10 Mechanical Properties of Fluids

Lesson at a Glance

• Fluids are the sustances which can flow *e.g.*, liquids and gases. It does not possess definite shape.

• Pressure

The thrust experienced per unit area of the surface of a liquid at rest is called pressure.

$$P = \frac{F}{A}$$

• Pascal's Law

According to Pascal's Law, the pressure appiied to an enclosed liquid is transmitted undiminished to every portion of the liquid and the walls of the containing vessel.

• Hydraulic system works on Pascal's law. Force exerted to area, ratio will be same at all cross-sections.

$$\frac{F_1}{a_1} = \frac{F_2}{a_2}$$

Note: A large force is experienced at larger cross-section if a smaller force is applied in smaller cross-section.

• A column of height h of a liquid of density ρ exerts a pressure P given by the relation

$$P = h\rho g$$

• Archimedes' Principle

When a body is partially or completely immersed in a liquid, it loses some of its weight. The loss in weight of the body in the liquid is equal to the weight of the liquid displaced by the immersed part of the body.

• Equation of Continuity

According to equation of continuity, if there is no fluid source or sink along the length of a pipe, then mass of the fluid crossing any section of the pipe per unit time remains constant.

i.e., $$a_1 v_1 \rho_1 = a_2 v_2 \rho_2$$

• Energy of a liquid

A liquid can possess three types of energies: (*i*) kinetic energy, (*ii*) potential energy and (*iii*) pressure energy

The energy possessed by a liquid due to its motion is called kinetic energy *i.e.,* $\frac{1}{2}mv^2$.

The potential energy of a liquid of mass m at a height h is given by

$$\text{P.E.} = mgh$$

• Bernoulli's Theorem

For an incompressible, non-viscous, irrotational liquid having streamlined flow, the sum of the pressure energy, kinetic energy and potential energy per unit mass is a constant *i.e.,*

$$\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{constant} \quad \text{or} \quad \frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

• For steady flow of a non-viscous fluid along a horizontal pipe, Bernoulli's equation is simplified as

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

• Viscosity

Viscosity is the property of the fluid (liquid or gas) by virtue of which an internal frictional force comes into play when the fluid is in motion in the form of layers having relative motion. It opposes the relative motion of the different layers. Viscosity is also called as fluid friction.

• The viscous force directly depends on the area of the layer and the velocity gradient.

$$F = -\eta A \frac{dv}{dx}$$

(*–ve* sign shows the opposing nature)

• Coefficient of Viscosity

Coefficient of viscosity of a liquid is equal to the tangential force required to maintain a unit velocity gradient between two parallel layers of liquid each of area unity.

$$\eta = \frac{F}{A\left(\frac{dv}{dx}\right)}$$

The SI unit of coefficient of viscosity is poiseuille (*Pl*) or *Pa* – s or Nm^{-2} s or kg m^{-1} s^{-1}. Dimensional formula of η is $[ML^{-1} T^{-1}]$.

• Stokes' Law

According to Stokes' law the backward dragging force acting on a small spherical body of radius *r* moving with a velocity *v* through a viscous medius of coefficient of viscosity η is given by

$$F = 6\pi\eta r v$$

• Terminal Velocity

It is maximum constant velocity acquired by the body while falling freely in a viscous medium.

This is attained when the apparent weight is compensated by the viscous force.

It is given by

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

where ρ be the density of the material of the body of radius *r* and σ be the density of the medium.

• Poiseuille's Equation

According to Poiseuille, if a pressure difference (*P*) is maintained across the two ends of a capillary tube of length '*l*' and radius '*r*', then the volume of liquid coming out of the tube per second is

$$V = \frac{\pi P r^4}{8\eta l}$$

• Reynold's Number

Reynold number R_e is a dimensionless number whose value gives an approximate idea whether the flow of a fluid will be streamline or turbulent. It is given by

$$R_e = \frac{\rho v d}{\eta},$$

where ρ = density of fluid flowing with a speed *v*, *d* stands for the diameter of the pipe and η is the viscosity of the fluid. Value of R_e remains same in any system of units.

• It is observed that flow is streamline or laminar for $R_e \leq 1000$ and the flow is turbulent for $R_e \geq 2000$. The flow becomes unsteady for R_e between 1000 and 2000. The critical value of R_e, at which turbulence sets, is same for the geometrically similar flows.

• Critical Velocity

The critical velocity is that velocity of liquid flow, upto which its flow is streamline and above which its flow becomes turbulent.
It is given by

$$v_c = \frac{K\eta}{\rho r}$$

where K is a dimensionless constant, η is coefficient of viscosity of liquid, ρ is density of liquid and r is the radius of tube.

• Surface Tension

It is the property of the liquid by virtue of which the free surface of liquid at rest tends to have minimum area and as such it behaves as a stretched elastic membrane.

$$T = \frac{F}{l}$$

The SI unit of surface tension is Nm^{-1} and its dimensional formula is $[MT^{-2}]$.

• Surface Energy

Energy possessed by the surface of the liquid is called surface energy. Change in surface energy is the product of surface tension and change in surface area under constant temperature.

• The height to which water rises in a capillary tube of radius r is given by

$$h = \frac{2T\cos\theta}{r\rho g}$$

where T is the surface tension of the liquid and θ is the angle of contact.

• Torricelli's Theorem

According to this theorem, velocity of efflux *i.e.*, the velocity with which the liquid flows out of on orifice (*i.e.*, a narrow hole) is equal to that which a freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid.

The velocity is given by

$$v = \sqrt{2gh}$$

TEXTBOOK QUESTIONS SOLVED

10.1. *Explain why*

(a) The blood pressure in humans is greater at the feet than at the brain.

(b) Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km.

(c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.

Ans. (*a*) The height of the blood column is more for the feet as compared to that for the brain. Consequently, the blood pressure in humans is greater at the feet than at the brain.

(*b*) The variation of air-density with height is not linear. So, pressure also does not reduce linearly with height. The air pressure at a height h is given by $P = P_0 e^{-\alpha h}$ where P_0 represents the pressure of air at sea-level and α is a constant.

(*c*) Due to applied force on liquid, the pressure is transmitted equally in all directions inside the liquid. That is why there is no fixed direction for the pressure due to liquid. Hence hydrostatic pressure is a scalar quantity.

10.2. *Explain why*

(a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.

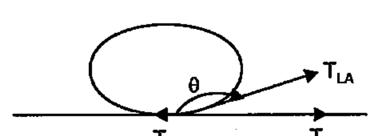
(b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)

(c) Surface tension of a liquid is independent of the area of the surface.

(d) Water with detergent dissolved in it should have small angles of contact.

(e) A drop of liquid under no external forces is always spherical in shape.

Ans. (*a*) Let a drop of a liquid L be poured on a solid surface S placed in air A. If T_{SL}, T_{LA} and T_{SA} be



the surface tensions corresponding to solid-liquid layer, liquid-air layer and solid-air layer respectively and θ be the angle of contact between the liquid and solid, then

$$T_{LA} \cos\theta + T_{SL} = T_{SA}$$

$$\Rightarrow \qquad \cos\theta = \frac{T_{SA} - T_{SL}}{T_{LA}}$$

For the mercury-glass interface, $T_{SA} < T_{SL}$. Therefore, cos θ is negative. Thus θ is an obtuse angle. For the water-glass interface, $T_{SA} > T_{SL}$. Therefore cos θ is positive. Thus, θ is an acute angle.

(*b*) Water on a clean glass surface tends to spread out *i.e.,* water wets glass because force of cohesion of water is much less than the force of adhesion due to glass. In case of mercury force of cohesion due to mercury molecules is quite strong as compared to adhesion force due to glass. Consequently, mercury does not wet glass and tends to form drops.

(*c*) Surface tension of liquid is the force acting per unit length on a line drawn tangentially to the liquid surface at rest. Since this force is independent of the area of liquid surface therefore, surface tension is also independent of the area of the liquid surface.

(*d*) We know that the clothes have narrow pores or spaces which act as capillaries. Also, we know that the rise of liquid in a capillary tube is directly proportional to cos θ (Here θ is the angle of contact). As θ is small for detergent, therefore cos θ will be large. Due to this, the detergent will penetrate more in the narrow pores of the clothes.

(*e*) We know that any system tends to remain in a state of minimum energy. In the absence of any external force for a given volume of liquid its surface area and consequently. Surface energy is least for a spherical shape. It is due to this reason that a liquid drop, in the absence of an external force is spherical in shape.

10.3. *Fill in the blanks using the words from the list appended with each statement:*

(a) Surface tension of liquids generally .... with temperature. (increases/decreases)

(*b*) *Viscosity of gases .... with temperature, whereas viscosity of liquids .... with temperature. (increases/decreases)*

(*c*) *For solids with elastic modulus of rigidity, the shearing force is proportional to .... while for fluids it is proportional to ....(shear strain/rate of shear strain)*

(*d*) *For a fluid in steady flow, the increases in flow speed at a constriction follows from .... while the decrease of pressure there follows from ....*
(conservation of mass/Bernoulli's principle)

(*e*) *For the model of a plane in a wind tunnel, turbulence occurs at a .... speed than the critical speed for turbulence for an actual plane. (greater/smaller)*

Ans. (*a*) decreases

(*b*) increases; decreases

(*c*) shear strain; rate of shear strain

(*d*) conservation of mass; Bernoulli's principle

(*e*) greater.

10.4. *Explain why*

(*a*) *To keep a piece of paper horizontal, you should blow over, not under, it.*

(*b*) *When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers.*

(*c*) *The size of a needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.*

(*d*) *A fluid flowing out of a small hole in a vessel results in a backward thurst on the vessel.*

(*e*) *A spinning cricket ball in air does not follow a parabolic trajectory.*

Ans. (*a*) When we blow over the piece of paper, the velocity of air increases. As a result, the pressure on it decreases in accordance with the Bernoulli's theorem whereas the pressure below remains the same (atmospheric pressure). Thus, the paper remains horizontal.

(*b*) By doing so the area of outlet of water jet is reduced, so velocity of water increases according to equation of continuity av = constant.

(*c*) For a constant height, the Bernoulli's theorem is expressed as

$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

In this equation, the pressure P occurs with a single power whereas the velocity occurs with a square power. Therefore, the velocity has more effect compared to the pressure. It is for this reason that needle of the syringe controls flow rate better than the thumb pressure exerted by the doctor.

(*d*) This is because of principle of conservation of momentum. While the flowing fluid carries forward momentum, the vessel gets a backward momentum.

(*e*) A spinning cricket ball would have followed a parabolic trajectory has there been no air. But because of air the Magnus effect takes place. Due to the Magnus effect the spinning cricket ball deviates from its parabolic trajectory.

10.5. *A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?*

Ans. Mass of girl, $m = 50$ kg.

$\therefore$ Force on the heel, $F = mg = 50 \times 9.8 = 490$ N

Diameter, $D = 1.0 \text{ cm} = 1 \times 10^{-2}$ m

$$\therefore \quad \text{Area, } A = \frac{\pi D^2}{4} = \frac{3.14 \times (1 \times 10^{-2})^2}{4} = 7.85 \times 10^{-5} \text{ m}^2$$

$$\therefore \quad \text{Pressure, } P = \frac{F}{A} = \frac{490}{7.85 \times 10^{-5}} = 6.24 \times 10^6 \text{ Pa.}$$

10.6. *Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m^{-3}. Determine the height of the wine column for normal atmospheric pressure.*

Ans. We know that atmospheric pressure, $P = 1.01 \times 10^5$ Pa.

If we use French wine of density, $\rho = 984$ kg m^{-3}, then height of wine column should be h_m, such that $P = h\rho g$

$$\Rightarrow \quad h_m = \frac{P}{\rho g} = \frac{1.01 \times 10^5}{984 \times 9.8} = 10.47\text{m} \simeq 10.5 \text{ m}$$

10.7. *A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.*

Ans. Here, Maximum stress $= 10^9$ Pa, $h = 3 \text{ km} = 3 \times 10^3$ m;

ρ (water) $= 10^3$ kg/m^3 and $g = 9.8$ m/s^2.

The structure will be suitable for putting upon top of an oil well provided the pressure exerted by sea water is less than the maximum stress it can bear.

Pressure due to sea water,

$$P = h\rho g = 3 \times 10^3 \times 10^3 \times 9.8 \text{ Pa} = 2.94 \times 10^7 \text{ Pa}$$

Since the pressure of sea water is less than the maximum stress of 10^9 Pa, the structure will be suitable for putting upon top of the oil well.

10.8. *A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm². What maximum pressure would the smaller piston have to bear?*

Ans. Pressure on the piston due to car

$$= \frac{\text{Weight of car}}{\text{Area of piston}}$$

$$P = \frac{3000 \times 9.8}{425 \times 10^{-4}} \text{Nm}^{-2} = \mathbf{6.92 \times 10^5 \text{ Pa}}$$

This is also the maximum pressure that the smaller piston would have to bear.

10.9. *A U tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the relative density of spirit?*

Ans. For water column in one arm of U tube, h_1 = 10.0 cm; ρ_1(density) = 1 g cm^{-3}

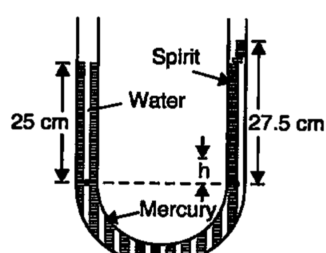
For spirit column in other arm of U tube, h_2 = 12.5 cm; ρ_2 =?

As the mercury columns in the two arms of U tube are in level, therefore pressure exerted by each is equal.

$$\text{Hence} \quad h_1\rho_1 g = h_2\rho_2 g \quad \text{or} \quad \rho_2 = \frac{h_1\rho_1}{h_2} = \frac{10 \times 1}{12.5} = 0.8 \text{ g cm}^{-3}$$

Therefore, relative density of spirit = ρ_2/ρ_1 = 0.8/1 = 0.8

10.10. *In Q.9, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (Relative density of mercury = 13.6)*



Ans. Let us select two points A and B lying in the same horizontal plane. Applying Pascal's law (taking into account the force of gravity),

$$\text{Pressure at } A = \text{Pressure at } B$$

$$\therefore \quad P_0 + h_w\rho_w g = P_0 + h_s\rho_s g + h_m\rho_m g$$

where P_0 is the atmospheric pressure,

$$\text{Now,} \quad h_w\rho_w = h_s\rho_s + h_m\rho_m$$

$$\therefore \quad 25 \times 1 = 27.5 \times 0.8 + h \times 13.6$$

$$\text{or} \quad h \times 13.6 = 25 - 27.5 \times 0.8$$

$$\text{or} \quad h = \frac{25-22}{13.6} \text{ cm} = 0.2206 \text{ cm}$$

Mercury will rise in the arm containing spirit; the difference in levels of mercury will be 0.2206 cm.

10.11. *Can Bernoulli's equation be used to describe the flow of water through a rapid motion in a river? Explain.*

Ans. Bernoulli's theorem is applicable only for ther it ideal fluids in streamlined motion. Since the flow of water in a river is rapid, way cannot be treated as streamlined motion, the theorem cannot be used.

10.12. *Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain.*

Ans. No, it does not matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation, provided the atmospheric pressure at the two points where Bernoulli's equation is applied are significantly different.

10.13. *Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0×10^{-3} kg s^{-1}, what is the pressure difference between the two ends of the tube? (Density of glycerine = 1.3×10^3 kg m^{-3} and viscosity of glycerine = 0.83 Pa s). [You may also like to check if the assumption of laminar flow in the tube is correct].*

Ans. $l = 1.5$ m, $r = 1 \times 10^{-2}$m,

$$\text{Volume/s,} \quad V = \frac{\text{Mass / s}}{\text{Density}} = \frac{4 \times 10^{-3}}{1.3 \times 10^{3}} \text{m}^3 \text{ s}^{-1}$$

$$= \frac{4}{1.3} \times 10^{-6} \text{m}^3\text{s}^{-1}$$

$$\eta = 0.83 \text{ Pa s}$$

Now, $V = \frac{\pi p r^4}{8\eta l}$,

where p is the pressure difference across the capillary.

or $p = \frac{8V\eta l}{\pi r^4}$

Substituting values,

$$p = 8 \times \frac{4}{1.3} \times 10^{-6} \times 0.83 \times 1.5 \times \frac{7}{22} \times \frac{1}{10^{-8}} \text{ Pa} = \mathbf{9.75 \times 10^2 \text{ Pa}}$$

The Reynolds number is 0.3. So, the flow is **laminar.**

10.14. *In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 ms^{-1} and 63 ms^{-1} respectively. What is the lift on the wing if its area is 2.5 m^2? Take the density of air to be 1.3 kg m^{-3}.*

Ans. Let v_1, v_2 be the speeds on the upper and lower surfaces of the wing of aeroplane, and P_1 and P_2 be the pressures on upper and lower surfaces of the wing respectively.

Then $v_1 = 70$ ms^{-1}; $v_2 = 63$ ms^{-1}; $\rho = 1.3$ kg m^{-3}.

From Bernoulli's theorem

$$\frac{P_1}{\rho} + gh + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh + \frac{1}{2}v_2^2$$

$$\therefore \quad \frac{P_1}{\rho} - \frac{P_2}{\rho} = \frac{1}{2}(v_2^2 - v_1^2)$$

or

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2} \times 1.3\,[(70)^2 - (63)^2] \text{ Pa}$$

$$= 605.15 \text{ Pa.}$$

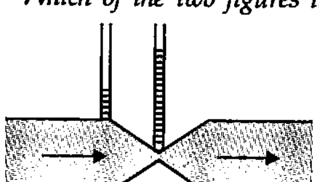
This difference of pressure provides the lift to the aeroplane.

So, lift on the aeroplane = pressure difference × area of wings

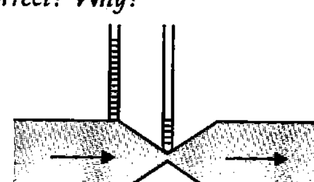
= 605.15 × 2.5 N = 1512.875 N

= 1.51 × 10^3 N.

10.15. *Figures (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures in incorrect? Why?*



(a)



(b)

Ans. Figure (*a*) is incorrect. It is because of the fact that at the kink, the velocity of flow of liquid is large and hence using the Bernoulli's theorem the pressure is less. As a result, the water should not rise higher in the tube where there is a kink (*i.e.*, where the area of cross-section is small).

10.16. *The cylindrical tube of a spare pump has a cross-section of 8.0 cm² one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min⁻¹, what is the speed of ejection of the liquid through the holes?*

Ans. Total cross-sectional area of 40 holes, a_2

$$= 40 \times \frac{22}{7} \times \frac{(1\times10^{-3})^2}{4} \text{m}^2$$

$$= \frac{22}{7} \times 10^{-5}\text{m}^2$$

Cross-sectional area of tube, $a_1 = 8 \times 10^{-4}$ m²

Speed inside the tube, v_1=1.5 m min⁻¹ $= \frac{1.5}{60}\text{ms}^{-1}$;

Speed of ejection, $v_2 = ?$

Using $a_2v_2 = a_1v_1$,

we get $$v_2 = \frac{a_1v_1}{a_2} = \frac{8\times10^{-4}\times\frac{1.5}{60}\times7}{22\times10^{-5}}\text{ms}^{-1} = 0.64 \text{ ms}^{-1}.$$

10.17. *A U-shaped wire is dipped in a soap solution, and removed. A thin soap film formed between the wire and a light slider supports a weight of 1.5 × 10⁻² N (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film?*

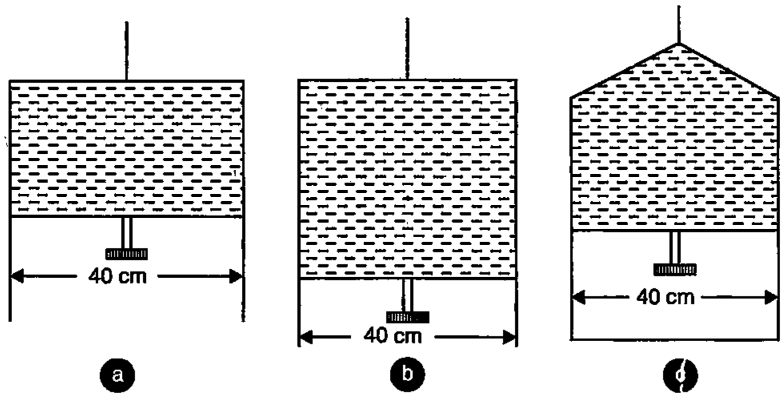
Ans. In present case force of surface tension is balancing the weight of 1.5 × 10⁻² N, hence force of surface tension, $F = 1.5 \times 10^{-2}$ N.

Total length of liquid film, $l = 2 \times 30$ cm = 60 cm = 0.6 m because the liquid film has two surfaces.

$$\therefore \quad \text{Surface tension, } T = \frac{F}{l} = \frac{1.5\times10^{-2}\text{N}}{0.6\text{ m}}$$

$$= 2.5 \times 10^{-2} \text{ Nm}^{-1}.$$

10.18. *Figure (a) below shows a thin film supporting a small weight = 4.5 × 10⁻²N. What is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c)? Explain your answer physically.*



Ans. (*a*) Here, length of the film supporting the weight = 40 cm = 0.4 m.

Total weight supported (or force) = 4.5×10^{-2} N.
Film has two free surfaces,

$$\therefore \quad \text{Surface tension, } S = \frac{4.5 \times 10^{-2}}{2 \times 0.4} = 5.625 \times 10^{-2} \text{ Nm}^{-1}.$$

Since the liquid is same for all the cases (*a*), (*b*) and (*c*), and temperature is also same, therefore surface tension for cases (*b*) and (*c*) will also be the same = 5.625×10^{-2}. In Fig. 7(*b*), 38(*b*) and (*c*), the length of the film supporting the weight is also the same as that of (*a*), hence the total weight supported in each case is 4.5×10^{-2} N.

10.19. *What is the pressure inside a drop of mercury of radius 3.0 mm at room temperature? Surface tension of mercury at that temperature (20°C) is 4.65×10^{-1} Nm^{-1}. The atmospheric pressure is 1.01×10^5 Pa. Also give the excess pressure inside the drop.*

Ans. Excess pressure $= \dfrac{2\sigma}{R} = \dfrac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} = 310$ Pa

Total pressure $= 1.01 \times 10^5 + \dfrac{2\sigma}{R}$

$= 1.01 \times 10^5 + 310 = 1.0131 \times 10^5$ Pa

Since data is correct upto three significant figures, we should write total pressure inside the drop as 1.01×10^5 Pa.

10.20. *What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20 °C) is 2.50×10^{-2} Nm^{-1}? If an air bubble of the same* *dimension were formed at depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atmospheric pressure is 1.01×10^5 Pa).*

Ans. Here surface tension of soap solution at room temperature

$T = 2.50 \times 10^{-2}\ Nm^{-1}$, radius of soap bubble,

$r = 5.00\ mm = 5.00 \times 10^{-3} m.$

$\therefore$ Excess pressure inside soap bubble, $P = P_i - P_0 = \dfrac{4T}{r}$

$$= \frac{4 \times 2.50 \times 10^{-2}}{5.00 \times 10^{-3}} = 20.0\ Pa$$

When an air bubble of radius $r = 5.00 \times 10^{-3}$ m is formed at a depth $h = 40.0$ cm $= 0.4$ m inside a container containing a soap solution of relative density 1.20 or density $\rho = 1.20 \times 10^3\ kg\ m^{-3}$, then excess pressure

$$P = P_i - P_0 = \frac{2T}{r}$$

$$\therefore \quad P_i = P_0 + \frac{2T}{r} = (P_a + h\rho g) + \frac{2T}{r}$$

$$= \left[1.01 \times 10^5 + 0.4 \times 1.2 \times 10^3 \times 9.8 + \frac{2 \times 2.50 \times 10^{-2}}{5.00 \times 10^{-3}}\right] Pa$$

$$= (1.01 \times 10^5 + 4.7 \times 10^3 + 10.0)\ Pa$$

$$\simeq 1.06 \times 10^5\ Pa.$$

10.21. *A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. Compute the force necessary to keep the door close.*

Ans. Pressure difference across the door

$= (4 \times 1700 \times 9.8 - 4 \times 1000 \times 9.8)$ Pa

$(6.664 \times 10^4 - 3.92 \times 10^4)\ Pa = 2.774 \times 10^4\ Pa$

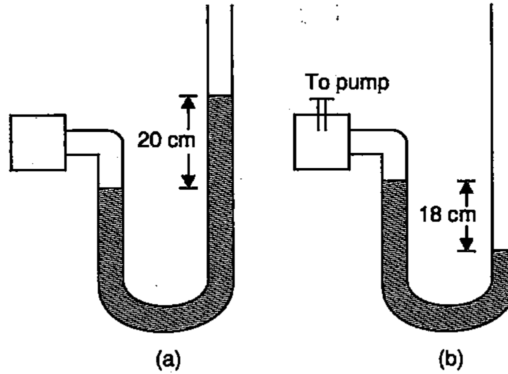
Force on the door = Pressure difference × Area of door

$= 2.774 \times 10^4 \times 20 \times 10^{-4}$ N

$= 54.88$ N $=$ **55 N.**

Note. Base area does not affect the answer.

10.22. *A manometer reads the pressure of a gas in an enclosure as shown in Fig. (a). When a pump removes some of the gas, the manometer reads as in Fig. (b). The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.*



(a) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of cm of mercury.

(b) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) is poured into the right limb of the manometer? Ignore the small change in the volume of the gas.

Ans. The atmospheric pressure, P = 76 cm of mercury

(*a*) From figure (*a*),

Pressure head, h = 20 cm of mercury

∴ Absolute pressure

$= p + h = 76 + 20 = 96$ cm of mercury

Also, Gauge pressure = h = 20 cm of mercury

From figure (*b*),

pressure head, h = –18 cm of mercury

∴ Absolute pressure = $p + h = 76 + (-18)$

= 58 cm of mercury

Also, Gauge pressure = h = –18 cm of mercury

(*b*) When 13.6 cm of water is poured into the right limb of the manometer of figure (*b*), then, using the relation:

Pressure = $\rho g h = \rho' g' h'$

We get $h' = \frac{\rho h}{\rho'} = \frac{1 \times 13.6}{13.6}$ = 1 cm of mercury

[ρ' = density of mercury]

Therefore, pressure at the point B,

$p_B = P + h' = 76 + 1$

= 77 cm of mercury

If h'' is the difference in the mercury levels in the two limbs, then taking $P_A = P_B$

$\Rightarrow$ $58 + h'' = 77$ $\Rightarrow$ $h'' = 77 - 58 = 19$ cm of mercury.

10.23. *Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill up to a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to that same height give different readings on a weighing scale?*

Ans. Pressure (and therefore force) on the two equal base areas are identical. But force is exerted by water on the sides of the vessels also, which has a non-zero vertical component when sides of the vessel are not perfectly normal to the base. This net vertical component of force by water on the sides of the vessel is greater for the first vessel than the second. Hence, the vessels weigh different even when the force on the base is the same in the two cases.

10.24. *During blood transfusion, the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein? Given: density of whole blood = 1.06×10^3 kg m^{-3}.*

Ans.
$$h = \frac{P}{\rho g} = \frac{2000}{1.06 \times 10^3 \times 9.8} = 0.1925 \text{ m}$$

The blood may just enter the vein if the height at which the blood container be kept must be slightly greater than 0.1925 m *i.e.,* 0.2 m.

10.25. *In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy. (a) What is the largest average velocity of blood flow in an artery of diameter 2×10^{-3} m if the flow must remain laminar? (b) Do the dissipative forces become more important as the fluid velocity increases? Discuss qualitatively.*

Ans. (*a*) If dissipative forces are present, then some forces in liquid flow due to pressure difference is spent against dissipative forces, due to which the pressure drop becomes large.

(*b*) The dissipative forces become more important with increasing flow velocity, because of turbulence.

10.26. *(a) What is the largest average velocity of blood flow in an artery of radius 2×10^{-3} m if the flow must remain laminar?*

(b) What is the corresponding flow rate? Take viscosity of blood to be 2.084×10^{-3} Pa–s. Density of blood is 1.06×10^3 kg/m³.

Ans. Here, $r = 2 \times 10^{-3}$ m; $D = 2r = 2 \times 2 \times 10^{-3} = 4 \times 10^{-3}$ m;

$\eta = 2.084 \times 10^{-3}$ Pa–s; $\rho = 1.06 \times 10^3$ kg m^{-3}.

For flow to be laminar, $N_R = 2000$

(a) Now, $$v_c = \frac{N_R \eta}{\rho D} = \frac{2000 \times (2.084 \times 10^{-3})}{(1.06 \times 10^3) \times (4 \times 10^{-3})} = 0.98 \text{ m/s.}$$

(b) Volume flowing per second $= \pi r^2 v_c = \frac{22}{7} \times (2 \times 10^{-3})^2 \times 0.98$

$= \mathbf{1.23 \times 10^{-5}\ m^3 s^{-1}}$.

10.27. *A plane is in level flight at constant speed and each of its wings has an area of 25m². If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. (Take air density to be 1 kg/m³). $g = 9.8$ m/s².*

Ans. Here speed of air over lower wing,

$$v_1 = 180 \text{ km/h} = 180 \times \frac{5}{18} = 50 \text{ ms}^{-1}$$

Speed over the upper wing,

$$v_2 = 234 \text{ km/h} = 234 \times \frac{5}{18} = 65 \text{ ms}^{-1}$$

$\therefore$ Pressure difference,

$$P_1 - P_2 = \frac{1}{2}\rho\,(v_2^2 - v_1^2) = \frac{1}{2} \times 1\,(65^2 - 50^2) = 862.5 \text{ Pa}$$

$\therefore$ Net upward force, $F = (P_1 - P_2)A$

This upward force balances the weight of the plane.

$\therefore$ $mg = F = (P_1 - P_2)A$ $\qquad$ [$A = 25 \times 2 = 50$ m²]

$$\therefore \quad m = \frac{(P_1 - P_2)\,A}{g} = \frac{862.5 \times 50}{9.8} = 4400 \text{ N.}$$

10.28. *In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^3 kg m^{-3}. Take the viscosity of air at the temperature of the experiment to be 1.8×10^{-5} Pa-s. How much is the viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.*

Ans. Here radius of drop, $r = 2.0 \times 10^{-5}$ m, density of drop, $\rho = 1.2 \times 10^3$ kg/m³, viscosity of air $\eta = 1.8 \times 10^{-5}$ Pa–s.

Neglecting upward thrust due to air, we find that terminal speed is

$$v_T = \frac{2}{9}\frac{r^2\rho g}{\eta} = \frac{2\times(2.0\times10^{-5})^2\times(1.2\times10^3)\times9.8}{9\times(1.8\times10^{-5})}$$

$= 5.81 \times 10^{-2}\ \text{ms}^{-1}$ or $5.81\ \text{cm s}^{-1}$

Viscous force at this speed,

$F = 6\pi\eta r v = 6 \times 3.14 \times (1.8 \times 10^{-5}) \times (2.0 \times 10^{-5}) \times (5.81 \times 10^{-2})$

$= 3.94 \times 10^{-10}$ N.

10.29. *Mercury has an angle of contact equal to 140° with soda-lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is 0.465 Nm^{-1}. Density of mercury = 13.6 × 10^3 kgm^{-3}.*

Ans. Radius of tube, $r = 1.00$ mm $= 10^{-3}$ m

Surface tension of mercury, $\sigma = 0.465\ \text{Nm}^{-1}$

Density of mercury, $\sigma = 13.6 \times 10^3\ \text{kg m}^{-3}$

Angle of contact, $\theta = 140°$

$$\therefore \quad h = \frac{2\sigma\cos\theta}{r\rho g} = \frac{2\times0.465\times\cos 140°}{10^{-3}\times13.6\times10^3\times9.8}$$

$$= \frac{2\times0.465\times(-0.7660)}{10^{-3}\times13.6\times10^3\times9.8} = -5.34 \times 10^{-3}\ \text{m} = -5.34\ \text{mm}$$

Negative sign shows that the mercury level is depressed in the tube.

10.30. *Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3 × 10^{-2} Nm^{-1}. Take the angle of contact to be zero and density of water to be 1.0 × 10^3 kg m^{-3} (g = 9.8 ms^{-2}).*

Ans. Let r_1 be the radius of one bore and r_2 be the radius of second bore of the U-tube. The, if h_1 and h_2 are the heights of water on two sides, then

$$h_1 = \frac{2S\cos\theta}{r_1\rho g} \quad \text{and} \quad h_2 = \frac{2S\cos\theta}{r_2\rho g}$$

On subtraction, we get

$$h_1 - h_2 = \frac{2S\cos\theta}{r_1\rho g} - \frac{2S\cos\theta}{r_2\rho g} = \frac{2S\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Here, $S = 7.3 \times 10^{-2}\ Nm^{-1}$, $\theta = 0$, $\rho = 1.0 \times 10^{3}\ kg\ m^{-3}$,

$g = 9.8\ ms^{-2}$, $r_1 = \frac{3}{2}$ min $= 1.5 \times 10^{-3}$ m and $r_2 = \frac{6}{2}$ mm $= 3 \times 10^{-3}$ m

$$\therefore h_1 - h_2 = \frac{2 \times 7.3 \times 10^{-2} \times \cos\theta}{1 \times 10^{3} \times 9.8}\left[\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right]$$

$$= 1.49 \times 10^{-5} \times \frac{1}{3 \times 10^{-3}} \approx 4.97 \times 10^{-3}\ m = 4.97\ mm$$

10.31. *(a) It is known that density ρ of air decreases with height y as*

$$\rho = \rho_0 e^{-y/y_0}$$

where $\rho_0 = 1.25\ kg\ m^{-3}$ is the density at sea level, and y_0 is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains a constant (isothermal conditions). Also assume that the value of g remains constant.

(b) A large He balloon of volume $1425\ m^3$ is used to lift a payload of 400 kg. Assume that the balloon maintains constant radius as it rises. How high does it rise?

[Take $y_0 = 8000$ m and $\rho_{He} = 0.18\ kg\ m^{-3}$].

Ans. (a) We know that rate of decrease of density ρ of air is directly proportional to the height y. It is given as

$$\frac{d\rho}{dy} = -\frac{\rho}{y_0},$$

where y is a constant of proportionality and $-ve$ sign signifies that density is decreasing with increase in height. On integration, we get

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = -\int_0^{y} \frac{1}{y_0} dy$$

$$\Rightarrow \quad [\log \rho]_{\rho_0}^{\rho} = -\left[\frac{y}{y_0}\right]_0^{y},$$

where, ρ_0 = density of air at sea level *i.e.*, $y = 0$

or $\log_e \frac{\rho}{\rho_0} = -\frac{y}{y_0}$ or $\rho = \rho_0 e^{-\frac{y}{y_0}}$.

Here dimensions and units of constant y_0 are same as of y.

(*b*) Here volume of He balloon, $V = 1425\ m^3$, mass of payload, $m = 400$ kg

$y_0 = 8000$ m, density of He $\rho_{He} = 0.18\ kgm^{-3}$

Mean density of balloon,

$$\rho = \frac{\text{Total mass of balloon}}{\text{Volume}} = \frac{m + V \cdot \rho_{He}}{V} \text{Pa}$$

$$= \frac{400 + 1425 \times 0.18}{1425} = 0.4608 = 0.46\ kgm^{-3}$$

As density of air at sea level $\rho_0 = 1.25\ kg\ m^{-3}$. The balloon will rise up to a height y where density of air = density of balloon $\rho = 0.46\ kgm^{-3}$

As $\rho = \rho_0 e^{-\frac{y}{y_0}}$ or $\frac{\rho_0}{\rho} = e^{\frac{y_0}{y}}$

$$\therefore \log_e\left(\frac{\rho_0}{\rho}\right) = \frac{y_0}{y} \quad \text{or} \quad y = \frac{y_0}{\log_e\left(\frac{\rho_0}{\rho}\right)} = \frac{8000}{\log_e\left(\frac{1.25}{0.46}\right)}$$

$$= 8002 \text{ m} \quad \text{or} \quad 8.0 \text{ km.}$$

❑❑❑