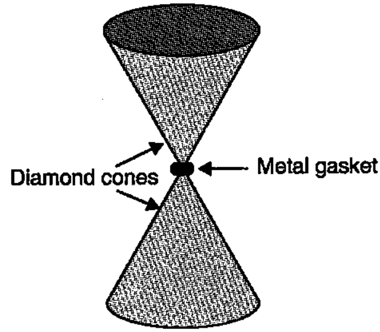
9.16. *How much should be pressure the a litre of water be changed to compress it by 0.10 %? Bulk modulus of elasticity of water = 2.2 × 10⁹ Nm⁻².*

Sol. Here, $V = 1 \text{ litre} = 10^{-3} \text{ m}^3; \quad \Delta V/V = 0.10/100 = 10^{-3}$

$$K = \frac{pV}{\Delta V}$$

or $$p = K\frac{\Delta V}{V} = (2.2 \times 10^9) \times 10^{-3} = 2.2 \times 10^6 \text{ Pa.}$$

9.17. *Anvils made of single crystals of diamond, with the shape as shown in figure are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.50 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?*



Sol. Diameter of the corner end of the anvil,

$$d = 0.50 \text{ mm} = 0.50 \times 10^{-3} \text{ m}$$

Area of cross-section of tip,

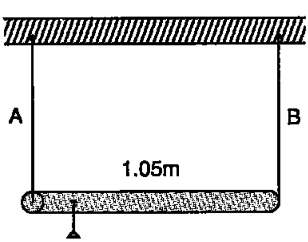
$$A = \frac{\pi d^2}{4} = \frac{22 \times (0.50 \times 10^{-3})^2}{7 \times 4} \text{m}^2$$

Stress (= pressure at the tip of the anvil)

$$= \frac{F}{A} = \frac{50{,}000 \times 4 \times 7}{22 \times (0.50)^2 \times 10^{-6}} \text{Nm}^{-2}$$

$$= 2.54 \times 10^{11} \text{ Nm}^{-2} \text{ (or Pa).}$$

9.18. *A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in figure. The cross-sectional areas of wires A and B are 1.0 mm² and 2.0 mm², respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires.*



Sol. For steel wire A, $l_1 = l$; $A_z = 1 \text{ mm}^2$; $Y_1 = 2 \times 10^{11} \text{ Nm}^{-2}$

For aluminium wire B, $l_2 = l$; $A_2 = 2 \text{mm}^2$; $Y_2 = 7 \times 10^{10} \text{ Nm}^{-2}$

(*a*) Let mass m be suspended from the rod at distance x from the end where wire A is connected. Let F_1 and F_2 be the tensions in two wires and there is equal stress in two wires, then

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \frac{F_1}{F_2} = \frac{A_1}{A_2} = \frac{1}{2} \quad ...(i)$$

Taking moment of forces about the point of suspension of mass from the rod, we have

$$F_1x = F_2(1.05 - x) \quad \text{or} \quad \frac{1.05-x}{x} = \frac{F_1}{F_2} = \frac{1}{2}$$

or $\quad 2.10 - 2x = x \Rightarrow x = 0.70 \text{ m} = 70 \text{ cm}$

(*b*) Let mass m be suspended from the rod at distance x from the end where wire A is connected. Let F_1 and F_2 be the tension in the wires and there is equal strain in the two wires *i.e.*,

$$\frac{F_1}{A_1Y_1} = \frac{F_2}{A_2Y_2}$$

$$\Rightarrow \frac{F_1}{F_2} = \frac{A_1Y_1}{A_2Y_2} = \frac{1}{2} \times \frac{2\times10^{11}}{7\times10^{10}} = \frac{10}{7}$$

As the rod is stationary, so

$$F_1x = F_2(1.05-x) \quad \text{or} \quad \frac{1.05-x}{x} = \frac{F_1}{F_2} = \frac{10}{7}$$

$$\Rightarrow 10x = 7.35 - 7x \quad \text{or} \quad x = 0.4324 \text{ m} = 43.2 \text{ cm.}$$

9.19. *A mild steel wire of length 1.0 m and cross-sectional area 0.50×10^{-2} cm² is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100g is suspended from the mid-point of the wire. Calculate the depression at the mid-point.*

Sol. Let AB be a mild steel wire of length $2L$ = 1m and its cross-section area A = 0.50×10^{-2} cm². A mass m = 100 g = 0.1 kg is suspended at mid-point C of wire as shown in figure. Let x be the depression at mid-point *i.e.*, CD = x

$$\therefore \quad AD = DB = \sqrt{AC^2+CD^2} = \sqrt{L^2+x^2}$$

$\therefore$ Increase in length

$$\Delta L = (AD + DB) - AB = 2\sqrt{L^2+x^2} - 2L$$

$$= 2L\left[\left(1+\frac{x^2}{L^2}\right)^{\frac{1}{2}} - 1\right] = 2L.\frac{x^2}{2L^2} = \frac{x^2}{L}$$

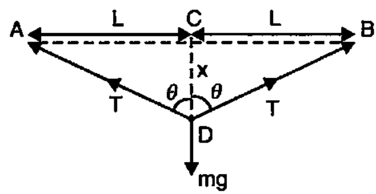
$\therefore$ Longitudinal strain $= \dfrac{\Delta L}{2L} = \dfrac{x^2}{2L^2}$.

If T be the tension in the wire as shown in Fig., then in equilibrium
$2T \cos \theta = mg$

or $$T = \frac{mg}{2\cos\theta} = \frac{mg}{2\cdot\frac{x}{\sqrt{x^2+L^2}}} = \frac{mg\sqrt{x^2+L^2}}{2x} = \frac{mgL}{2x}$$

[Since $x << L$]

$\therefore$ Stress $= \dfrac{T}{A} = \dfrac{mgL}{2xA}$



As Young's modulus

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{\left(\frac{mgL}{2xA}\right)}{\left(\frac{x^2}{2L^2}\right)} = \frac{mgL}{2xA} \times \frac{2L^2}{x^2} = \frac{mgL^3}{Ax^3}$$

$$\Rightarrow \quad x = \left[\frac{mgL^3}{YA}\right]^{\frac{1}{3}} = L\left[\frac{mg}{YA}\right]^{\frac{1}{3}}$$

$$= \frac{1}{2}\left[\frac{0.1\times 9.8}{2\times 10^{11}\times 0.50\times 10^{-2}\times 10^{-4}}\right]^{\frac{1}{3}} = 1.074 \times 10^{-2}\text{ m}$$

$= 1.074$ cm $\simeq 1.07$ cm or 0.01 m.

9.20. *Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9×10^7 Pa? Assume that each rivet is to carry one quarter of the load.*

Sol. Diameter = 6mm; Radius, $r = 3 \times 10^{-3}$ m;

Maximum stress $= 6.9 \times 10^7$ Pa

Maximum load on a rivet

= Maximum stress × cross-sectional area

$= 6.9 \times 10^7 \times \dfrac{22}{7}(3 \times 10^{-3})^2$ N $= 1952$ N

Maximum tension $= 4 \times 1951.7$ N

$= 7.8 \times 10^3$ N.

9.21. *The Marina trench is located in the Pacific Ocean, and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume $0.32\ m^3$ is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom?*

Sol. Given, $P = 1.1 \times 10^8$ Pa, $V = 0.32$ m^3, $K = 1.6 \times 10^{11}$ Nm^{-2}

Bulk modulus for steel $= 1.6 \times 10^{11}$ Nm^{-2}

Using relation, $$K = \frac{P}{\frac{\Delta V}{V}} = \frac{PV}{\Delta V} \quad \text{or,} \quad \Delta V = \frac{PV}{K}$$

$$\Rightarrow \qquad \Delta V = \frac{1.1 \times 10^8 \times 0.32}{1.6 \times 10^{11}} \text{ m}^3 = 2.2 \times 10^{-4} \text{ m}^3.$$

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