

11 Thermal Properties of Matter

Lesson at a Glance

- Heat is the form of energy transferred between two (or more) systems or a system and its surroundings by virtue of temperature difference.
- Temperature of a substance is a physical quantity which measures the degree of hotness or coldness of the substance.
- A branch of science which deals with the measurement of temperature of a substance is known as thermometry. A device used to measure the temperature of a body is called thermometer.
- The two familiar temperature scales are the Fahrenheit temperature scale and the Celsius temperature scale.

• Thermal Expansion

The increase of size of a body due to the increase in the temperature is called thermal expansion.

Three types of expansions can take place in solids viz. linear, superficial and volume expansion.

- The relation among coefficients of linear expansion (α), superficial expansion (β) and volume expansion (γ) is given as

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

• Thermal Capacity

The thermal capacity of a body is the quantity of heat required to raise the temperature of the whole of the body through a unit degree. It is measured in calorie per °C or joule per K.

If Q be the amount of heat needed to produce a change in temperature (Δt) of the substance, then thermal capacity of the substance is given by

$$S = \frac{Q}{\Delta t}$$

Dimensional formula of heat capacity is $[ML^2T^{-2}K^{-1}]$.

• Specific Heat Capacity

The specific heat capacity (also referred to as specific heat) of a substance is the amount of heat required to raise the temperature of

a unit mass of substance through 1 °C. It is measured in cal g⁻¹(°C)⁻¹ or J kg⁻¹ K⁻¹.

The specific heat capacity of a substance is given by

$$S = \frac{1}{m} \frac{Q}{\Delta t}$$

where m is mass of substance and Q is the heat required to change its temperature Δt .

• Calorimetry

Calorimetry is concerned with the measurement of heat, the basic apparatus for this purpose being called the *calorimeter*.

• The Basic Heat Formula

The heat Q required to raise the temperature of a mass m of a substance of specific heat capacity s through t degrees is given by

$$Q = m \times S \times t$$

i.e., Heat required = mass \times specific heat \times change in temperature

• Latent Heat

Latent heat of a substance is the amount of heat energy required to change the state of unit mass of the substance from solid to liquid or from liquid to gas/vapour without any change in temperature.

• Newton's Law of Cooling

Newton's law of cooling states that the rate of loss of heat of a body is directly proportional to the difference in temperature of the body and the surroundings, provided the difference in temperature is small, not more than 40 °C.

i.e.,
$$\frac{dT}{dt} = -K(T - T_s).$$

-ve sign implies that as time passes, temperature T decreases.

• Black Body Radiation

- (i) **Wein's Displacement Law** : This law states that as the temperature increases, the maximum value of the radiant energy emitted by the black body, move towards shorter wavelengths. Wein found that "The product of the peak wavelength (λ_m) and the Kelvin temperature (T) of the black body should remain constant."

$$\lambda_m \times T = b$$

Where b is constant known as Wein's constant. Its value is 2.898×10^{-3} mk.

- (ii) **Stefan's Law** : This law states that the thermal radiations energy emitted per second from the surface of a black body is directly proportional to its surface area A and to the fourth power of its absolute temperature T .

If H be the thermal radiation energy emitted per unit time, then

$$\begin{aligned} H &\propto AT^4 \\ &= \sigma \times AT^4 \end{aligned}$$

Where σ is a universal constant known as Stephan-Boltzman constant. Its value in S.I. unit is 5.67×10^{-8} W m⁻²k⁻⁴.

- (iii) **The Solar Constant** : The average energy emitted from the surface of the sun, absorbed per unit area, per minute by the earth is constant which is called as solar constant which is represented by S whose value is 8.135 Jm⁻² min⁻¹.

■ TEXTBOOK QUESTIONS SOLVED ■

- 11.1. The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.

Sol. The relation between kelvin scale and Celsius scale is

$$T_K - 273.15 = T_C \Rightarrow T_C = T_K - 273.15$$

For neon, $T_K = 24.57$ K

$\therefore T_C = 24.57 - 273.15 = -248.58$ °C

For CO₂, $T_K = 216.55$ K

$\therefore T_C = 216.55 - 273.15 = -56.60$ °C

Also, the relation between Kelvin scale and Fahrenheit scale is

$$\frac{T_K - 273.15}{100} = \frac{T_F - 32}{180}$$

$\therefore T_F = \frac{9}{5} (T_K - 273.15) + 32$

Now, for neon, $T_K = 24.57$ K

$\therefore T_F = \frac{9}{5} [24.57 - 273.15] + 32 = -415.44$ °F

For CO₂, $T_K = 216.55$ K

$\therefore T_F = \frac{9}{5} [216.55 - 273.15] + 32 = -69.88$ °F

- 11.2. Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between T_A and T_B ?
- Sol. As we know, triple point of water on absolute scale = 273.16 K,
Size of one degree of kelvin scale on absolute scale A

$$= \frac{273.16}{200}$$

Value of temperature T_A on absolute scale A

$$= \frac{273.16}{200} T_A$$

Value of temperature T_B on absolute scale B

$$= \frac{273.16}{350} T_B$$

Since T_A and T_B represent the same temperature,

$$\therefore \frac{273.16}{200} T_A = \frac{273.16}{350} T_B \quad \text{or} \quad T_A = \frac{200}{350} T_B = \frac{4}{7} T_B.$$

- 11.3. The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law:

$$R = R_0 [1 + \alpha (T - T_0)].$$

The resistances is 101.6 Ω at the triple-point of water 273.16 K, and 165.5 Ω at the normal melting point of lead (600.5K). What is the temperature when the resistance is 123.4 Ω ?

Sol. Here, $R_0 = 101.6 \Omega$; $T_0 = 273.16 \text{ K}$

Case (i) $R_1 = 165.5 \Omega$; $T_1 = 600.5 \text{ K}$

Case (ii) $R_2 = 123.4 \Omega$; $T_2 = ?$

Using the relation

$$R = R_0 [1 + \alpha (T - T_0)]$$

$$\text{Case (i)} \quad 165.5 = 101.6 [1 + \alpha (600.5 - 273.16)]$$

$$\alpha = \frac{165.5 - 101.6}{101.6 \times (600.5 - 273.16)} = \frac{63.9}{101.6 \times 327.34}$$

$$\text{Case (ii)} \quad 123.4 = 101.6 [1 + \alpha (T_2 - 273.16)]$$

$$\text{or} \quad 123.4 = 101.6 \left[1 + \frac{63.9}{101.6 \times 327.34} (T_2 - 273.16) \right]$$

$$= 101.6 + \frac{63.9}{327.34} (T_2 - 273.16)$$

$$\begin{aligned} \text{or} \quad T_2 &= \frac{(123.4 - 101.6) \times 327.34}{63.9} + 273.16 \\ &= 111.67 + 273.16 = 384.83 \text{ K} \end{aligned}$$

11.4. Answer the following:

- The triple-point of water is a standard fixed point in modern thermometry. Why? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale)?
- There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0°C and 100°C respectively. On the absolute scale, one of the fixed points is the triple-point of water, which on the Kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (Kelvin) Scale?
- The absolute temperature (Kelvin scale) T is related to the temperature t_c on the Celsius scale by

$$t_c = T - 273.15$$

Why do we have 273.15 in this relation, and not 273.16?

- What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale?

- Sol.
- Triple point of water has a unique value i.e., 273.16 K. The melting point and boiling points of ice and water respectively do not have unique values and change with the change in pressure.
 - On Kelvin's absolute scale, there is only one fixed point, namely, the triple-point of water and there is no other fixed point.
 - On Celsius scale 0°C corresponds to the melting point of ice at normal pressure and the value of absolute temperature is 273.15 K. The temperature 273.16 K corresponds to the triple point of water.

- The Fahrenheit scale and Absolute scale are related as

$$\frac{T_F - 32}{180} = \frac{T_K - 273}{100} \quad \dots(i)$$

For another set of temperature T'_F and T'_K ,

$$\frac{T'_F - 32}{180} = \frac{T'_K - 273}{100} \quad \dots(ii)$$

Subtracting (i) from (ii)

$$\frac{T'_F - T_F}{180} = \frac{T'_K - T_K}{100}$$

$$\therefore T'_F - T_F = \frac{180}{100} (T'_K - T_K)$$

$$\text{For } T'_K - T_K = 1 \text{ K,}$$

$$T'_F - T_F = \frac{180}{100}$$

\therefore For a temperature of triple point i.e., 273.16 K, the temperature on the new scale is

$$= 273.16 \times \frac{180}{100} = 491.688.$$

- 11.5. Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made:

Temperature	Pressure thermometer A	Pressure thermometer B
Triple-point of water	$1.250 \times 10^5 \text{ Pa}$	$0.200 \times 10^5 \text{ Pa}$
Normal melting point of sulphur	$1.797 \times 10^5 \text{ Pa}$	$0.287 \times 10^5 \text{ Pa}$

- (a) What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B ?
 (b) What do you think is the reason behind the slight difference in answers of thermometers A and B ? (The thermometers are not faulty). What further procedure is needed in the experiment to reduce the discrepancy between the two readings ?

Sol. (a) Let T be the melting point of sulphur.

For thermometer A

$$P_{tr} = 1.250 \times 10^5 \text{ Pa; } P = 1.797 \times 10^5 \text{ Pa}$$

$$\text{Now, } T_A = T_{tr} \times \frac{P}{P_{tr}}$$

$$T_A = 273.16 \times \frac{1.797 \times 10^5}{1.250 \times 10^5} \text{ K} = 392.69 \text{ K}$$

For thermometer B

$$P_{tr} = 0.200 \times 10^5 \text{ Pa; } P = 0.287 \times 10^5 \text{ Pa}$$

$$T_B = T_{tr} \times \frac{P}{P_{tr}} = \frac{273.16 \times 0.287 \times 10^5}{0.200 \times 10^5} \text{ K}$$

$$= 391.98 \text{ K}$$

- (b) The value of the melting point of sulphur found from the two thermometers differ slightly due to the reason that in practice, the gases do not behave strictly as perfect gases *i.e.*, gases are not perfectly ideal.

To reduce the discrepancy, readings should be taken for lower and lower pressures and the plot between temperature measured versus absolute pressure of the gas at triple point should be extrapolated to obtain the temperature in the limit pressure tends to zero (if $P \rightarrow 0$), when the gases approach ideal gas behaviour.

- 11.6.** A steel tape 1m long is correctly calibrated for a temperature of 27.0 °C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0 °C. What is the actual length of the steel rod on that day ? What is the length of the same steel rod on a day when the temperature is 27.0 °C ? Coefficient of linear expansion of steel = $1.20 \times 10^{-5} \text{K}^{-1}$.

Sol. On a day when the temperature is 27 °C, the length of 1 cm division on the steel tape is exactly 1 cm, because the tape has been calibrated for 27 °C.

When the temperature rises to 45 °C (that is, $\Delta T = 45 - 27 = 18$ °C), the increase in the length of 1 cm division is

$$\Delta l = \alpha \Delta T = (1.2 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}) \times 1 \text{ cm} \times 18 \text{ } ^\circ\text{C} = 0.000216 \text{ cm}$$

Therefore, the length of 1 cm division on the tape becomes 1.000216 cm at 45 °C. As the length of the steel rod is read to be 63.0 cm on the steel tape at 45 °C, the actual length of the rod at 45 °C is

$$63.0 \times 1.000216 \text{ cm} = 63.0136 \text{ cm}$$

The length of the same rod at 27 °C is 63.0 cm, because 1 cm mark on the steel tape is exactly 1 cm at 27 °C.

- 11.7.** A large steel wheel is to be fitted on to a shaft of the same material. At 27 °C, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft ? Assume coefficient of linear expansion of the steel to be constant over the required temperature range:

$$\alpha_{\text{steel}} = 1.20 \times 10^{-5} \text{ K}^{-1}$$

Sol. Here at temperature $T_1 = 27$ °C, diameter of shaft $D_1 = 8.70$ cm
Let at temperature T_2 , the diameter of shaft changes to $D_2 = 8.69$ cm and for steel

$$\alpha = 1.20 \times 10^{-5} \text{ K}^{-1} = 1.20 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$$

∴ Change in diameter

$$\Delta D = D_2 - D_1 = D_1 \times \alpha \times (T_2 - T_1)$$

$$\therefore 8.69 - 8.70 = 8.70 \times 1.20 \times 10^{-5} \times (T_2 - 27)$$

$$\Rightarrow T_2 = 27 - \frac{0.01}{8.70 \times 1.20 \times 10^{-5}} = 27 - 95.8$$

$$= -68.8 \text{ }^\circ\text{C} \text{ or } -69 \text{ }^\circ\text{C}.$$

- 11.8.** A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0 °C. What is the change in the diameter of the hole when the sheet is heated to 227 °C ? Coefficient of linear expansion of copper = $1.70 \times 10^{-5} \text{ K}^{-1}$.

Sol. In this problem superficial expansion of copper sheet will be involved on heating.

Here, area of hole at 27° C, $A_1 = \frac{\pi D_1^2}{4} = \frac{\pi}{4} \times (4.24)^2 \text{ cm}^2$

If D_2 cm is the diameter of the hole at 227 °C, then area of the hole at 227 °C,

$$A_2 = \frac{\pi D_2^2}{4} \text{ cm}^2$$

Coefficient of superficial expansion of copper is,

$$\beta = 2 \alpha = 2 \times 1.70 \times 10^{-5} = 3.4 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$$

Increase in area = $A_2 - A_1 = \beta A_1 \Delta T$

or $A_2 = A_1 + \beta A_1 \Delta T = A_1 (1 + \beta \Delta T)$

$$\frac{\pi D_2^2}{4} = \frac{\pi}{4} (4.24)^2 [1 + 3.4 \times 10^{-5} (228 - 27)]$$

$$\Rightarrow D_2^2 = (4.24)^2 \times 1.0068$$

or $D_2 = 4.2544 \text{ cm}$

Change in diameter

$$= D_2 - D_1 = 4.2544 - 4.24 = 0.0144 \text{ cm}.$$

- 11.9.** A brass wire 1.8 m long at 27 °C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of -39 °C, what is the tension developed in the wire, if its diameter is 2.0 mm ? Co-efficient of linear expansion of brass = $2.0 \times 10^{-5} \text{ K}^{-1}$; Young's modulus of brass = $0.91 \times 10^{11} \text{ Pa}$.

Sol. Here,

$$l = 1.8 \text{ m},$$

$$\Delta t = (-39 - 27) \text{ }^\circ\text{C} = -66 \text{ }^\circ\text{C}$$

$$\alpha = 2.0 \times 10^{-5} \text{ K}^{-1}$$

$$Y = 0.91 \times 10^{11} \text{ Pa}$$

$$A = \frac{\pi D^2}{4} = \frac{22}{7} \times \frac{1}{4} (2 \times 10^{-3})^2 \text{ m}^2$$

Now, $Y = \frac{Fl}{A\Delta l} \Rightarrow \Delta l = \frac{Fl}{AY}$ or $l\alpha\Delta t = \frac{Fl}{AY}$
or $F = -YA\alpha\Delta t$

or $F = -0.91 \times 10^{11} \times \frac{22}{7} \times \frac{1}{4} (2 \times 10^{-3})^2 \times 2.0$
 $\times 10^{-5} \times 66 \text{ N}$
 $= -3.77 \times 10^2 \text{ N.}$

- 11.10.** A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250 °C, if the original lengths are at 40.0 °C? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand (Co-efficient of linear expansion of brass = $2.0 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, steel = $1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$).

Sol. Here, $l_{\text{brass}} = l_{\text{steel}} = 50 \text{ cm}$, $d_{\text{brass}} = d_{\text{steel}} = 3 \text{ mm} = 0.3 \text{ cm}$,

$$\Delta l_{\text{brass}} = ?, \Delta l_{\text{steel}} = ?$$

$$\Delta T = 250 - 40 = 210 \text{ }^\circ\text{C.}$$

$$\alpha_{\text{brass}} = 2 \times 10^{-5} \text{ }^\circ\text{C}^{-1} \text{ and } \alpha_{\text{steel}} = 1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$$

Now $\Delta l_{\text{brass}} = \alpha_{\text{brass}} \times l_{\text{brass}} \times \Delta T$
 $= 2 \times 10^{-5} \times 50 \times 210 = 0.21 \text{ cm}$

Now $\Delta l_{\text{steel}} = \alpha_{\text{steel}} \times l_{\text{steel}} \times \Delta T$
 $= 1.2 \times 10^{-5} \times 50 \times 210 = 0.126 \text{ cm} \approx 0.13 \text{ cm}$

\therefore Total change in length,

$$\Delta l = \Delta l_{\text{brass}} + \Delta l_{\text{steel}} = 0.21 + 0.13 = 0.34 \text{ cm}$$

Since the rod is not clamped at its ends, no thermal stress is developed at the junction.

- 11.11.** The coefficient of volume expansion of glycerine is $49 \times 10^{-5} \text{ K}^{-1}$. What is the fractional change in its density for a 30 °C rise in temperature?

Sol. Here, $\gamma = 49 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, $\Delta T = 30 \text{ }^\circ\text{C}$

As $V = V + \Delta V = V(1 + \gamma\Delta T)$

$\therefore V' = V(1 + 49 \times 10^{-5} \times 30) = 1.0147 V$

Since $\rho = \frac{m}{V}$, $\rho' = \frac{m}{V'} = \frac{m}{1.0147V} = 0.9855 \rho$

Fractional change in density = $\frac{\rho - \rho'}{\rho} = \frac{\rho - 0.9855\rho}{\rho} = 0.0145.$

11.12. A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings? Specific heat of aluminium = $0.91 \text{ J g}^{-1} \text{ K}^{-1}$.

Sol. Power = 10 kW = 10^4 W
 Mass, $m = 8.0 \text{ kg} = 8 \times 10^3 \text{ g}$

Rise in temperature,

$$\Delta T = ?$$

$$\text{Time, } t = 2.5 \text{ min} = 2.5 \times 60 = 150 \text{ s}$$

$$\text{Specific heat, } C = 0.91 \text{ Jg}^{-1} \text{ K}^{-1}$$

$$\begin{aligned} \text{Total energy} &= \text{Power} \times \text{Time} = 10^4 \times 150 \text{ J} \\ &= 15 \times 10^5 \text{ J} \end{aligned}$$

As 50% of energy is lost,

\therefore Thermal energy available,

$$\Delta Q = \frac{1}{2} \times 15 \times 10^5 = 7.5 \times 10^5 \text{ J}$$

Since $\Delta Q = mc\Delta T$

$$\therefore \Delta T = \frac{\Delta Q}{mc} = \frac{7.5 \times 10^5}{8 \times 10^3 \times 0.91} = 103^\circ \text{ C.}$$

11.13. A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt? Specific heat of copper is $0.39 \text{ Jg}^{-1} \text{ }^\circ\text{C}^{-1}$. Heat of fusion of water = 335 Jg^{-1} .

Sol. Here, mass of copper block, $m = 2.5 \text{ kg} = 2500 \text{ g}$

Fall in temperature, $\Delta T = 500 - 0 = 500^\circ\text{C}$

Specific heat of copper, $c = 0.39 \text{ J g}^{-1} \text{ }^\circ\text{C}^{-1}$

Latent heat of fusion, $L = 335 \text{ J g}^{-1}$

Let the mass of ice melted be m'

As, Heat gained by ice = Heat lost by copper

$$\therefore m' L = mc \Delta T$$

$$m' = \frac{mc \Delta T}{L}$$

$$m' = \frac{2500 \times 0.39 \times 500}{335} = 1500 \text{ g} = 1.5 \text{ kg}$$

- 11.14.** In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm³ of water at 27 °C. The final temperature is 40° C. Compute the specific heat of the metal. If heat losses to the surroundings are not negligible, is your answer greater or smaller than the actual value for specific heat of the metal?

Sol. Mass of metal block, $m = 0.20 \text{ kg} = 200\text{g}$

Fall in the temperature of metal block,

$$\Delta T = (150 - 40) \text{ }^\circ\text{C} = 110 \text{ }^\circ\text{C}$$

If C be the specific heat of metal, then heat lost by the metal block = $200 \times C \times 110 \text{ cal}$

$$\text{Volume of water} = 150 \text{ cm}^3$$

$$\text{mass of water} = 150 \text{ g}$$

Increase in temperature of water = $(40 - 27) \text{ }^\circ\text{C} = 13^\circ\text{C}$

$$\text{Heat gained by water} = 150 \times 13 \text{ cal}$$

Water equivalent of calorimeter, $w = 0.025 \text{ kg} = 25\text{g}$

Heat gained by calorimeter,

$$= w \times \text{increase in temperature of calorimeter}$$

$$= 25 \times 13 \text{ cal}$$

Heat lost by metal block

$$= \text{Heat gained by water} + \text{Heat gained by calorimeter}$$

$$200 \times C \times 110 = (150 + 25) 13$$

$$C = \frac{175 \times 13}{200 \times 110} = 0.1 \text{ Cal g}^{-1} \text{ }^\circ\text{C}^{-1} = 0.43 \text{ Jg}^{-1} \text{ K}^{-1}$$

If heat is lost to the surroundings, C will be smaller than the actual value.

- 11.15.** Given below are observations on molar specific heats at room temperature of some common gases.

Gas	Molar specific heat (C_v) ($\text{cal mol}^{-1} \text{ K}^{-1}$)
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon monoxide	5.01
Chlorine	6.17

The measured molar specific heats of these gases are markedly different from those for monoatomic gases. Typically, molar specific heat of a

monoatomic gas is 2.92 cal/mol K. Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine ?

Sol. The gases which are listed in the above table are diatomic gases and not monoatomic gases. For diatomic gases, molar specific

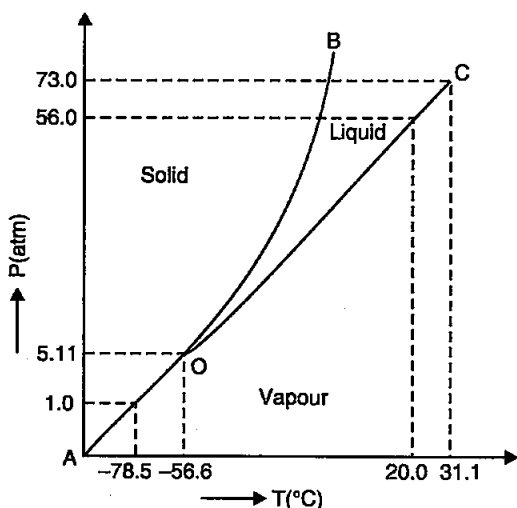
$$\text{heat} = \frac{5}{2}R = \frac{5}{2} \times 1.98 = 4.95, \text{ which agrees fairly well with all}$$

observations listed in the table except for chlorine. A monoatomic gas molecule has only the translational motion. A diatomic gas molecule, apart from translational motion, the vibrational as well as rotational motion is also possible. Therefore, to raise the temperature of 1 mole of a diatomic gas through 1°C, heat is to be supplied to increase not only translational energy but also rotational and vibrational energies. Hence, molar specific heat of a diatomic gas is greater than that for monoatomic gas. The higher value of molar specific heat of chlorine as compared to hydrogen, nitrogen, oxygen etc. shows that for chlorine molecule, at room temperature vibrational motion also occurs along with translational and rotational motions, whereas other diatomic molecules at room temperature usually have rotational motion apart from their translational motion. This is the reason that chlorine has somewhat larger value of molar specific heat.

- 11.16. (a) At what temperature and pressure can the solid, liquid and vapour phases of CO_2 co-exist in equilibrium ?
 (b) What is the effect of decrease of pressure on the fusion and boiling point of CO_2 ?
 (c) What are the critical temperature and pressure for CO_2 ? What is their significance ?
 (d) Is CO_2 solid, liquid or gas at (a) -70°C under 1 atm (b) -60°C under 10 atm (c) 15°C under 56 atm?

- Sol.** (a) At the triple point, temperature = -56.6°C and pressure = 5.11 atm.
 (b) Both the boiling point and freezing point of CO_2 decrease if pressure decreases.
 (c) The critical temperature and pressure of CO_2 are 31.1°C and 73.0 atm respectively. Above this temperature, CO_2 will not liquefy even if compressed to high pressures.
 (d) (i) The point (-70°C , 1.0 atm) lies in the vapour region. Hence, CO_2 is vapour at this point.
 (ii) The point (-60°C , 10 atm) lies in the solid region. Hence, CO_2 is solid at this point.

- (iii) The point (15 °C, 56 atm) lies in the liquid region. Hence, CO₂ is liquid at this point.



- 11.17.** Answer the following questions based on the $P - T$ phase diagram of CO₂ (Fig. of question 17 given above)
- CO₂ at 1 atm pressure and temperature -60°C is compressed isothermally. Does it go through a liquid phase?
 - What happens when CO₂ at 4 atm pressure is cooled from room temperature at constant pressure?
 - Describe qualitatively the changes in a given mass of solid CO₂ at 10 atm pressure and temperature -65°C as it is heated up at room temperature at constant pressure.
 - CO₂ is heated to a temperature 70°C and compressed isothermally. What changes in its properties do you expect to observe?

- Sol.**
- No, the CO₂ does not go through the liquid phase. The point (1.00 atm, -60°C) is to the left of the triple-point O and below the sublimation curve OA. Therefore, when CO₂ is compressed at this point at constant temperature, the point moves perpendicular to the temperature-axis and enters the solid phase region. Hence, the CO₂ vapour condenses to solid directly without going through the liquid phase.
 - CO₂ at 4.0 atm pressure and room temperature (say, 27°C) is in vapour phase. This point (4.0 atm, 27°C) lies below the vaporization curve OC and to the right of the triple point O. Therefore, when CO₂ is cooled at this point at constant pressure, the point moves perpendicular to the pressure-axis and enters the solid phase region. Hence, the CO₂ vapour

condenses directly to solid phase without going through the liquid phase.

- (c) When the solid CO_2 at -65°C is heated at 10 atm pressure, it is first converted into liquid. A further increase in its temperature brings it into the vapour phase. If a horizontal line at $P = 10$ atm is drawn parallel to the T-axis, then the points of intersection of line with the fusion and vaporization curve give the fusion and boiling points at 10 atm.
- (d) Above 31.1°C , the gas cannot be liquefied. Therefore, on being compressed isothermally at 70°C , there will be no transition to the liquid region. However, the gas will depart more and more from its perfect gas behaviour with the increase in pressure.

11.18. A child running a temperature of 101°F is given an antipyryn (i.e., a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98°F in 20 minutes, what is the average rate of extra evaporation caused by the drug? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about 580 cal g^{-1} .

Sol. Decrease of temperature, Δt

$$= 101^\circ\text{F} - 98^\circ\text{F} = 3^\circ\text{F} = 3 \times \frac{5}{9}^\circ\text{C} = 1.67^\circ\text{C}$$

specific heat of water = $1000\text{ cal kg}^{-1}^\circ\text{C}^{-1}$

latent heat of vaporisation, $L = 580 \times 10^3\text{ cal kg}^{-1}$

heat lost

$$= 30\text{ kg} \times 1000\text{ cal kg}^{-1}^\circ\text{C}^{-1} \times 1.67^\circ\text{C} = 50100\text{ cal}$$

If m' be the mass of water evaporated, then

$$m' = \frac{50100\text{ cal}}{580 \times 10^3\text{ cal kg}^{-1}} = 0.086\text{ kg}$$

This much water has taken 20 minutes to evaporate.

$$\text{So, rate of evaporation} = \frac{0.086\text{ kg}}{20\text{ min}} = \frac{86\text{ g}}{20\text{ min}} = 4.3\text{ g min}^{-1}.$$

11.19. A 'thermacole' icebox is a cheap and efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6 h. The outside temperature

is 45°C , and coefficient of thermal conductivity of thermacole is $0.01 \text{ Js}^{-1} \text{ m}^{-1} \text{ }^{\circ}\text{C}^{-1}$. [Heat of fusion of water = $335 \times 10^3 \text{ J kg}^{-1}$].

- Sol.** Each side of the cubical box (having 6 faces) is $30 \text{ cm} = 0.30 \text{ m}$. Therefore, the total surface area of the icebox exposed to outside air is $A = 6 \times (0.30 \text{ m})^2 = 0.54 \text{ m}^2$. The thickness of the icebox is $d = 5.0 \text{ cm} = 0.05 \text{ m}$, time of exposure $t = 6\text{h} = 6 \times 3600 \text{ s}$ and temperature difference $T_1 - T_2 = 45^{\circ}\text{C} - 0^{\circ}\text{C} = 45^{\circ}\text{C}$.

\therefore Total heat entering the icebox in 6 h is given by

$$\begin{aligned} Q &= \frac{KA(T_1 - T_2)t}{d} \\ &= \frac{0.01 \text{ Js}^{-1} \text{ m}^{-1} \text{ }^{\circ}\text{C}^{-1} \times 0.54 \text{ m}^2 \times 45^{\circ}\text{C} \times (6 \times 3600 \text{ s})}{0.05 \text{ m}} \\ &= 1.05 \times 10^5 \text{ J} \end{aligned}$$

Suppose a mass m of ice melts with this heat. Then $Q = mL$, where L is latent heat of fusion of water. Thus,

$$1.05 \times 10^5 \text{ J} = m (335 \times 10^3 \text{ J kg}^{-1})$$

$$\text{or } m = \frac{1.05 \times 10^5 \text{ J}}{335 \times 10^3 \text{ J kg}^{-1}} = 0.313 \text{ kg}$$

The initial mass of ice in the box is 4.0 kg . Therefore, the ice remaining in the box after 6 h is

$$= (4.0 - 0.313) \text{ kg} = 3.687 \text{ kg}.$$

- 11.20.** A brass boiler has a base area 0.15 m^2 and thickness 1.0 cm . It boils water at the rate of 6.0 kg/min when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = $109 \text{ Js}^{-1} \text{ m}^{-1} \text{ K}^{-1}$.

(Heat of vaporization of water = $2256 \times 10^3 \text{ J kg}^{-1}$)

- Sol.** Here, $K = 109 \text{ Js}^{-1} \text{ m}^{-1} \text{ K}^{-1}$
 $A = 0.15 \text{ m}^2$
 $d = 1.0 \text{ cm} = 10^{-2} \text{ m}$
 $T_2 = 100^{\circ}\text{C}$

Let T_1 = temperature of the part of the boiler in contact with the stove.

If Q be the amount of heat flowing per second through the base of the boiler, then

$$Q = \frac{KA(T_1 - T_2)}{d}$$

$$\text{or } Q = \frac{109 \times 0.15 \times (T_1 - 100)}{10^{-2}} = 1635 (T_1 - 100) \text{ Js}^{-1} \dots (i)$$

Also heat of vaporisation of water

$$L = 2256 \times 10^3 \text{ J kg}^{-1}$$

Rate of boiling of water in the boiler,

$$M = 6.0 \text{ kg min}^{-1} = \frac{6.0}{60} = 0.1 \text{ kg}^{-1} \text{ s.}$$

∴ Heat received by water per second, $Q = ML$

$$\Rightarrow Q = 0.1 \times 2256 \times 10^3 \text{ Js}^{-1} \quad \dots(ii)$$

∴ From eqn. (i) and (ii), we get

$$1635 (T_1 - 100) = 2256 \times 10^2$$

$$\text{or } T_1 - 100 = \frac{2256 \times 10^2}{1635} = 138$$

$$T_1 = 138 + 100 = 238^\circ\text{C.}$$

11.21. Explain why:

- a body with large reflectivity is a poor emitter.
- a brass tumbler feels much colder than a wooden tray on a chilly day.
- an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace.
- the earth without its atmosphere would be inhospitably cold.
- heat systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.

- Sol.**
- According to Kirchhoff's law of black body radiations, good emitters are good absorbers and bad emitters are bad absorbers. A body with large reflectivity is a poor absorber of heat and consequently, it is also a poor emitter.
 - Brass is a good conductor of heat, while wood is a bad conductor. When we touch the brass tumbler on a chilly day, heat starts flowing from our body to the tumbler and we feel it cold. However, when the wooden tray is touched, heat does not flow from our hands to the tray and we do not feel cold.
 - An optical pyrometer is based on the principle that the brightness of a glowing surface of a body depends upon its temperature. Therefore, if the temperature of the body is less than 600°C , the image formed by the optical pyrometer is not brilliant and we do not get the reliable result. It is for

this reason that the pyrometer gives a very low value for the temperature of red hot iron in the open.

- (d) The lower layers of earth's atmosphere reflect infrared radiations from earth back to the surface of earth. Thus the heat radiation received by the earth from the sun during the day are kept trapped by the atmosphere. If atmosphere of earth were not there, its surface would become too cold to live.
- (e) Steam at 100°C possesses more heat than the same mass of water at 100°C. One gram of steam at 100°C possesses 540 calories of heat more than that possessed by 1 gm of water at 100°C. That is why heating systems based on circulation of steam are more efficient than those based on circulation of hot water.

11.22. A body cools from 80 °C to 50°C in 5 minutes. Calculate the time it takes to cool from 60 °C to 30°C. The temperature of the surroundings is 20 °C.

Sol. According to Newton's law of cooling, the rate of cooling is proportional to the difference in temperature.

Here average of 80 °C and 50 °C = 65 °C

Temperature of surroundings = 20°C

∴ Difference = 65 – 20 = 45 °C

Under these conditions, the body cools 30°C in time 5 minutes

∴ $\frac{\text{Change in temp.}}{\text{Time}} = K \Delta T$ or $\frac{30}{5} = K \times 45^\circ$... (i)

The average of 60°C and 30°C is 45°C which is 25°C (45 – 20) above the room temperature and the body cools by 30°C (60 – 30) in a time t (say)

∴ $\frac{30}{t} = K \times 25$... (ii)

where K is same for this situation as for the original.

Dividing eqn. (i) by (ii), we get

$$\frac{30/5}{30/t} = \frac{K \times 45}{K \times 25}$$

or $\frac{t}{5} = \frac{9}{5}$

⇒ $t = 9 \text{ min.}$

