15 Waves

Lesson at a Glance

• Waves

Wave is a form of disturbance which travels through a material medium due to the repeated periodic motion of the particles of the medium about their mean positions without any actual transportation of matter. Waves are mainly of three types: (*a*) mechanical or elastic waves, (*b*) electromagnetic waves and (*c*) matter waves.

• Wavelength

The distance travelled by the disturbance during the time of one vibration by a medium particle is called the wavelength (λ). In case of a transverse wave the wavelength may also be defined as the distance between two successive crests or troughs. In case of a longitudinal wave, the wavelength (λ) is equal to distance from centre of one compression (or rarefraction) to another.

• Wave Velocity

Wave velocity is the time rate of propagation of wave motion in the given medium. It is different from particle velocity. Wave velocity depends upon the nature of medium.

Wave velocity (v) = frequency (ν) × wavelength (λ)

• Amplitude

The amplitude of a wave is the maximum displacement of the particles of the medium from their mean position.

• Frequency

The number of vibrations made by a particle in one second is called Frequency. It is represented by ν. Its unit is hertz (Hz) $\nu = \frac{1}{T}$

• Time Period

The time taken by a particle to complete one vibration is called time period.

$$T = \frac{1}{\nu}, \text{ it is expressed in seconds.}$$

- The velocity of transverse waves in a stretched string is given by

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension in the string and μ is the mass per unit length of the string. μ is also called linear mass density of the string. SI unit of μ is kg m^{-1}.

- The velocity of the longitudinal wave in an elastic medium is given by

$$v = \sqrt{\frac{E}{\rho}}$$

where E is the modulus of elasticity of the medium and ρ is the density of the medium.

In case of solids, E is Young's modulus of elasticity (Y), then

$$v = \sqrt{\frac{Y}{\rho}}$$

In case of fluids, E is replaced by the bulk modulus of elasticity (B), then

$$v = \sqrt{\frac{B}{\rho}}$$

• General Equation of Progressive Waves

"A progressive wave is one which travels in a given direction with constant amplitude, *i.e.*, without attenuation."

$$y\,(x,t) = A \sin (kx - \omega t + \phi)$$

• The Principle of Superposition of Wave

When any number of waves meet simultaneously at a point in a medium, the net displacement at a given time is the algebraic sum of the displacements due to each wave at that time.

$$\vec{y} = \vec{y}_1 + \vec{y}_2$$

- The frequency of fundamental mode of vibration (or first harmonic) is given by

$$\nu = \frac{v}{\lambda} = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

- In closed pipes, only odd harmonics are developed, *i.e.*, $\nu = \frac{(2n-1)v}{4l}$,

with fundamental frequency of $\left(\frac{v}{4l}\right)$.

- In open pipes, all harmonics with fundamental or first harmonic $\left(\frac{v}{2l}\right)$ are developed.

$$\nu = \frac{nv}{2l}, \text{ where } v \text{ is the velocity of sound.}$$

• Frequency of the Stretched String

In general, if the string vibrates in P loops, the frequency of the string under that mode is given by

$$\nu = \frac{P}{2L}\sqrt{\frac{T}{\mu}}$$

Based on this relation three laws of transverse vibrations of stretched strings arise. They are law of length, law of tension and law of mass.

• Beats

The phenomenon of regular rise and fall in the intensity of sound, when two waves of nearly equal frequencies travelling along the same line and in the same direction superimpose each other is called beats. One rise and one fall in the intensity of sound constitutes one beat and the number of beats per second is called beat frequency. It is given as:

$$\nu_b = (\nu_1 - \nu_2)$$

where ν_1 and ν_2 are the frequencies of the two interfering waves; ν_1 being greater than ν_2.

• Doppler Effect

According to Doppler's effect, whenever there is a relative motion between a source of sound and listener, the apparent frequencies of sound heard by the listener is different from the actual frequency of sound emitted by the source.

For sound the observed frequency ν' is given by

$$\nu' = \left(\frac{v+v_0}{v+v_s}\right).\nu$$

Here ν = true frequency of wave emitted by the source, v = speed of sound through the medium, v_0 the velocity of observer relative to the medium and v_s the velocity of source relative to the medium.

TEXTBOOK QUESTIONS SOLVED

15.1. *A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?*

Ans. Tension, $T = 200$ N;

Length, $l = 20.0$ m; Mass, $M = 2.50$ kg

Mass per unit length, $\mu = \frac{2.50}{20.0}\text{ kg m}^{-1} = 0.125\text{ kgm}^{-1}$

Wave velocity, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200\text{ N}}{0.125\text{ kg m}^{-1}}}$

or $v = \sqrt{1600}\text{ ms}^{-1} = 40\text{ ms}^{-1}$

Time, $t = \frac{l}{v} = \frac{20.0}{40}s = \frac{1}{2}s = 0.5$ s.

15.2. *A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 ms^{-1}? (g = 9.8 ms^{-2})*

Ans. Here, $h = 300$ m, $g = 9.8\text{ ms}^{-2}$ and velocity of sound, $v = 340\text{ ms}^{-1}$

Let t_1 be the time taken by the stone to reach at the surface of pond.

Then, using $s = ut + \frac{1}{2}at^2 \frac{1}{2}at^2 \Rightarrow h = 0 \times t + \frac{1}{2}gt_1^2$

$\therefore$ $t_1 = \sqrt{\frac{2\times 300}{9.8}} = 7.82$ s

Also, if t_2 is the time taken by the sound to reach at a height h, then

$$t_2 = \frac{h}{v} = \frac{300}{340} = 0.88\text{ s}$$

∴ Total time after which sound of splash is heard

$$= t_1 + t_2$$
$$= 7.82 + 0.88 = 8.7s.$$

15.3. *A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C = 340 ms¹.*

Ans. Here, l = 12.0 m, M = 2.10 kg, v = 343 ms^{-1}

Mass per unit length $= \frac{M}{l} = \frac{2.10}{12.0} = 0.175$ kg m^{-1}

As $$v = \sqrt{\frac{T}{m}}$$

∴ $$T = v^2 . m = (343)^2 \times 0.175 = 2.06 \times 10^4 \text{ N}.$$

15.4. *Use the formula* $v = \sqrt{\frac{\gamma P}{\rho}}$ *to explain why the speed of sound in air*

(a) is independent of pressure. (b) increases with temperature.
(c) increases with humidity.

Ans. We are given that $$v = \sqrt{\frac{\gamma P}{\rho}}$$

We know $PV = nRT$ (for n moles of ideal gas)

⇒ $$PV = \frac{m}{M} RT$$

where m is the total mass and M is the molecular mass of the gas.

∴ $$P = \frac{m}{M} \cdot \frac{RT}{M} = \frac{\rho RT}{M} \Rightarrow \frac{P}{\rho} = \frac{RT}{M}$$

(*a*) For a gas at constant temperature, $\frac{P}{\rho}$ = constant

∴ As P increase, ρ also increases and vice versa. This implies

that $v = \sqrt{\frac{\gamma P}{\rho}}$ = constant, *i.e.*, velocity is independent of pressure of the gas.

(*b*) Since $\frac{P}{\rho} = \frac{RT}{M}$, therfore, $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

Clearly $v \propto \sqrt{T}$ *i.e.*, speed of sound in air increases with increase in temperature.

(*c*) Increase in humidity decreases the effective density of air. Therefore the velocity $\left(v \propto \frac{1}{\sqrt{\rho}}\right)$ increases.

15.5. *You have learnt, that a travelling wave in one dimension is represented by a function $y = f(x, t)$, where x and t must appear in the combination $x - vt$ or $x + vt$ i.e., $y = f(x \pm vt)$. Is the converse true? That is, does every function of $(x - vt)$ or $(x + vt)$ represent a travelling wave? Examine, if the following functions for y can possibly represent a travelling wave?*

(*a*) $(x - vt)^2$ (*b*) $\log\left[\frac{(x+vt)}{x_0}\right]$ (*c*) $\frac{1}{x+vt}$

Ans. No, the converse is not true. The basic requirement for a wave function to represent a travelling wave is that for all values of x and t, wave function must have a finite value. Out of the given functions for y none satisfies this condition. Therefore, none can represent a travelling wave.

15.6. *A bat emits ultrasonic sound of frequency 1000 kHz in air. If this sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air = 340 ms^{-1} and in water = 1486 ms^{-1}.*

Ans. Here, $\nu = 1000 \times 10^3$ Hz $= 10^6$ Hz, $v_a = 340$ ms^{-1},

$v_w = 1486$ ms^{-1}

Wavelength of reflected sound, λ_a

$$= \frac{v_a}{\nu} = \frac{340}{10^6} \text{m} = 3.4 \times 10^{-4} \text{ m}$$

Wavelength of transmitted sound, λ_ω

$$= \frac{v_w}{\nu} = \frac{1486}{10^6} \text{m} = 1.486 \times 10^{-3} \text{m}$$

15.7. *A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in a tissue in which the speed of sound is 1.7 km s^{-1}? The operating frequency of the scanner is 4.2 MHz.*

Ans. Here speed of sound $\Rightarrow$ $v = 1.7$ km s^{-1} = 1700 ms^{-1}

and frequency $\nu = 4.2$ MHz $= 4.2 \times 10^6$ Hz

$\therefore$ Wavelength, $\lambda = \dfrac{v}{\nu} = \dfrac{1700}{4.2\times10^6} = 4.1 \times 10^{-4}$ m.

15.8. *A transverse harmonic wave on a string is described by*

$$y(x, t) = 3.0 \sin (36t + 0.018\,x + \pi/4)$$

where x and y are in cm and t in s. The positive direction of x is from left to right.

(a) Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation?

(b) What are its amplitude and frequency?

(c) What is the initial phase at the origin?

(d) What is the least distance between two successive crests in the wave?

Ans. The given equation is $y(x, t) = 3.0 \sin (36t + 0.018x + \dfrac{\pi}{4})$, where x and y are in cm and t in s.

(*a*) The equation is the equation of a travelling wave, travelling from right to left (*i.e.*, along *-ve* direction of x because it is an equation of the type

$$y(x, t) = A \sin (\omega t + kx + \phi)$$

Here, $A = 3.0$ cm, $\omega = 36$ rad s^{-1}, $k = 0.018$ cm and $\phi = \dfrac{\pi}{4}$.

$\therefore$ Speed of wave propagation,

$$v = \frac{\omega}{k} = \frac{36 \text{ rad s}^{-1}}{0.018 \text{ cm}^{-1}} = \frac{36 \text{ rad s}^{-1}}{0.018\times10^2 \text{ ms}^{-1}} = 20 \text{ ms}^{-1}$$

(*b*) Amplitude of wave, $A = 3.0$ cm $= 0.03$ m

Frequency of wave $\nu = \dfrac{\omega}{2\pi} = \dfrac{36}{2\pi} = 5.7$ Hz

(*c*) Initial phase at the origin, $\phi = \dfrac{\pi}{4}$

(*d*) Least distance between two successive crests in the wave

$$= \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.018} = 349 \text{ cm} = 3.5 \text{ m}$$

15.9. *For the wave described in Exercise 8, plot the displacement (y) versus (t) graphs for x = 0, 2 and 4 cm. What are the shape of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another : amplitude, frequency or phase?*

Ans. The transverse harmonic wave is

$$y\ (x,\ t)\ =\ 3.0\ \sin\left(36t + 0.018x + \frac{\pi}{4}\right)$$

For $x = 0$,

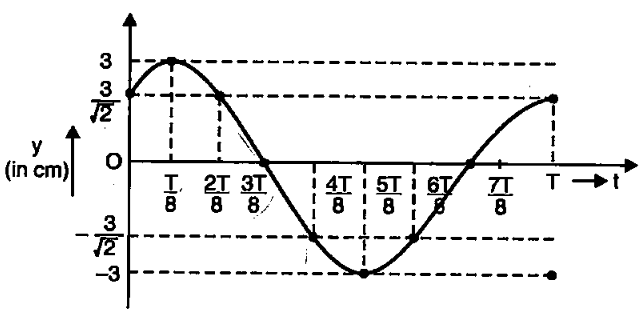
$$y\ (0,\ t) = 3\ \sin\left(36t + 0 + \frac{\pi}{4}\right) = 3\ \sin\left(36t + \frac{\pi}{4}\right) \quad ...(1)$$

Here $\omega = \frac{2\pi}{T} = 36 \Rightarrow T = \frac{2\pi}{36}$

To plot a (y) versus (t) graph, different values of y corresponding to the values of t may be tabulated as under (by making use of eqn. (1)).

t	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$	T
y	$\frac{3}{\sqrt{2}}$	3	$\frac{3}{\sqrt{2}}$	0	$\frac{-3}{\sqrt{2}}$	-3	$\frac{-3}{\sqrt{2}}$	0	$\frac{3}{\sqrt{2}}$

Using the values of t and y (as in the table), a graph is plotted as under.



The graph obtained is sinusoidal. Similar graphs are obtained for $x = 2$ cm and $x = 4$ cm. The oscillatory motion in the travelling wave only differs in respect of phase. Amplitude and frequency of oscillatory motion remains the same in all the cases.

15.10. *For the travelling harmonic wave*

$$y(x,\ t) = 2.0\ \cos 2\pi\ (10t - 0.0080x + 0.35)$$

where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of

(a) 4 m *(b) 0.5 m*

(a) $\lambda/2$ *(b) $3\lambda/4$.*

Ans. The given equation can be rewritten as under:

$$y(x, t) = 2.0 \cos [2\pi (10t - 0.0080\, x) + 2\pi \times 0.35]$$

or
$$y(x, t) = 2.0 \cos \left[2\pi \times 0.0080 \left(\frac{10t}{0.0080} - x\right) + 0.7\pi\right]$$

Comparing this equation with the standard equation of a travelling harmonic wave,

$$\frac{2\pi}{\lambda} = 2\pi \times 0.0080 \quad \text{or} \quad \lambda = \frac{1}{0.0080}\,\text{cm} = 125\,\text{cm}$$

The phase difference between oscillatory motion of two points separated by a distance Δx is given by

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

(*a*) When $\Delta x = 4\text{ m} = 400\text{ cm}$, then

$$\Delta\phi = \frac{2\pi}{125} \times 400 = 6.4\,\pi\text{ rad}$$

(*b*) When $\Delta x = 0.5\text{ m} = 50\text{ cm}$, then

$$\Delta\phi = \frac{2\pi}{125} \times 50 = 0.8\,\pi\text{ rad}$$

(*c*) When $\Delta x = \frac{\lambda}{2} = \frac{125}{2}$ cm, then

$$\Delta\phi = \frac{2\pi}{125} \times \frac{125}{2} = \pi\text{ rad}$$

(*d*) When $\Delta x = \frac{3\lambda}{4} = \frac{3 \times 125}{4}$ cm, then

$$\Delta\phi = \frac{2\pi}{125} \times \frac{3 \times 125}{4} = \frac{3\pi}{2}\text{ rad.}$$

15.11. *The transverse displacement of a string (clamped at its two ends) is given by*

$$y(x, t) = 0.06 \sin \frac{2\pi}{3} x \cos (120\,\pi t)$$

where x, y are in m and t in s. The length of the string is 1.5 m and its mass is 3 × 10^{-2} kg. Answer the following:

(i) *Does the function represent a travelling or a stationary wave?*

(ii) *Interpret the wave as a superimposition of two waves travelling in opposite directions. What are the wavelength, frequency and speed of propagation of each wave?*

(iii) *Determine the tension in the string.*

Ans. The given equation is

$$y(x, t) = 0.06 \sin \frac{2\pi}{3} x \cos 120\, \pi t \qquad ...(1)$$

(*i*) As the equation involves harmonic functions of x and t separately, it represents a stationary wave.

(*ii*) We know that when a wave pulse

$$y_1 = r \sin \frac{2\pi}{\lambda} (vt - x)$$

travelling along + direction of x-axis is superimposed by the reflected wave

$$y_2 = -r \sin \frac{2\pi}{\lambda} (vt + x)$$

travelling in opposite direction, a stationary wave

$$y = y_1 + y_2 = -2r \sin \frac{2\pi}{\lambda} x \cos \frac{2\pi}{\lambda} vt$$

is formed. ...(2)

Comparing eqns. (1) and (2), we find that

$$\frac{2\pi}{\lambda} = \frac{2\pi}{3} \Rightarrow \lambda = 3\text{m}$$

Also, $\frac{2\pi}{\lambda} v = 120\,\pi$ or $v = 60\lambda = 60 \times 3 = 180 \text{ ms}^{-1}$

Frequency, $\nu = \frac{v}{\lambda} = \frac{180}{3} = 60 \text{ Hz}$

Note that both the waves have same wavelength, same frequency and same speed.

(*iii*) Velocity of transverse waves is

$$v = \sqrt{\frac{T}{m}} \text{ or } v^2 = \frac{T}{m}$$

$$T = mv^2, \text{ where } m = \frac{3 \times 10^{-2}}{1.5} = 2 \times 10^{-2} \text{ kg/m}$$

$$\therefore \quad T = (180)^2 \times 2 \times 10^{-2} = 648 \text{ N.}$$

15.12. *(i) For the wave on a string described in Question 11, do all the points on the string oscillate with the same (a) frequency, (b) phase, (c) amplitude? Explain your answers. (ii) What is the amplitude of a point 0.375 m away from one end?*

Ans. (*i*) For the wave on the string described in questions we have seen that $l = 1.5$ m and $\lambda = 3$ m. So, it is clear that $\lambda \doteq \frac{\lambda}{2}$

and for a string clamped at both ends, it is possible only when both ends behave as nodes and there is only one antinode in between *i.e.,* whole string is vibrating in one segment only.

(*a*) Yes, all the sring particles, except nodes, vibrate with the same frequency ν = 60 Hz.

(*b*) As all string particles lie in one segment, all of them are in same phase.

(*c*) Amplitude varies from particle to particle. At antinode, amplitude = 2A = 0.06 m. It gradually falls on going towards nodes and at nodes, and at nodes, it is zero.

(*ii*) Amplitude at a point x = 0.375 m will be obtained by putting cos (120 πt) as + 1 in the wave equation.

$$\therefore \qquad A(x) = 0.06 \sin\left(\frac{2\pi}{3}\times 0.375\right)\times 1$$

$$= 0.06 \sin\frac{\pi}{4} = 0.042 \text{ m.}$$

15.13. *Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all*

(*a*) $y = 2\cos(3x)\sin(10t)$

(*b*) $y = 2\sqrt{x - vt}$

(*c*) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$

(*d*) $y = \cos x \sin t + \cos 2x \sin 2t$.

Ans. (*a*) It represents a stationary wave.

(*b*) It does not represent either a travelling wave or a stationary wave.

(*c*) It is a representation for the travelling wave.

(*d*) It is a superposition of two stationary wave.

15.14. *A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5 × 10^{-2} kg and its linear mass density is 4.0 × 10^{-2} kg m^{-1}. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?*

Ans. Here, n = 45 Hz, M = 3.5 × 10^{-2} kg

Mass per unit length = m = 4.0 × 10^{-2} kg m^{-1}

$$\therefore \qquad l = \frac{M}{m} = \frac{3.5\times10^{-2}}{4.0\times10^{-2}} = \frac{7}{8}$$

As $\frac{l}{2} = \lambda = \frac{7}{8}$ $\therefore$ $\lambda = \frac{7}{4}$ m = 1.75 m.

(*a*) The speed of the transverse wave, $v = \nu\lambda$

$= 45 \times 1.75 = 78.75$ m/s

(*b*) As $v = \sqrt{\frac{T}{m}}$

$\therefore$ $T = v^2 \times m = (78.75)^2 \times 4.0 \times 10^{-2} = 248.06$ N.

15.15. *A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a turning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effect may be neglected.*

Ans. Frequency of nth mode of vibration of the closed organ pipe of length

$$l_1 = (2n-1)\frac{v}{4l_1}$$

Frequency of $(n+1)^{th}$ mode of vibration of closed pipe of length

$$'l_2' = [2(n+1)-1]\frac{v}{4l_2} = (2n+1)\frac{v}{4l_2}$$

Both the modes are given to resonate with a frequency of 340 Hz.

$$\therefore \quad (2n-1)\frac{v}{4l_1} = (2n+1)\frac{v}{4l_2}$$

or $$\frac{2n-1}{2n+2} = \frac{l_1}{l_2} = \frac{25.5}{79.3} = \frac{1}{3}$$

[Approximation has been used because edge effect is being ignored. Moreover, we know that in the case of a closed organ pipe, the second resonance length is three times the first resonance length.]

On simplification, $n = 1$

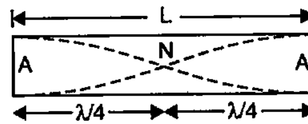
Now, $(2n-1)\frac{v}{4l_1} = 340$. Substituting values

$$(2\times1-1)\frac{v\times100}{4\times25.5} = 340 \quad \text{or} \quad v = 346.8 \text{ ms}^{-1}.$$

15.16. *A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 k Hz. What is the speed of sound in steel?*

Ans. Here, L = 100 cm = 1m, $\nu = 2.53\ k$ Hz $= 2.53 \times 10^3$ Hz

When the rod is clamped at the middle, then in the fundamental mode of vibration of the rod, a node is formed at the middle and antinode is formed at each end.



Therefore, as is clear from Fig.

$$L = \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$$

$$\lambda = 2\,L = 2\text{ m}$$

As $$v = \nu\lambda$$

$\therefore$ $$v = 2.53 \times 10^3 \times 2 = \mathbf{5.06 \times 10^3\ ms^{-1}}$$

15.17. *A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 ms^{-1}).*

Ans. Here length of pipe, l = 20 cm = 0.20 m, frequency ν = 430 Hz and speed of sound in air v = 340 ms^{-1}

For closed end pipe, $\nu = \dfrac{(2n-1)\,v}{4l}$, where n = 1, 2, 3..........

$\therefore$ $$(2n-1) = \frac{4\nu l}{v} = \frac{4 \times 430 \times 0.20}{340} = 1.02$$

$\Rightarrow$ $$2n = 1.02 + 1 = 2.02 \Rightarrow n = \frac{0.20}{2} = 1.01$$

Hence, resonance can occur only for first (or fundamental) mode of vibration.

As for an open pipe $\nu = \dfrac{nv}{2l}$, where n = 1, 2, 3...........

$\therefore$ $$n = \frac{2l\nu}{v} = \frac{2 \times 430 \times 0.20}{340} = 0.51.$$

As $n < 1$, hence, in this case resonance position cannot be obtained.

15.18. *Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3Hz. If the original frequency of A is 324 Hz, what is the frequency of B?*

Ans. Let v_1 and v_2 be the frequencies of strings A and B respectively.

Then, $$\nu_1 = 324\text{ Hz}, \quad \nu_2 = ?$$

Number of beats, $b = 6$

$\therefore \quad \nu_2 = \nu_1 \pm b = 324 \pm 6$

i.e., $\nu_2 = 330$ Hz or 318 Hz

Since the frequency is directly proportional to square root of tension, on decreasing the tension in the string *A*, its frequency ν_1 will be reduced *i.e.,* number of beats will increase if $\nu_2 = 330$ Hz. This is not so because number of beats become 3.

Therefore, it is concluded that the frequency $\nu_2 = 318$ Hz. because on reducing the tension in the string *A*, its frequency may be reduced to 321 Hz, thereby giving 3 beats with $\nu_2 = 318$ Hz.

15.19. *Explain why (or how):*

(*a*) *in a sound wave, a displacement node is a pressure antinode and vice versa.*

(*b*) *bats can ascertain distances, directions, nature and sizes of the obstacles without any "eyes".*

(*c*) *a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes.*

(*d*) *solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and*

(*e*) *the shape of a pulse gets distorted during propagation in a dispersive medium.*

Ans. (*a*) In a sound wave, a decrease in displacement *i.e.,* displacement node causes an increase in the pressure there *i.e.,* a pressure antinode is formed. Also, an increase in displacement is due to the decrease in pressure.

(*b*) Bats emit ultrasonic waves of high frequency from their mouths. These waves after being reflected back from the obstacles on their path are observed by the bats. These waves give them an idea of distance, direction, nature and size of the obstacles.

(*c*) The quality of a violin note is different from the quality of sitar. Therefore, they emit different harmonics which can be observed by human ear and used to differentiate between the two notes.

(*d*) This is due to the fact that gases have only the bulk modulus of elasticity whereas solids have both, the shear modulus as well as the bulk modulus of elasticity.

(*e*) A pulse of sound consists of a combination of waves of different wavelength. In a dispersive medium, these waves travel with different velocities giving rise to the distortion in the wave.

15.20. *A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of 10 ms^{-1}. (b) recedes from the platform with a speed of 10 ms^{-1}?(ii) What is the speed of sound in each case? The speed of sound in still air can be taken as 340 ms^{-1}.*

Ans. Frequency of whistle, ν = 400 Hz; speed of sound, v = 340 ms^{-1} speed of train, v_s = 10 ms^{-1}

(*i*) (*a*) When the train approaches the platform (*i.e.*, the observer at rest),

$$\nu' = \frac{v}{v-v_s}\times\nu = \frac{340}{340-10}\times 400 = 412 \text{ Hz}.$$

(*b*) When the train recedes from the platform (*i.e.*, from the observer at rest),

$$\nu' = \frac{v}{v+v_s}\times\nu = \frac{340}{340+10}\times 400 = 389 \text{ Hz}.$$

(*ii*) The speed of sound in each case does not change.

∴ It is 340 ms^{-1} in each case.

15.21. *A train, standing in a station-yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with a speed of 10ms^{-1}. What are the frequency, wavelength, and speed of sound for an observer standing on the station's platform? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of 10 ms^{-1}? The speed of sound in still air can be taken as 340 ms^{-1}?*

Ans. Here actual frequency of whistle of train ν = 400 Hz, speed of sound in still air v = 340 ms^{-1}.

As wind is blowing in the direction from the yard to the station with a speed of v_m = 10 ms^{-1}

∴ For an observer standing on the platform, the effective speed of sound $v' = v + v_m$ = 340 + 10 = 350 ms^{-1}

As there is no relative motion between the sound source (rail engine) and the observer, the frequency of sound for the observer, ν = 400 Hz

∴ Wavelength of sound heard by the observer

$$\lambda' = \frac{v'}{\nu} = \frac{350}{400} = 0.875 \text{ m}.$$

The situation is not identical to the case when the air is still and observer runs towards the yard at a speed of

$v_0 = 10\ ms^{-1}$. In this situation as medium is at rest. Hence $v' = v = 340\ ms^{-1}$.

$$\nu' = \frac{v+v_0}{v}\nu = \frac{340+10}{340} \times 400 = 412\ Hz$$

and $$\lambda' = \lambda = \frac{v}{\nu} = \frac{340}{400} = 0.85\ m$$

15.22. *A travelling harmonic wave on a string is described by $y(x, t) = 7.5 \sin (0.0050x + 12t + \pi/4)$*

(a) what are the displacement and velocity of oscillation of a point at $x = 1$ cm, and $t = 1$s? Is this velocity equal to the velocity of wave propagation?

(b) Locate the points of the string which have the same transverse displacement and velocity as the $x = 1$cm point at $t = 2$s, 5s and 11s.

Ans. (*a*) The travelling harmonic wave is

$$y(x, t) = 7.5 \sin (0.0050x + 12t + \pi/4)$$

At $x = 1$ cm and $t = 1$ sec,

$$y(1, 1) = 7.5 \sin (0.005 \times 1 + 12 \times 1\ \pi/4)$$
$$= 7.5 \sin (12.005 + \pi/4) \quad ...(i)$$

Now, $$\theta = (12.005 + \pi/4) \text{ radian}$$
$$= \frac{180}{\pi}(12.005 + \pi/4) \text{ degree}$$
$$= \frac{12.005 \times 180}{\frac{22}{7}} + 45 = 732.55°.$$

$\therefore$ From (*i*), $$y(1, 1) = 7.5 \sin (732.55°) = 7.5 \sin (720 + 12.55°)$$
$$= 7.5 \sin 12.55° = 7.5 \times 0.2173 = 1.63 \text{ cm}$$

- Velocity of oscillation, $$v = \frac{dy}{dt}(1, 1)$$
$$= \frac{d}{dt}\left[7.5 \sin\left(0.005x + 12 + \frac{\pi}{4}\right)\right]$$
$$= 7.5 \times 12 \cos\left[0.005x + 12t + \frac{\pi}{4}\right]$$

At $x = 1$ cm, $t = 1$sec.

$$v = 7.5 \times 12 \cos (0.005 + 12 + \pi/4)$$
$$= 90 \text{ ccs } (732.35°)$$
$$= 90 \cos (720 + 12.55)$$
$$v = 90 \cos (12.55°) = 90 \times 0.9765 = 87.89 \text{ cm/s}.$$

Comparing the given eqn. with the standard form

$$y\ (x,\ t) = t\ \sin\left[\frac{\pi}{4}(vt + x) + \phi_0\right]$$

We get $r = 7.5$ cm, $\frac{2\pi v}{\lambda} = 12$ or $2\pi\nu = 12$

$$\nu = \frac{6}{\pi}$$

$$\frac{2\pi}{\lambda} = 0.005.$$

$$\therefore \quad \lambda = \frac{2\pi}{0.005} = \frac{2 \times 3.14}{0.005} = 1256 \text{ cm} = 12.56 \text{ m}$$

Velocity of wave propagation, $v = \nu\lambda = \frac{6}{\pi} \times 12.56 =$ **24 m/s.**

We find that velocity at $x = 1$ cm, $t = 1$ sec is not equal to velocity of wave propagation.

(*b*) Now, all points which are at a distance of $\pm\lambda$, $\pm 2\lambda$, $\pm 3\lambda$ from $x = 1$ cm will have same transverse displacement and velocity. As $\lambda = 12.56$ m, therefore, all points at distances ± 12.6 m, ± 25.2 m, ± 37.8 m.... from $x = 1$ cm will have same displacement and velocity, as at $x = 1$ point $t = 2$ s, 5 *s* and 11 s.

15.23. *A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium. (a) Does the pulse have a definite (i) frequency, (ii) wavelength, (iii) speed of propagation? (b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz?*

Ans. (*a*) In a non dispersive medium, the wave propagates with definite speed but its wavelength of frequency is not definite.

(*b*) No, the frequency of the note is not $\frac{1}{20}$ or 0.50 Hz. 0.005 Hz is only the frequency of repetition of the pip of the whistle.

15.24. *One end of a long string of linear mass density 8.0×10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency* 256 Hz. *The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At $t = 0$, the left end (fork end) of the string $x = 0$ has zero transverse displacement ($y = 0$) and is moving along positive y-direction. The amplitude of the* *wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.*

Ans. Here, mass per unit length, μ = linear mass density

$= 8 \times 10^{-3}$ kg m^{-1};

Tension in the string, $T = 90$ kg $= 90 \times 9.8$ N $= 882$ N;

Frequency, $\nu = 256$ Hz

and amplitude, $A = 5.0$ cm $= 0.05$ m

As the wave propagating along the string is a transverse travelling wave, the velocity of the wave,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{882}{8 \times 10^{-3}}}\ \text{ms}^{-1} = 3.32 \times 10^2\ \text{ms}^{-1}$$

Now, $\omega = 2\pi\nu = 2 \times 3.142 \times 256 = 1.61 \times 10^3$ rad s^{-1}

Also, $v = \nu\lambda$ or $\lambda = \frac{v}{\nu} = \frac{3.32 \times 10^2}{256}$ m

Propagation constant, $k = \frac{2\pi}{\lambda} = \frac{2 \times 3.142 \times 256}{3.32 \times 10^2} = 4.84\ m^{-1}$

∴ The equation of the wave is,

$$v(x, t) = A \sin(\omega t - kx)$$
$$= 0.05 \sin(1.61 \times 10^3 t - 4.84 x)$$

Here, x, y are in metre and t is in second.

15.25. *A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km h^{-1}. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 ms^{-1}.*

Ans. Here, frequency of SONAR (source) = 40.0 kHz = 40×10^3 Hz

Speed of sound waves, $v = 1450\ ms^{-1}$

Speed of observers, $v_0 = 360$ km/h $= 360 \times \frac{5}{18} = 100\ ms^{-1}$.

Since the source is at rest and observer moves towards the source (SONAR),

$$\therefore \quad \nu' = \frac{v + v_0}{v}.\nu = \frac{1450 + 100}{1450} \times 40 \times 10^3 = 4.276 \times 10^4\ \text{Hz}.$$

This frequency (ν') is reflected by the enemy ship and is observed by the SONAR (which now acts as observer). Therefore, in this case, $v_s = 360$ km/h $= 100\ ms^{-1}$.

∴ Apparent frequency, $\nu'' = \frac{v}{v - v_s}\nu'$

$$= \frac{1450}{1450-100} \times 4.276 \times 10^4 = 4.59 \times 10^4 \text{ Hz} = 45.9 \text{ kHz}.$$

15.26. *Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 kms^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?*

Ans. Here speed of S wave, v_s = 4.0 km s^{-1} and speed of P wave, v_p = 8.0 km s^{-1}. Time gap between P and S waves reaching the resimograph, t = 40 min = 240 s.

Let distance of earthquake centre = sKm

$$\therefore \quad t = t_s - t_p = \frac{S}{v_s} - \frac{S}{v_p} = \frac{S}{4.0} - \frac{S}{8.0} = \frac{S}{8.0} = 240 \text{ s}$$

or $\quad s = 240 \times 8.0 = 1920$ km.

15.27. *A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?*

Ans. Here, the frequency of sound emitted by the bat, ν = 40 kHz.
Velocity of bat, $v_s = 0.03\, v$, where v is velocity of sound.
Apparent frequency of sound striking the wall

$$\nu' = \frac{v}{v - v_s} \times \nu = \frac{v}{v - 0.03v} \times 40 \text{ kHz} = \frac{40}{0.97} \text{kHz}$$

This frequency is reflected by the wall and is received by the bat moving towards the wall. So $v_s = 0$,

$v_L = 0.03\, v$.

$$\nu'' = \frac{(v + v_L)}{v} \times \nu' = \frac{(v + 0.03v)}{v}\left(\frac{40}{0.97}\right)$$

$$= \frac{1.03}{0.97} \times 40 \text{ kHz} = 42.47 \text{ kHz}.$$

□□□