3 Matrices

3.3 EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. If a matrix has 28 elements, what are the possible orders it can have? What if it has 13 elements?

Sol. The possible orders that a matrix having 28 elements are $\{28 \times 1, 1 \times 28, 2 \times 14, 14 \times 2, 4 \times 7, 7 \times 4\}$. The possible orders of a matrix having 13 elements are $\{1 \times 13, 13 \times 1\}$.

Q2. In the matrix $A = \begin{bmatrix} a & 1 & x \\ 2 & \sqrt{3} & x^2 - y \\ 0 & 5 & \frac{-2}{5} \end{bmatrix}$, write:

(*i*) The order of the matrix A (*ii*) The number of elements

(*iii*) Write elements a_{23}, a_{31}, a_{12}

Sol. (*i*) The order of the given matrix A is 3×3

(*ii*) The number of elements in matrix $A = 3 \times 3 = 9$

(*iii*) a_{ij} = the elements of i^{th} row and j^{th} column.

So, $a_{23} = x^2 - y$, $a_{31} = 0$, $a_{12} = 1$.

Q3. Construct $a_{2\times2}$ matrix where

(*i*) $a_{ij} = \frac{(i-2j)^2}{2}$ (*ii*) $a_{ij} = |-2i + 3j|$

Sol. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$

(*i*) Given that $a_{ij} = \frac{(i-2j)^2}{2}$

$$a_{11} = \frac{(1-2\times1)^2}{2} = \frac{1}{2};\ a_{12} = \frac{(1-2\times2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2-2\times1)^2}{2} = 0;\ a_{22} = \frac{(2-2\times2)^2}{2} = 2$$

Hence, the matrix $A = \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{bmatrix}$

(*ii*) Given that $a_{ij} = |-2i + 3j|$

$$a_{11} = |-2 \times 1 + 3 \times 1| = 1;\ a_{12} = |-2 \times 1 + 3 \times 2| = 4$$

$$a_{21} = |-2 \times 2 + 3 \times 1| = -1; a_{22} = |-2 \times 2 + 3 \times 2| = 2$$

Hence, the matrix $A = \begin{bmatrix} 1 & 4 \\ -1 & 2 \end{bmatrix}$

Q4. Construct a 3 × 2 matrix whose elements are given by $a_{ij} = e^{ix} \sin jx$.

Sol. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times 2}$

Given that $a_{ij} = e^{ix} \sin jx$

$$a_{11} = e^{x} \sin x \qquad a_{12} = e^{x} \sin 2x$$

$$a_{21} = e^{2x} \sin x \qquad a_{22} = e^{2x} \sin 2x$$

$$a_{31} = e^{3x} \sin x \qquad a_{32} = e^{3x} \sin 2x$$

Hence, the matrix $A = \begin{bmatrix} e^{x} \sin x & e^{x} \sin 2x \\ e^{2x} \sin x & e^{2x} \sin 2x \\ e^{3x} \sin x & e^{3x} \sin 2x \end{bmatrix}$

Q5. Find the values of a and b if A = B, where

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix},\quad B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

Sol. Given that A = B

$$\Rightarrow \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

Equating the corresponding elements, we get

$a + 4 = 2a + 2,\quad 3b = b^2 + 2$ and $b^2 - 5b = -6$

$\Rightarrow\ 2a - a = 2,\quad b^2 - 3b + 2 = 0,\quad b^2 - 5b + 6 = 0$

$\therefore\ a = 2$

$\therefore\ b^2 - 3b + 2 = 0$	$\therefore\ b^2 - 5b + 6 = 0$
$\Rightarrow\ b^2 - 2b - b + 2 = 0,$	$b^2 - 3b - 2b + 6 = 0$
$\Rightarrow\ b(b-2) - 1(b-2) = 0,$	$\Rightarrow\ b(b-3) - 2(b-3) = 0$
$\Rightarrow\ (b-1)(b-2) = 0,$	$\Rightarrow\ (b-2)(b-3) = 0$
$\therefore\ b = 1, 2$	$\Rightarrow\ b = 2, 3$

but here 2 is common.

Hence, the value of $a = 2$ and $b = 2$.

Q6. If possible, find the sum of the matrices A and B, where

$$A = \begin{bmatrix} \sqrt{3} & 1 \\ 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} x & y & z \\ a & b & 6 \end{bmatrix}.$$

Sol. The order of matrix $A = 2 \times 2$ and the order of matrix $B = 2 \times 3$

Addition of matrices is only possible when they have same order. So, A + B is not possible.

Q7. If $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$, find

(*i*) X + Y (*ii*) 2X – 3Y

(*iii*) A matrix Z such that X + Y + Z is a zero matrix.

Sol. Given that $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$

(*i*) $X + Y = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 3+2 & 1+1 & -1-1 \\ 5+7 & -2+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$$

(*ii*) $2X - 3Y = 2\begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} - 3\begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 2\times3 & 2\times1 & -2\times1 \\ 2\times5 & -2\times2 & -2\times3 \end{bmatrix} - \begin{bmatrix} 3\times2 & 1\times3 & -1\times3 \\ 3\times7 & 3\times2 & 3\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 2 & -2 \\ 10 & -4 & -6 \end{bmatrix} - \begin{bmatrix} 6 & 3 & -3 \\ 21 & 6 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 6-6 & 2-3 & -2+3 \\ 10-21 & -4-6 & -6-12 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ -11 & -10 & -18 \end{bmatrix}$$

(*iii*) $X + Y + Z = 0$

$$\Rightarrow \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix} + \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $Z = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 3+2+a & 1+1+b & -1-1+c \\ 5+7+d & -2+2+e & -3+4+f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5+a & 2+b & -2+c \\ 12+d & e & 1+f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Equating the corresponding elements, we get

$5+a=0 \Rightarrow a=-5, \quad 2+b=0 \Rightarrow b=-2, \quad -2+c=0 \Rightarrow c=2$

$12+d=0 \Rightarrow d=-12, \quad e=0, \quad 1+f=0 \Rightarrow f=-1$

Hence, the matrix $Z=\begin{bmatrix}-5 & -2 & 2\\ -12 & 0 & -1\end{bmatrix}$

Q8. Find non-zero values of x satisfying the matrix equation:

$$x\begin{bmatrix}2x & 2\\ 3 & x\end{bmatrix}+2\begin{bmatrix}8 & 5x\\ 4 & 4x\end{bmatrix}=2\begin{bmatrix}(x^2+8) & 24\\ (10) & 6x\end{bmatrix}$$

Sol. The given equation can be written as

$$\begin{bmatrix}2x^2 & 2x\\ 3x & x^2\end{bmatrix}+\begin{bmatrix}16 & 10x\\ 8 & 8x\end{bmatrix}=\begin{bmatrix}(2x^2+16) & 48\\ 20 & 12x\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2x^2+16 & 12x\\ 3x+8 & x^2+8x\end{bmatrix}=\begin{bmatrix}2x^2+16 & 48\\ 20 & 12x\end{bmatrix}$$

Equating the corresponding elements we get

$12x=48, \qquad 3x+8=20, \qquad x^2+8x=12x$

$\therefore\ x=\frac{48}{12}=4, \qquad 3x=20-8=12, \qquad \Rightarrow\ x^2=12x-8x=4x$

$\therefore \qquad x=4, \qquad \Rightarrow \qquad x^2-4x=0$

$x=0, x=4$

Hence, the non-zero values of x is 4.

Q9. If $A=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$, show that

$(A+B)(A-B) \neq A^2-B^2$

Sol. Given that $A=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

$$A+B=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}+\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

$$\Rightarrow \quad A+B=\begin{bmatrix}0+0 & 1-1\\ 1+1 & 1+0\end{bmatrix} \Rightarrow A+B=\begin{bmatrix}0 & 0\\ 2 & 1\end{bmatrix}$$

$$A-B=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}-\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

$$\Rightarrow \quad A-B=\begin{bmatrix}0-0 & 1+1\\ 1-1 & 1-0\end{bmatrix} \Rightarrow A-B=\begin{bmatrix}0 & 2\\ 0 & 1\end{bmatrix}$$

$\therefore (A+B)\cdot(A-B) = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 4+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$

Now, R.H.S. $= A^2 - B^2$

$= A \cdot A - B \cdot B$

$= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+1 & 0+1 \\ 0+1 & 1+1 \end{bmatrix} - \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1+1 & 1-0 \\ 1-0 & 2+1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

Hence, $\begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

Hence, $(A+B).(A-B) \neq A^2 - B^2$

Q10. Find the value of x if

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$$

Sol. Given that $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} 1+2x+15 & 3+5x+3 & 2+x+2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} 2x+16 & 5x+6 & x+4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$

$\Rightarrow [2x+16+10x+12+x^2+4x] = 0; \Rightarrow x^2+16x+28 = 0$

$\Rightarrow x^2+14x+2x+28 = 0; \Rightarrow x(x+14)+2(x+14) = 0$

$\Rightarrow (x+2)(x+14) = 0;\ x+2 = 0$ or $x+14 = 0$

$\therefore x = -2$ or $x = -14$

Hence, the values of x are -2 and -14.

Q11. Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $A^2 - 3A - 7I = O$

and hence find A^{-1}.

Sol. Given that $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$

$A^2 = A\cdot A$

$$=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}25-3 & 15-6\\ -5+2 & -3+4\end{bmatrix}=\begin{bmatrix}22 & 9\\ -3 & 1\end{bmatrix}$$

$A^2-3A-7I=O$

L.H.S. $\begin{bmatrix}22 & 9\\ -3 & 1\end{bmatrix}-3\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}-7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix}22 & 9\\ -3 & 1\end{bmatrix}-\begin{bmatrix}15 & 9\\ -3 & -6\end{bmatrix}-\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix}22-15-7 & 9-9-0\\ -3+3-0 & 1+6-7\end{bmatrix} \Rightarrow \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ R.H.S.

We are given $A^2-3A-7I=O$

$\Rightarrow \quad A^{-1}[A^2-3A-7I]=A^{-1}O$

[Pre-multiplying both sides by A^{-1}]

$\Rightarrow \quad A^{-1}A\cdot A-3A^{-1}\cdot A-7A^{-1}I=O \qquad [A^{-1}O=O]$

$\Rightarrow \quad I\cdot A-3I-7A^{-1}I=O$

$\Rightarrow \quad A-3I-7A^{-1}=O$

$\Rightarrow \quad -7A^{-1}=3I-A$

$\Rightarrow \quad A^{-1}=\frac{1}{-7}[3I-A]$

$\Rightarrow \quad A^{-1}=\frac{1}{-7}\left[3\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}-\begin{pmatrix}5 & 3\\ -1 & -2\end{pmatrix}\right]$

$$=\frac{1}{-7}\left[\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}-\begin{pmatrix}5 & 3\\ -1 & -2\end{pmatrix}\right]$$

$$=\frac{1}{-7}\begin{bmatrix}3-5 & 0-3\\ 0+1 & 3+2\end{bmatrix}=\frac{1}{-7}\begin{bmatrix}-2 & -3\\ 1 & 5\end{bmatrix}$$

Hence, $A^{-1}=-\frac{1}{7}\begin{bmatrix}-2 & -3\\ 1 & 5\end{bmatrix}$

Q12. Find the matrix A satisfying the matrix equation:

$$\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}_{2\times 2} \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

$$\Rightarrow \quad \begin{bmatrix} 2a+c & 2b+d \\ 3a+2c & 3b+2d \end{bmatrix}_{2\times 2} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

$$\Rightarrow \quad \begin{bmatrix} -6a-3c+10b+5d & 4a+2c-6b-3d \\ -9a-6c+15b+10d & 6a+4c-9b-6d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating the corresponding elements, we get,

$$-6a-3c+10b+5d = 1 \qquad ...(1)$$

$$-9a-6c+15b+10d = 0 \qquad ...(2)$$

$$4a+2c-6b-3d = 0 \qquad ...(3)$$

$$6a+4c-9b-6d = 1 \qquad ...(4)$$

Multiplying eq. (1) by 2 and subtracting eq. (2), we get,

$$\begin{array}{rrrrrl} -12a & -\ 6c & +\ 20b & +\ 10d & = & 2 \\ 9a & -\ 6c & +\ 15b & +\ 10d & = & 0 \\ (+) & (+) & (-) & (-) & (-) & \\ \hline -\ 3a & & +\ 5b & & = & 2 \end{array}$$

$$-3a+5b = 2 \qquad ...(5)$$

Now, multiplying eq. (3) by 2 and subtracting eq. (4), we get

$$\begin{array}{rrrrrl} 8a & +\ 4c & -\ 12b & -\ 6d & = & 0 \\ 6a & +\ 4c & -\ 9b & -\ 6d & = & 1 \\ (-) & (-) & (+) & (+) & (-) & \\ \hline 2a & & -\ 3b & & = & -1 \end{array}$$

$$2a-3b = -1 \qquad ...(6)$$

Solving eq. (5) and (6) *i.e.*,

$$-3a+5b = 2$$

$$2a-3b = -1$$

$2\times(-3a+5b=2) \quad \Rightarrow \quad -6a+10b = 4$

$3\times(2a-3b=-1) \quad \Rightarrow \quad 6a-9b = -3$

Adding $\quad b = 1$

Putting the value of b in eq. (6), we get,

$$2a-3\times 1 = -1$$

$\Rightarrow \quad 2a-3=-1 \quad \Rightarrow \quad 2a = 3-1 \quad \Rightarrow \quad 2a = 2$

$\therefore \quad a = 1$

Now, putting the values of a and b in equations (1) and (3)

$$-6\times 1 - 3c + 10\times 1 + 5d = 1$$

$\Rightarrow \quad -6-3c+10+5d = 1$

$\Rightarrow \quad -3c + 5d + 4 = 1 \quad \Rightarrow -3c + 5d = -3$ ...(7)

and from eq. (3)

$4 \times 1 + 2c - 6 \times 1 - 3d = 0; \Rightarrow 4 + 2c - 6 - 3d = 0$

$\Rightarrow \quad -2 + 2c - 3d = 0 \quad \Rightarrow \quad 2c - 3d = 2$ ...(8)

Solving eq. (7) and (8) we get,

$2 \times (-3c + 5d = -3) \quad \Rightarrow \quad -6c + 10d = -6$

$3 \times (2c - 3d = 2) \quad \Rightarrow \quad 6c - 9d = 6$

Adding $\quad d = 0$

Putting the value of d in eq. (8) we get,

$2c - 3 \times 0 = 2 \quad \Rightarrow \quad 2c = 2 \Rightarrow c = 1$

Hence, $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

Q13. Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$

Sol. Order of $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$ is 3×1 and order of $\begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$ is 3×3. So, the order of matrix A must be 1×3.

Let $A = [a \quad b \quad c]_{1\times 3}$

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3\times 1} [a \quad b \quad c]_{1\times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times 3}$$

$$\Rightarrow \quad \begin{bmatrix} 4a & 4b & 4c \\ a & b & c \\ 3a & 3b & 3c \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

Equating the corresponding elements to get the values of a, b and c

$4a = -4, \quad 4b = 8, \quad 4c = 4$

$\therefore \quad a = -1 \quad \therefore \quad b = 2 \quad \therefore c = 1$

Hence, matrix $A = [-1 \quad 2 \quad 1]$

Q14. If $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then verify $(BA)^2 \neq B^2A^2$.

Sol. Here, $B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3}$ and $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}_{3\times 2}$

$$\therefore \quad BA = \begin{bmatrix} 6+1+4 & -8+1+0 \\ 3+2+8 & -4+2+0 \end{bmatrix}_{2\times 2} \Rightarrow BA = \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix}$$

L.H.S. $(BA)^2 = (BA)\cdot(BA) = \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix}\begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 121-91 & -77+14 \\ 143-26 & -91+4 \end{bmatrix} \Rightarrow \begin{bmatrix} 30 & -63 \\ 117 & -87 \end{bmatrix}$$

R.H.S. $B^2 = B\cdot B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3} \cdot \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3}$

Here, number of columns of first *i.e.,* 3 is not equal to the number of rows of second matrix *i.e.,* 2.

So, B^2 is not possible. Similarly, A^2 is also not possible.

Hence, $(BA)^2 \neq B^2A^2$

Q15. If possible, find BA and AB, where

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Sol. $BA = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3\times 2} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3}$

$$BA = \begin{bmatrix} 8+1 & 4+2 & 8+4 \\ 4+3 & 2+6 & 4+12 \\ 2+2 & 1+4 & 2+8 \end{bmatrix}_{3\times 3} = \begin{bmatrix} 9 & 6 & 12 \\ 7 & 8 & 16 \\ 4 & 5 & 10 \end{bmatrix}_{3\times 3}$$

Now $AB = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3} \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3\times 2}$

$$= \begin{bmatrix} 8+2+2 & 2+3+4 \\ 4+4+4 & 1+6+8 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 12 & 9 \\ 12 & 15 \end{bmatrix}_{2\times 2}$$

Hence, $BA = \begin{bmatrix} 9 & 6 & 12 \\ 7 & 8 & 16 \\ 4 & 5 & 10 \end{bmatrix}$ and $AB = \begin{bmatrix} 12 & 9 \\ 12 & 15 \end{bmatrix}$.

Q16. Show by an example that for $A \neq O$ and $B \neq O$, $AB = O$.

Sol. Let $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow \quad AB = \begin{bmatrix} 1-1 & 1-1 \\ -1+1 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Hence, $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

Q17. Given $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$. Is $(AB)' = B'A'$?

Sol. Here, $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+8+0 & 8+32+0 \\ 3+18+6 & 12+72+18 \end{bmatrix} = \begin{bmatrix} 10 & 40 \\ 27 & 102 \end{bmatrix}$$

L.H.S. $(AB)' = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix}$

Now $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix} \Rightarrow B' = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 8 & 3 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 & 3 \\ 4 & 9 \\ 0 & 6 \end{bmatrix}$

R.H.S. $B'A' = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 8 & 3 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 9 \\ 0 & 6 \end{bmatrix}$

$$= \begin{bmatrix} 2+8+0 & 3+18+6 \\ 8+32+0 & 12+72+18 \end{bmatrix} = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix} = \text{L.H.S.}$$

Hence, L.H.S. = R.H.S.

Q18. Solve for x and y: $x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = O$

Sol. Given that: $x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ 11 \end{bmatrix} =$

L.H.S. $x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = O$

$$\Rightarrow \begin{bmatrix} 2x \\ x \end{bmatrix} + \begin{bmatrix} 3y \\ 5y \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = O \Rightarrow \begin{bmatrix} 2x+3y-8 \\ x+5y-11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Comparing the corresponding elements of both sides, we get,

$$2x+3y-8=0 \quad \Rightarrow \quad 2x+3y=8 \qquad ...(1)$$
$$x+5y-11=0 \quad \Rightarrow \quad x+5y=11 \qquad ...(2)$$

Multiplying eq. (1) by 1 and eq. (2) by 2, and then on subtracting, we get,

$$\begin{array}{r} 2x + 3y = 8 \\ 2x + 10y = 22 \\ (-) \quad (-) \quad (-) \\ \hline -7y = -14 \end{array}$$

$\therefore \quad y = 2$

Putting y = 2 in eq. (2) we get,

$$x + 5 \times 2 = 11 \Rightarrow x + 10 = 11$$

$\therefore \qquad x = 11 - 10 = 1$

Hence, the values of x and y are 1 and 2 respectively.

Q19. If X and Y are 2×2 matrices, then solve the following matrix equations for X and Y.

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}, 3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}.$$

Sol. Given that:

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \qquad ...(1)$$

$$3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \qquad ...(2)$$

Multiplying eq. (1) by 3 and eq. (2) by 2, we get,

$$3\,[2X+3Y] = 3\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix} \Rightarrow 6X+9Y = \begin{bmatrix}6 & 9\\12 & 0\end{bmatrix} \quad ...(3)$$

$$2\,[3X+2Y] = 2\begin{bmatrix}-2 & 2\\1 & -5\end{bmatrix} \Rightarrow 6X+4Y = \begin{bmatrix}-4 & 4\\2 & -10\end{bmatrix} \quad ...(4)$$

On subtracting eq. (4) from eq. (3) we get

$$5Y = \begin{bmatrix}6+4 & 9-4\\12-2 & 0+10\end{bmatrix}$$

$$5Y = \begin{bmatrix}10 & 5\\10 & 10\end{bmatrix} \Rightarrow Y = \begin{bmatrix}2 & 1\\2 & 2\end{bmatrix}$$

Now, putting the value of Y in equation (1) we get,

$$2X+3\begin{bmatrix}2 & 1\\2 & 2\end{bmatrix} = \begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}$$

$$\Rightarrow 2X+\begin{bmatrix}6 & 3\\6 & 6\end{bmatrix} = \begin{bmatrix}2 & 3\\4 & 0\end{bmatrix} \Rightarrow 2X = \begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}-\begin{bmatrix}6 & 3\\6 & 6\end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix}2-6 & 3-3\\4-6 & 0-6\end{bmatrix} \Rightarrow 2X = \begin{bmatrix}-4 & 0\\-2 & -6\end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2}\begin{bmatrix}-4 & 0\\-2 & -6\end{bmatrix} \Rightarrow X = \begin{bmatrix}-2 & 0\\-1 & -3\end{bmatrix}$$

Hence, $X = \begin{bmatrix}-2 & 0\\-1 & -3\end{bmatrix}$ and $Y = \begin{bmatrix}2 & 1\\2 & 2\end{bmatrix}$.

Q20. If $A = [3 \quad 5], B = [7 \quad 3]$, then find a non-zero matrix C such that AC = BC.

Sol. Given that: $A = [3 \quad 5]_{1\times 2}, B = [7 \quad 3]_{1\times 2}$

Let $$C = \begin{bmatrix}\alpha\\ \beta\end{bmatrix}_{2\times 1}$$

$$AC = [3 \quad 5]_{1\times 2}\begin{bmatrix}\alpha\\ \beta\end{bmatrix}_{2\times 1} = [3\alpha+5\beta]$$

$$BC = [7 \quad 3]_{1\times 2}\begin{bmatrix}\alpha\\ \beta\end{bmatrix}_{2\times 1} = [7\alpha+3\beta]$$

$$AC = BC \quad \text{(Given)}$$

$$\Rightarrow [3\alpha+5\beta] = [7\alpha+3\beta]$$

$$\Rightarrow 3\alpha+5\beta = 7\alpha+3\beta$$

$$\Rightarrow \quad 3\alpha - 7\alpha = 3\beta - 5\beta$$

$$\Rightarrow \quad -4\alpha = -2\beta$$

$$\therefore \quad \frac{\alpha}{\beta} = \frac{1}{2}$$

So, let $\alpha = K$ and $\beta = 2K$, K is some real number.

Hence, matrix $C = \begin{bmatrix} K \\ 2K \end{bmatrix}_{2\times1}$ or $\begin{bmatrix} K & K \\ 2K & 2K \end{bmatrix}_{2\times2}$ etc.

Q21. Give an example of matrices A, B and C such that AB = AC, where A is non-zero matrix, but $B \neq C$.

Sol. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 1+0 & 2+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 1+0 & 2+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

Hence, AB = AC for matrix A is non-zero and $B \neq C$.

Q22. If $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

verify: (*i*) (AB)C = A(BC) (*ii*) A(B + C) = AB + AC

Sol. Given that $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

(*i*) To verify: (AB)C = A(BC)

$$AB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 2+6 & 3-8 \\ -4+3 & -6-4 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}$$

L.H.S.

$$(AB)C = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 8+5 & 0+0 \\ -1+10 & 0+0 \end{bmatrix} = \begin{bmatrix} 13 & 0 \\ 9 & 0 \end{bmatrix}$$

$$BC = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 2-3 & 0+0 \\ 3+4 & 0+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix}$$

R.H.S.

$$A(BC) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix} = \begin{bmatrix} -1+14 & 0+0 \\ 2+7 & 0+0 \end{bmatrix} = \begin{bmatrix} 13 & 0 \\ 9 & 0 \end{bmatrix}$$

L.H.S. = R.H.S.

So, (AB)C = A(BC)

(*ii*) To verify: A(B + C) = AB + AC

$$\text{L.H.S.}\quad B + C = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 2+1 & 3+0 \\ 3-1 & -4+0 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 2 & -4 \end{bmatrix}$$

$$\text{L.H.S.}\,A(B + C) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+4 & 3-8 \\ -6+2 & -6-4 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ -4 & -10 \end{bmatrix}$$

$$\text{R.H.S.}\quad AB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 2+6 & 3-8 \\ -4+3 & -6-4 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 0+0 \\ -2-1 & 0+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix}$$

$$\text{R.H.S.}\quad AB + AC = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 8-1 & -5+0 \\ -1-3 & -10+0 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -5 \\ -4 & -10 \end{bmatrix}$$

L.H.S. = R.H.S.

Hence, A(B + C) = AB + AC

Q23. If $P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ and $Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, prove that

$$PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP$$

Sol. Given that: $P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ and $Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

$$PQ = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$PQ = \begin{bmatrix} xa+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+yb+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+zc \end{bmatrix}$$

$$PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix}$$

Now $$QP = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

$$QP = \begin{bmatrix} xa+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+yb+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+zc \end{bmatrix}$$

$$QP = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix}$$

Hence, PQ = QP.

Q24. If $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$ find A.

Sol. Given that: $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$

L.H.S. $$\begin{bmatrix} 2 & 1 & 3 \end{bmatrix}_{1\times 3}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{3\times 3}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}_{3\times 1}$$

$$\Rightarrow \quad \begin{bmatrix}-2-1+0 & 0+1+3 & -2+0+3\end{bmatrix}_{1\times3}\begin{bmatrix}1\\0\\-1\end{bmatrix}_{3\times1}$$

$$\Rightarrow \quad \begin{bmatrix}-3 & 4 & 1\end{bmatrix}_{1\times3}\begin{bmatrix}1\\0\\-1\end{bmatrix}_{3\times1}$$

$$\Rightarrow \quad \begin{bmatrix}-3+0-1\end{bmatrix}_{1\times1} = \begin{bmatrix}-4\end{bmatrix}_{1\times1}$$

Hence, matrix $A = [-4]$

Q25. If $A = \begin{bmatrix}2 & 1\end{bmatrix}, B = \begin{bmatrix}5 & 3 & 4\\8 & 7 & 6\end{bmatrix}$ and $C = \begin{bmatrix}-1 & 2 & 1\\1 & 0 & 2\end{bmatrix}$,

verify that $A(B + C) = (AB + AC)$.

Sol. Given that: $A = \begin{bmatrix}2 & 1\end{bmatrix}, B = \begin{bmatrix}5 & 3 & 4\\8 & 7 & 6\end{bmatrix}$ and $C = \begin{bmatrix}-1 & 2 & 1\\1 & 0 & 2\end{bmatrix}$.

L.H.S. $\quad (B + C) = \begin{bmatrix}5 & 3 & 4\\8 & 7 & 6\end{bmatrix} + \begin{bmatrix}-1 & 2 & 1\\1 & 0 & 2\end{bmatrix}$

$$= \begin{bmatrix}5-1 & 3+2 & 4+1\\8+1 & 7+0 & 2+6\end{bmatrix} = \begin{bmatrix}4 & 5 & 5\\9 & 7 & 8\end{bmatrix}$$

$$A(B+C) = \begin{bmatrix}2 & 1\end{bmatrix}_{1\times2}\begin{bmatrix}4 & 5 & 5\\9 & 7 & 8\end{bmatrix}_{2\times3}$$

$$= \begin{bmatrix}8+9 & 10+7 & 10+8\end{bmatrix}_{1\times3}$$

$$A(B+C) = \begin{bmatrix}17 & 17 & 18\end{bmatrix}$$

R.H.S. $\quad AB = \begin{bmatrix}2 & 1\end{bmatrix}_{1\times2}\begin{bmatrix}5 & 3 & 4\\8 & 7 & 6\end{bmatrix}_{2\times3}$

$$= \begin{bmatrix}10+8 & 6+7 & 8+6\end{bmatrix}_{1\times3} = \begin{bmatrix}18 & 13 & 14\end{bmatrix}_{1\times3}$$

$$AC = \begin{bmatrix}2 & 1\end{bmatrix}_{1\times2}\begin{bmatrix}-1 & 2 & 1\\1 & 0 & 2\end{bmatrix}_{2\times3}$$

$$= \begin{bmatrix}-2+1 & 4+0 & 2+2\end{bmatrix}_{1\times3} = \begin{bmatrix}-1 & 4 & 4\end{bmatrix}_{1\times3}$$

$$AB + AC = \begin{bmatrix}18 & 13 & 14\end{bmatrix}_{1\times3} + \begin{bmatrix}-1 & 4 & 4\end{bmatrix}_{1\times3}$$

$$= \begin{bmatrix}18-1 & 13+4 & 14+4\end{bmatrix}_{1\times3}$$

$$AB + AC = \begin{bmatrix}17 & 17 & 18\end{bmatrix}_{1\times3}$$

L.H.S. = R.H.S.

Hence, $A(B + C) = (AB + AC)$ is verified.

Q26. If $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ then verify that $A^2 + A = A(A + I)$, where I is 3×3 unit matrix.

Sol. Given that: $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 0+0-1 & -1+0-1 \\ 2+2+0 & 0+1+3 & -2+3+3 \\ 0+2+0 & 0+1+1 & 0+3+1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix}$$

L.H.S. $\quad A^2 + A = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+1 & -1+0 & -2-1 \\ 4+2 & 4+1 & 4+3 \\ 2+0 & 2+1 & 4+1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix}$$

R.H.S. $\quad A(A + I) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\left[\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+0 & 0+0-1 & -1+0-2 \\ 4+2+0 & 0+2+3 & -2+3+6 \\ 0+2+0 & 0+2+1 & 0+3+2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix}$$

L.H.S. = R.H.S.

$A^2 + A = A(A + I)$. Hence verified.

Q27. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$, then verify that:

(*i*) $(A')' = A$ (*ii*) $(AB)' = B'A'$ (*iii*) $(kA)' = (kA')$

Sol. Given that: $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$

(*i*) $$A' = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}'_{2\times 3} = \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix}_{3\times 2}$$

$$(A')' = \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix}'_{3\times 2} = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}_{2\times 3} = A$$

Hence, $(A')' = A$

(*ii*) L.H.S. $$AB = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}_{2\times 3} \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} 0-1+4 & 0-3+12 \\ 16+3-8 & 0+9-24 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 3 & 9 \\ 11 & -15 \end{bmatrix}_{2\times 2}$$

$$(AB)' = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix}_{2\times 2}$$

R.H.S. $$B' = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}' = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}' = \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix}_{2\times 3} \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} 0-1+4 & 16+3-8 \\ 0-3+12 & 0+9-24 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix}_{2\times 2}$$

L.H.S. = R.H.S.

Hence, $(AB)' = B'A'$ is verified.

(iii) L.H.S. $kA = k\begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & -k & 2k \\ 4k & 3k & -4k \end{bmatrix}$

$$(kA)' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix}$$

R.H.S. $kA' = k\begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}' = k\begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix}$

Hence, L.H.S. = R.H.S.

$(kA)' = (kA')$ is verified.

Q28. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$, then verify that:

(i) $(2A + B)' = 2A' + B'$ (ii) $(A - B)' = A' - B'$

Sol. Given that: $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$

(i) To verify that: $(2A + B)' = 2A' + B'$

L.H.S. $(2A + B)' = \left[2\begin{pmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{pmatrix}\right]' = \left[\begin{pmatrix} 2 & 4 \\ 8 & 2 \\ 10 & 12 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{pmatrix}\right]'$

$$= \begin{bmatrix} 2+1 & 4+2 \\ 8+6 & 2+4 \\ 10+7 & 12+3 \end{bmatrix}' = \begin{bmatrix} 3 & 6 \\ 14 & 6 \\ 17 & 15 \end{bmatrix}' = \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix}$$

R.H.S. $2A' + B' = 2\begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}' + \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}'$

$$= 2\begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 8 & 10 \\ 4 & 2 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 8+6 & 10+7 \\ 4+2 & 2+4 & 12+3 \end{bmatrix} = \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix}$$

Hence, L.H.S. = R.H.S.

$(2A + B)' = 2A' + B'$ is verified.

(*ii*) To verify that: $(A - B)' = A' - B'$

$$\text{L.H.S. } (A-B)' = \left[\begin{pmatrix}1 & 2\\4 & 1\\5 & 6\end{pmatrix} - \begin{pmatrix}1 & 2\\6 & 4\\7 & 3\end{pmatrix}\right]'$$

$$= \begin{bmatrix}1-1 & 2-2\\4-6 & 1-4\\5-7 & 6-3\end{bmatrix}' = \begin{bmatrix}0 & 0\\-2 & -3\\-2 & 3\end{bmatrix}' = \begin{bmatrix}0 & -2 & -2\\0 & -3 & 3\end{bmatrix}$$

$$\text{R.H.S. } A' - B' = \begin{bmatrix}1 & 2\\4 & 1\\5 & 6\end{bmatrix}' - \begin{bmatrix}1 & 2\\6 & 4\\7 & 3\end{bmatrix}' = \begin{bmatrix}1 & 4 & 5\\2 & 1 & 6\end{bmatrix} - \begin{bmatrix}1 & 6 & 7\\2 & 4 & 3\end{bmatrix}$$

$$= \begin{bmatrix}1-1 & 4-6 & 5-7\\2-2 & 1-4 & 6-3\end{bmatrix} = \begin{bmatrix}0 & -2 & -2\\0 & -3 & 3\end{bmatrix}$$

Hence, L.H.S. = R.H.S.

$(A - B)' = A' - B'$ is verified.

Q29. Show that $A'A$ and AA' are both symmetric matrices for any matrix A.

Sol. Let $P = A'A$

$\Rightarrow$ $P' = (A'A)'$

$\Rightarrow$ $P' = A'(A')'$ $\quad [(AB)' = B'A']$

$\Rightarrow$ $P' = A'A$ $\quad [\because (A')' = A]$

$\Rightarrow$ $P' = P$

Hence, $A'A$ is a symmetric matrix.

Now, Let $Q = AA'$

$\Rightarrow$ $Q' = (AA')'$

$\Rightarrow$ $Q' = (A')'A'$ $\quad [(AB)' = B'A']$

$\Rightarrow$ $Q' = AA'$ $\quad [\because (A')' = A]$

$\Rightarrow$ $Q' = Q$

Hence, AA' is also a symmetric matrix.

Q30. Let A and B be square matrices of the order 3×3. Is $(AB)^2 = A^2B^2$? Give reasons.

Sol. Given that A and B are the matrices of the order 3×3.

$$(AB)^2 = AB \cdot AB$$
$$= AA \cdot BB$$
$$= A^2 \cdot B^2$$

Hence, $(AB)^2 = A^2B^2$

Q31. Show that if A and B are square matrices such that AB = BA then $(A+B)^2 = A^2 + 2AB + B^2$.

Sol. To prove that $(A+B)^2 = A^2 + 2AB + B^2$

L.H.S. $(A+B)^2 = (A+B)\cdot(A+B)$ $\quad [\because A^2 = A\cdot A]$

$= A\cdot A + AB + BA + B\cdot B$

$= A^2 + AB + AB + B^2$ $\quad [AB = BA]$

$= A^2 + 2AB + B^2$ R.H.S.

So, L.H.S. = R.H.S.

Q32. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$ and $a = 4$, $b = -2$.

Show that:

(*a*) $A + (B + C) = (A + B) + C$

(*b*) $A(BC) = (AB)C$

(*c*) $(a + b)B = aB + bB$

(*d*) $a(C - A) = aC - aA$

(*e*) $(A^T)^T = A$

(*f*) $(bA)^T = bA^T$

(*g*) $(AB)^T = B^T A^T$

(*h*) $(A - B)C = AC - BC$

(*i*) $(A - B)^T = A^T - B^T$

Sol. (*a*) To prove that: $A + (B + C) = (A + B) + C$

L.H.S. $A + (B + C) = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} + \left[\begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 1 & -2 \end{pmatrix}\right]$

$= \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 4+2 & 0+0 \\ 1+1 & 5-2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+6 & 2+0 \\ -1+2 & 3+3 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

R.H.S. $(A + B) + C = \left[\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\right] + \begin{pmatrix} 2 & 0 \\ 1 & -2 \end{pmatrix}$

$= \begin{bmatrix} 1+4 & 2+0 \\ -1+1 & 3+5 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 5 & 2 \\ 0 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 5+2 & 2+0 \\ 0+1 & 8-2 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

Hence, L.H.S. = R.H.S.

$A + (B + C) = (A + B) + C$ Hence proved.

(*b*) To prove that: A(BC) = (AB)C

$$\text{L.H.S. } A(BC) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\left[\begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & -2 \end{pmatrix}\right]$$

$$= \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 8+0 & 0+0 \\ 2+5 & 0-10 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 8+14 & 0-20 \\ -8+21 & 0-30 \end{bmatrix} = \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix}$$

$$\text{R.H.S. } (AB)C = \left[\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\right]\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2 & 0+10 \\ -4+3 & 0+15 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+10 & 0-20 \\ -2+15 & 0-30 \end{bmatrix} = \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix}$$

Hence, L.H.S. = R.H.S.

A(BC) = (AB)C Hence proved.

(*c*) To prove that: $(a+b)B = aB + bB$

Here, $a = 4$ and $b = -2$

$$\text{L.H.S. } (a+b)B = (4-2)\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = 2\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$$

$$\text{R.H.S. } aB + bB = 4\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} - 2\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 16-8 & 0-0 \\ 4-2 & 20-10 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$$

Hence, L.H.S. = R.H.S.

$(a+b)B = aB + bB$ Hence proved.

(*d*) To prove that: $a(C - A) = aC - aA$

$$\text{L.H.S. } a(C-A) = 4\left[\begin{pmatrix} 2 & 0 \\ 1 & -2 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}\right]$$

$$= 4\begin{bmatrix} 2-1 & 0-2 \\ 1+1 & -2-3 \end{bmatrix} = 4\begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ 8 & -20 \end{bmatrix}$$

R.H.S. $aC - aA = 4\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 8 & 0 \\ 4 & -8 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ -4 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 8-4 & 0-8 \\ 4+4 & -8-12 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ 8 & -20 \end{bmatrix}$$

Hence, L.H.S. = R.H.S.

$a(C - A) = aC - aA$ Hence proved.

(*e*) To prove that: $(A^T)^T = A$

L.H.S. $A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$$(A^T)^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} = A \quad \text{R.H.S.}$$

Hence, $(A^T)^T = A$

(*f*) To prove that: $(bA)^T = bA^T$

L.H.S. $(bA)^T = \left[-2\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}\right]^T = \begin{bmatrix} -2 & -4 \\ 2 & -6 \end{bmatrix}^T = \begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}$

R.H.S. $bA^T = -2\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}^T = -2\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}$

Hence, L.H.S. = R.H.S.

$(bA)^T = bA^T$ Hence proved.

(*g*) To prove that: $(AB)^T = B^TA^T$

L.H.S. $(AB)^T = \left[\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\right]^T$

$$= \begin{bmatrix} 4+2 & 0+10 \\ -4+3 & 0+15 \end{bmatrix}^T = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}^T = \begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix}$$

R.H.S. $B^TA^T = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}^T \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}^T$

$$= \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4+2 & -4+3 \\ 0+10 & 0+15 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix}$$

Hence, L.H.S. = R.H.S.

$(AB)^T = B^TA^T$ Hence proved.

(*h*) To prove that: $(A-B)C = AC - BC$

L.H.S. $(A-B)C = \left[\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\right]\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 1-4 & 2-0 \\ -1-1 & 3-5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6+2 & 0-4 \\ -4-2 & 0+4 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$$

R.H.S. $AC - BC = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 2+2 & 0-4 \\ -2+3 & 0-6 \end{bmatrix} - \begin{bmatrix} 8+0 & 0+0 \\ 2+5 & 0-10 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -4 \\ 1 & -6 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 4-8 & -4-0 \\ 1-7 & -6+10 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$$

Hence, L.H.S. = R.H.S.

$(A-B)C = AC - BC$

(*i*) To prove that: $(A-B)^T = A^T - B^T$

L.H.S. $(A-B)^T = \left[\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}\right]^T$

$$= \begin{bmatrix} 1-4 & 2-0 \\ -1-1 & 3-5 \end{bmatrix}^T = \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}^T = \begin{bmatrix} -3 & -2 \\ 2 & -2 \end{bmatrix}$$

R.H.S. $A^T - B^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}^T - \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 1-4 & -1-1 \\ 2-0 & 3-5 \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ 2 & -2 \end{bmatrix}$$

Hence, L.H.S. = R.H.S.

$(A-B)^T = A^T - B^T$ Hence proved.

Q33. If $A = \begin{bmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{bmatrix}$, then show that

$$A^2 = \begin{bmatrix} \cos 2q & \sin 2q \\ -\sin 2q & \cos 2q \end{bmatrix}$$

Sol. Given that

$$A = \begin{bmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{bmatrix}$$

$$A = A \cdot A = \begin{bmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{bmatrix} \begin{bmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 q - \sin^2 q & \cos q \sin q + \sin q \cos q \\ \sin q \cos q - \cos q \sin q & -\sin^2 q + \cos^2 q \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2q & \sin 2q \\ -\sin 2q & \cos 2q \end{bmatrix} \quad \begin{bmatrix} \because \cos^2 A - \sin^2 A = \cos 2A \\ 2\sin A \cos A = \sin 2A \end{bmatrix}$$

Hence proved.

Q34. If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$, then show that $(A + B)^2 = A^2 + B^2$.

Sol. Given that: $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

L.H.S. $(A + B)^2 = (A + B) \cdot (A + B)$

$$= \left[\begin{pmatrix} 0 & -x \\ x & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right] \cdot \left[\begin{pmatrix} 0 & -x \\ x & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right]$$

$$= \begin{bmatrix} 0+0 & -x+1 \\ x+1 & 0+0 \end{bmatrix} \cdot \begin{bmatrix} 0+0 & -x+1 \\ x+1 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + (-x+1)(x+1) & 0+0 \\ 0+0 & (x+1)(-x+1)+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix}$$

Put $x^2 = -1$ (given)

R.H.S. $= \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$$A^2 + B^2 = A \cdot A + B \cdot B$$

$$= \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}\begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0-x^2 & 0+0 \\ 0+0 & -x^2+0 \end{bmatrix} + \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -x^2+1 & 0+0 \\ 0+0 & -x^2+1 \end{bmatrix}$$

$$= \begin{bmatrix} -x^2+1 & 0 \\ 0 & -x^2+1 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} \quad [\because x^2 = -1]$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Hence, L.H.S. = R.H.S.

$$(A+B)^2 = A^2 + B^2$$

Q35. Verify that $A^2 = I$ when $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$.

Sol. Given that: $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

L.H.S. $A^2 = A \cdot A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 0+4-3 & 0-3+3 & 0+4-4 \\ 0-12+12 & 4+9-12 & -4-12+16 \\ 0-12+12 & 3+9-12 & -3-12+16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad \text{R.H.S.}$$

Hence, $A^2 = I$ is verified.

Q36. Prove by Mathematical Induction that $(A')^n = (A^n)'$, where $n \in N$ for any square matrix A.

Sol. To prove that $(A')^n = (A^n)'$

Let $P(n)$: $(A')^n = (A^n)'$

Step 1: Put $n = 1$, P(1): $A' = A'$ which is true for $n = 1$

Step 2: Put $n = K$, P(K): $(A')^K = (A^K)'$ Let it be true for $n = K$

Step 3: Put $n = K + 1$, P(K + 1): $(A')^{K+1} = (A^{K+1})'$

L.H.S. $(A')^{K+1} = (A')^K \cdot (A')$

$= (A^K)' \cdot (A')$ (From step 2)

$= (A^K \cdot A)'$

$= (A^{K+1})'$ R.H.S.

The given statement is true for P(K+1) whenever it is true for P(K), where $K \in N$.

Q37. Find inverse, by elementary row operations (if possible), of the following matrices.

(*i*) $\begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$ (*ii*) $\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$

Sol. (*i*) Let $A = \begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$

$|A| = 1 \times 7 - (-5) \times 3 = 7 + 15 = 22 \neq 0$

So, A is invertible.

Let $A = IA$

$$\begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + 5R_1$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} A$$

$R_2 \to \frac{1}{22} R_2$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{5}{22} & \frac{1}{22} \end{bmatrix} A$$

$R_1 \to R_1 - 3R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{7}{22} & \frac{-3}{22} \\ \frac{5}{22} & \frac{1}{22} \end{bmatrix} A$$

So $$A^{-1} = \begin{bmatrix} \frac{7}{22} & \frac{-3}{22} \\ \frac{5}{22} & \frac{1}{22} \end{bmatrix} \Rightarrow \frac{1}{22}\begin{bmatrix} 7 & -3 \\ 5 & 1 \end{bmatrix}$$

Hence, inverse of $\begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$ is $\frac{1}{22}\begin{bmatrix} 7 & -3 \\ 5 & 1 \end{bmatrix}$

(*ii*) Let $$A = \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$$

$|A| = 1 \times 6 - (-3)(-2) = 6 - 6 = 0$

$|A| = 0$ so A is not invertible.

Hence, inverse of $\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$ is not possible.

Q38. If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, then find the values of x, y, z and w.

Sol. Given that: $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

Equating the corresponding elements,

$xy = 8, w = 4, z + 6 = 0 \Rightarrow z = -6, x + y = 6$

Now, solving $x + y = 6$ ...(*i*)

and $xy = 8$ ...(*ii*)

From eqn. (*i*), $y = 6 - x$ ...(*iii*)

Putting the value of y in eqn. (*ii*) we get,

$x(6 - x) = 8 \Rightarrow 6x - x^2 = 8$

$\Rightarrow x^2 - 6x + 8 = 0 \Rightarrow x^2 - 4x - 2x + 8 = 0$

$\Rightarrow x(x - 4) - 2(x - 4) = 0 \Rightarrow (x - 4)(x - 2) = 0$

$\therefore x = 4, 2$

From eqn. (*iii*), $y = 2, 4.$

Hence, $x = 4$ or 2, $y = 2$ or 4, $z = -6$ and $w = 4$.

Q39. If $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, find a matrix C such that $3A + 5B + 2C$ is a null matrix.

Sol. Order of matrices A and B is 2×2.

$\therefore$ Order of matrix C must be 2×2.

Let $$C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\therefore$ $$3A + 5B + 2C = 0$$

$$\Rightarrow 3\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} + 5\begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix} + 2\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+45+2a & 15+5+2b \\ 21+35+2c & 36+40+2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 48+2a & 20+2b \\ 56+2c & 76+2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Equating the corresponding elements, we get,

$48 + 2a = 0 \Rightarrow 2a = -48 \Rightarrow a = -24$

$20 + 2b = 0 \Rightarrow 2b = -20 \Rightarrow b = -10$

$56 + 2c = 0 \Rightarrow 2c = -56 \Rightarrow c = -28$

$76 + 2d = 0 \Rightarrow 2d = -76 \Rightarrow d = -38$

Hence, $$C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$$

Q40. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, find A^3.

Sol. Given that: $$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

$$\therefore A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 29-29 & -25+25 \\ -20+20 & 24-24 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, $A^2 - 5A - 14I = O$

Now, multiplying both sides by A, we get,

$A^2 . A - 5A . A - 14IA = OA$

$\Rightarrow \quad A^3 - 5A^2 - 14A = 0$

$\Rightarrow \quad A^3 = 5A^2 + 14A$

$$\Rightarrow \quad A^3 = 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

$$= \begin{bmatrix} 145+42 & -125-70 \\ -100-56 & 120+28 \end{bmatrix} = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

Hence, $A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$

Q41. Find the values of *a*, *b*, *c* and *d* if

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}.$$

Sol. Given that: $3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}$

$$\begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \begin{bmatrix} a+4 & 6+a+b \\ -1+c+d & 2d+3 \end{bmatrix}$$

Equating the corresponding elements, we get,

$3a = a + 4 \quad \Rightarrow 3a - a = 4 \quad \Rightarrow 2a = 4 \quad \Rightarrow a = 2$

$3b = 6 + a + b \quad \Rightarrow 3b - b - a = 6 \quad \Rightarrow 2b - a = 6 \quad \Rightarrow 2b - 2 = 6$

$\Rightarrow 2b = 8$

$\Rightarrow b = 4$

$3c = -1 + c + d \quad \Rightarrow 3c - c - d = -1 \Rightarrow 2c - d = -1$

and $3d = 2d + 3 \quad \Rightarrow 3d - 2d = 3 \quad \Rightarrow d = 3$

Now $\;2c - d = -1$

$\Rightarrow \quad 2c-3=-1 \quad \Rightarrow \quad 2c=3-1 \quad \Rightarrow 2c=2$

$\therefore \quad c=1$

$\therefore a=2, b=4, c=1$ and $d=3$.

Q42. Find the matrix A such that

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

Sol. Order of matrix $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}$ is 3×2 and the matrix

$\begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$ is 3×3

$\therefore$ Order of matrix A must be 2×3

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times 3}$

So, $$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2a-d & 2b-e & 2c-f \\ a+0 & b+0 & c+0 \\ -3a+4d & -3b+4e & -3c+4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

Equating the corresponding elements, we get,

$2a-d=-1$ and $a=1 \quad \Rightarrow 2\times 1-d=-1 \quad \Rightarrow d=2+1 \Rightarrow d=3$

$2b-e=-8$ and $b=-2 \Rightarrow 2(-2)-e= -8 \quad \Rightarrow -4-e=-8$

$\Rightarrow e=4$

$2c-f=-10$ and $c=-5 \Rightarrow 2(-5) -f=-10 \Rightarrow -10-f=-10$

$\Rightarrow f=0$

$a=1, b=-2, c=-5, d=3, e=4$ and $f=0$

Hence, $$A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}.$$

Q43. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$, find $A^2 + 2A + 7I$.

Sol. Given that: $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$

$$A^2 = A . A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 1+8 & 2+2 \\ 4+4 & 8+1 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix}$$

$$A^2 + 2A + 7I = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix} + 2\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2+7 & 4+4+0 \\ 8+8+0 & 9+2+7 \end{bmatrix} = \begin{bmatrix} 18 & 8 \\ 16 & 18 \end{bmatrix}$$

Hence, $A^2 + 2A + 7I = \begin{bmatrix} 18 & 8 \\ 16 & 18 \end{bmatrix}$.

Q44. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, and $A^{-1} = A'$, find value of α.

Sol. Here, $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Given that: $A^{-1} = A'$

Pre-multiplying both sides by A

$AA^{-1} = AA'$

$\Rightarrow$ $I = AA'$ $[\because AA^{-1} = I]$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha + \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence, it is true for all values of α.

Q45. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew symmetric matrix, find the values of a, b and c.

Sol. Let $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ $A' = \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix}$

For skew symmetric matrix, $A' = -A$.

$$\begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & -3 \\ -2 & -b & 1 \\ -c & -1 & 0 \end{bmatrix}$$

Equating the corresponding elements, we get

$a = -2$, $b = -b \Rightarrow 2b = 0 \Rightarrow b = 0$ and $c = -3$

Hence, $a = -2$, $b = 0$ and $c = -3$.

Q46. If $P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then show that

$P(x).P(y) = P(x+y) = P(y).P(x)$

Sol. Given that:

$$P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$P(y) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix} \qquad \text{[Replacing } x \text{ by } y\text{]}$$

$$P(x).P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos x\cos y - \sin x \sin y & \cos x \sin y + \sin x \cos y \\ -\sin x \cos y - \cos x \sin y & -\sin x \sin y + \cos x \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix}$$

$$= P(x+y)$$

Now

$$P(y).P(x) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$= \begin{bmatrix} \cos x\cos y - \sin x \sin y & \sin x \cos y + \cos x \sin y \\ -\cos x \sin y - \cos y \sin x & -\sin x \sin y + \cos x \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix}$$

$= P(x+y)$

Hence, $P(x).P(y) = P(x+y) = P(y).P(x)$.

Q47. If A is a square matrix such that $A^2 = A$, show that $(I + A)^3 = 7A + I$.

Sol. To show that: $(I + A)^3 = 7A + I$

L.H.S. $(I + A)^3 = I^3 + A^3 + 3I^2A + 3IA^2$

$\Rightarrow I + A^2.A + 3IA + 3IA^2$

$\Rightarrow I + A.A + 3IA + 3IA$ $\quad [\because A^2 = A]$

$\Rightarrow I + A^2 + 3IA + 3IA$

$\Rightarrow I + A + 3IA + 3IA$ $\quad [\because A^2 = A]$

$\Rightarrow I + A + 3A + 3A \Rightarrow 7A + I$ R.H.S.

L.H.S. = R.H.S. Hence, Proved.

Q48. If A and B are square matrices of the same order and B is a skew symmetric matrix, show that A′BA is a skew symmetric.

Sol. Given that B is a skew symmetric matrix $\therefore B' = -B$

Let $P = A'BA$

$\Rightarrow$ $P' = (A'BA)'$

$= A'B'(A')'$ $\quad [(AB)' = B'A']$

$= A'(-B)A$

$= -A'BA = -P$

So $P' = -P$

Hence, A′BA is a skew symmetric matrix.

LONG ANSWER TYPE QUESTIONS

Q49. If AB = BA for any two square matrices, prove by mathematical induction that $(AB)^n = A^nB^n$.

Sol. Let $P(n) : (AB)^n = A^nB^n$

Step 1:

Put $n = 1$, $P(1) : AB = AB$ (True)

Step 2:

Put $n = k$, $P(k) : (AB)^k = A^kB^k$ (Let it be true for any $k \in N$)

Step 3:

Put $n = k + 1$, $P(k+1) : (AB)^{k+1} = A^{k+1}B^{k+1}$

L.H.S. $(AB)^{k+1} = (AB)^k.AB$

$= A^kB^k.AB$ $\quad$ [from Step 2]

$= A^{k+1}B^{k+1}$ R.H.S.

L.H.S. = R.H.S.

Hence, if P(n) is true for P(k) then it is true for P(k + 1).

Q50. Find x, y, z if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$.

Sol. Given that: $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ and $A' = A^{-1}$

Pre-multiplying both sides by A we get,

$$AA' = AA^{-1}$$

$$\Rightarrow \quad AA' = I$$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0+4y^2+z^2 & 0+2y^2-z^2 & 0-2y^2+z^2 \\ 0+2y^2-z^2 & x^2+y^2+z^2 & x^2-y^2-z^2 \\ 0-2y^2+z^2 & x^2-y^2-z^2 & x^2+y^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equating the corresponding elements, we get,

$$4y^2 + z^2 = 1 \qquad ...(i)$$

$$2y^2 - z^2 = 0 \qquad ...(ii)$$

Adding (*i*) and (*ii*) we get, $6y^2 = 1 \Rightarrow y^2 = \frac{1}{6} \Rightarrow y = \pm\frac{1}{\sqrt{6}}$

From eqn. (*i*), we get,

$$4y^2 + z^2 = 1$$

$$\Rightarrow 4\left(\frac{1}{\sqrt{6}}\right)^2 + z^2 = 1 \Rightarrow \frac{2}{3} + z^2 = 1 \quad \Rightarrow z^2 = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\therefore \qquad z = \pm\frac{1}{\sqrt{3}}$$

$$x^2 + y^2 + z^2 = 1 \qquad ...(iii)$$

$$x^2 - y^2 - z^2 = 0 \qquad ...(iv)$$

Adding (*iii*) and (*iv*) we get,

$$2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \quad \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

Hence, $x = \pm\frac{1}{\sqrt{2}}$, $y = \pm\frac{1}{\sqrt{6}}$ and $z = \pm\frac{1}{\sqrt{3}}$.

Q51. If possible, using elementary row transformation, find the inverse of the following matrices.

(i) $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Sol. (i) Here, $A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$ for elementary row transformation we put

$$A = IA$$

$$\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 4 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A$$

$R_1 \to R_1 + R_2$

$$\begin{bmatrix} -1 & 1 & 7 \\ -3 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 3R_1$

$$\begin{bmatrix} -1 & 1 & 7 \\ 0 & -1 & -17 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ -5 & -2 & 0 \\ -1 & -1 & 1 \end{bmatrix} A$$

$R_1 \to R_1 + R_2$ and $R_3 \to -1 . R_3$

$$\begin{bmatrix} -1 & 0 & -10 \\ 0 & -1 & -17 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -1 & 0 \\ -5 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix} A$$

$R_1 \to R_1 + 10R_3$ and $R_2 \to R_2 + 17R_3$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ 1 & 1 & -1 \end{bmatrix} A$$

$R_1 \to -1.R_1$ and $R_2 \to -1.R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix} A$$

Hence, $$A^{-1} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$$

(*ii*) Here, $$A = \begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

Put $A = IA$

$$\begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 - 2R_3$ and $R_2 \to R_2 + R_1$

$$\begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 + R_2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A$$

First row on L.H.S. contains all zeros, so the inverse of the given matrix A does not exist.
Hence, matrix A has no inverse.

(*iii*) Here, $$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Put $$A = IA$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \to 3R_1 - R_2$

$$\begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 5R_1$

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 6 & 15 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 5R_3$

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 15 & -6 & 6 \end{bmatrix} A$$

$R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 15 & -6 & 6 \end{bmatrix} A$$

$R_3 \to \frac{1}{3}R_3$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_1 \to R_1 + 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Q52. Express the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$ as the sum of symmetric and a skew symmetric matrix.

Sol. We know that any square matrix can be expressed as the sum of symmetric and skew symmetric matrix *i.e.*

$A = \frac{1}{2}[A + A'] + \frac{1}{2}[A - A']$.

Let $P = \frac{1}{2}[A + A']$ and $Q = \frac{1}{2}[A - A']$

So $$P = \frac{1}{2}\left[\begin{pmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 1 & 4 \\ 3 & -1 & 1 \\ 1 & 2 & 2 \end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{bmatrix} 2+2 & 3+1 & 1+4 \\ 1+3 & -1-1 & 2+1 \\ 4+1 & 1+2 & 2+2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 & 4 & 5 \\ 4 & -2 & 3 \\ 5 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 5/2 \\ 2 & -1 & 3/2 \\ 5/2 & 3/2 & 2 \end{bmatrix}$$

$$P' = \begin{bmatrix} 2 & 2 & 5/2 \\ 2 & -1 & 3/2 \\ 5/2 & 3/2 & 2 \end{bmatrix} = P$$

As $P' = P$ $\therefore$ P is a symmetric matrix.

Now $Q = \frac{1}{2}[A - A']$

$$= \frac{1}{2}\left[\begin{pmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{pmatrix} - \begin{pmatrix} 2 & 1 & 4 \\ 3 & -1 & 1 \\ 1 & 2 & 2 \end{pmatrix}\right]$$

$$= \frac{1}{2}\left[\begin{pmatrix} 2-2 & 3-1 & 1-4 \\ 1-3 & -1+1 & 2-1 \\ 4-1 & 1-2 & 2-2 \end{pmatrix}\right] = \frac{1}{2}\left[\begin{pmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{pmatrix}\right]$$

$$= \begin{bmatrix} 0 & 1 & -3/2 \\ -1 & 0 & 1/2 \\ 3/2 & -1/2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & 3/2 \\ 1 & 0 & -1/2 \\ -3/2 & 1/2 & 0 \end{bmatrix} = -Q.$$

As $Q = -Q$ $\therefore$ Q is a skew symmetric matrix.

So $A = P + Q$

$$A = \begin{bmatrix} 2 & 2 & 5/2 \\ 2 & -1 & 3/2 \\ 5/2 & 3/2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & -3/2 \\ -1 & 0 & 1/2 \\ 3/2 & -1/2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & 2+1 & \frac{5}{2}-\frac{3}{2} \\ 2-1 & -1+0 & \frac{3}{2}+\frac{1}{2} \\ \frac{5}{2}+\frac{3}{2} & \frac{3}{2}-\frac{1}{2} & 2+0 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix} = A$$

Hence, the required relation is

$$A = \begin{bmatrix} 2 & 2 & 5/2 \\ 2 & -1 & 3/2 \\ 5/2 & 3/2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & -3/2 \\ -1 & 0 & 1/2 \\ 3/2 & -1/2 & 0 \end{bmatrix}$$

OBJECTIVE TYPE QUESTIONS

Choose the correct answer from the given four options in each of the Exercises 53 to 67.

Q53. The matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a

(*a*) square matrix (*b*) diagonal matrix
(*c*) unit matrix (*d*) None

Sol. Given that $A = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$

Here number of columns and the number of rows are equal *i.e.*, 3. So, A is a square matrix.

Hence, the correct option is (*a*).

Q54. Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is

(*a*) 9 (*b*) 27 (*c*) 81 (*d*) 512

Sol. Total number of possible matrices of order 3 × 3 with each entry 0 or $2 = 2^{3\times3} = 2^9 = 512$.

Hence, the correct option is (*d*).

Q55. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of x and y is

(*a*) $x=3, y=1$ (*b*) $x=2, y=3$

(*c*) $x=2, y=4$ (*d*) $x=3, y=3$

Sol. Given that: $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$

Equating the corresponding elements, we get,

$$2x + y = 7 \quad \text{...(i)}$$

and $$4x = x + 6 \quad \text{...(ii)}$$

from eqn. (*ii*) $4x - x = 6$

$3x = 6$

$\therefore$ $x = 2$

from eqn. (*i*) $2 \times 2 + y = 7$

$4 + y = 7 \quad \therefore y = 7 - 4 = 3$

Hence, the correct option is (*b*).

Q56. If $$A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$$

$$B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$$

then A – B is equal to

(*a*) I (*b*) O (*c*) 2I (*d*) $\frac{1}{2}$I

Sol. Given that: $$A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$$

and $$B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$$

$$A - B = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix} - \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$$

$$= \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) + \cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) - \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) - \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) + \tan^{-1}(\pi x) \end{bmatrix}$$

$$= \frac{1}{\pi}\begin{bmatrix} \frac{\pi}{2} & 0 \\ 0 & \frac{\pi}{2} \end{bmatrix} \qquad \begin{bmatrix} \because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \\ \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \end{bmatrix}$$

$$= \frac{1}{\pi} \times \frac{\pi}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2} I$$

Hence, the correct option is (*d*).

Q57. If A and B are two matrices of the order $3 \times m$ and $3 \times n$, respectively and $m = n$, then the order of matrix $(5A - 2B)$ is

(*a*) $m \times 3$ (*b*) 3×3 (*c*) $m \times n$ (*d*) $3 \times n$

Sol. As we know that the addition and subtraction of two matrices is only possible when they have same order. It is also given that $m = n$.

$\therefore$ Order of $(5A - 2B)$ is $3 \times n$

Hence, the correct option is (*d*).

Q58. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to

(*a*) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (*b*) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (*c*) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (*d*) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Sol. Given that $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence, the correct option is (*d*).

Q59. If matrix $A = [a_{ij}]_{2 \times 2}$, where $a_{ij} = 1$ if $i \neq j$
$= 0$ if $i = j$

then A^2 is equal to

(*a*) I (*b*) A (*c*) 0 (*d*) None of these

Sol. Given that: $A = [a_{ij}]_{2 \times 2}$

Let $$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2 \times 2}$$

$$a_{11} = 0 \quad [\because i = j]$$
$$a_{12} = 1 \quad [\because i \neq j]$$
$$a_{21} = 1 \quad [\because i \neq j]$$
$$a_{22} = 0 \quad [\because i = j]$$

$\therefore$ $$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Now, $$A^2 = A \,.\, A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence, the correct option is (*a*).

Q60. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a

(*a*) Identity matrix (*b*) symmetric matrix

(*c*) skew symmetric matrix (*d*) none of these

Sol. Let $$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} = A$$

$A' = A$, so A is a symmetric matrix.

Hence, the correct option is (*b*).

Q61. The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a

(*a*) diagonal matrix (*b*) symmetric matrix

(*c*) skew symmetric matrix (*d*) scalar matrix

Sol. Let $$A = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & 5 & -8 \\ -5 & 0 & -12 \\ 8 & 12 & 0 \end{bmatrix}$$

$$\Rightarrow \quad A' = -\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix} = -A$$

$A' = -A$, so A is a skew symmetric matrix.
Hence, the correct option is (*c*).

Q62. If A is a matrix of order $m \times n$ and B is a matrix such that AB′ and B′A are both defined, then order of B is

(*a*) $m \times m$ (*b*) $n \times n$ (*c*) $n \times m$ (*d*) $m \times n$

Sol. Order of matrix $A = m \times n$
Let order of matrix B be $K \times P$
Order of matrix $B' = P \times K$
If AB′ is defined then the order of AB′ is $m \times K$ if $n = P$
If B′A is defined then order of B′A is $P \times n$ when $K = m$

Now, order of $B' = P \times K$

$\therefore$ Order of $B = K \times P$

$= m \times n$ $[\because K = m, P = n]$

Hence, the correct option is (*d*).

Q63. If A and B are matrices of same order, then $(AB' - BA')$ is a

(*a*) skew symmetric matrix (*b*) null matrix
(*c*) symmetric matrix (*d*) unit matrix

Sol. Let $P = (AB' - BA')$

$P' = (AB' - BA')'$
$= (AB')' - (BA')'$
$= (B')'A' - (A')'B'$ $[\because (AB)' = B'A']$
$= BA' - AB'$
$= -(AB' - BA') = -P$

$P' = -P$, so it is a skew symmetric matrix.

Hence, the correct option is (*a*).

Q64. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

(*a*) A (*b*) $I - A$ (*c*) $I + A$ (*d*) 3A

Sol. $(A-I)^3+(A+I)^3-7A = A^3-I^3-3A^2I+3AI^2+A^3+I^3+3A^2I$
$+3AI^2-7A$
$= 2A^3+6AI^2-7A$
$= 2A.A^2+6AI-7A$
$= 2AI+6AI-7A \quad [A^2=I]$
$= 8AI-7A = 8A-7A$
$= A$

Hence, the correct option is (*a*).

65. For any two matrices A and B, we have

(*a*) $AB = BA$ (*b*) $AB \neq BA$
(*c*) $AB = O$ (*d*) None of the above

Sol. We know that for any two matrices A and B, we may have $AB = BA$, $AB \neq BA$ and $AB = 0$, but it is not always true.
Hence, the correct option is (*d*).

Q66. On using elementary column operation $C_2 \to C_2 - 2C_1$, in the following matrix equation

$\begin{bmatrix}1 & -3\\ 2 & 4\end{bmatrix} = \begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 1\\ 2 & 4\end{bmatrix}$, we have:

(*a*) $\begin{bmatrix}1 & -5\\ 0 & 4\end{bmatrix} = \begin{bmatrix}1 & -1\\ -2 & 2\end{bmatrix}\begin{bmatrix}3 & -5\\ 2 & 0\end{bmatrix}$

(*b*) $\begin{bmatrix}1 & -5\\ 0 & 4\end{bmatrix} = \begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & -5\\ -0 & 2\end{bmatrix}$

(*c*) $\begin{bmatrix}1 & -5\\ 2 & 0\end{bmatrix} = \begin{bmatrix}1 & -3\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 1\\ -2 & 4\end{bmatrix}$

(*d*) $\begin{bmatrix}1 & -5\\ 2 & 0\end{bmatrix} = \begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & -5\\ 2 & 0\end{bmatrix}$

Sol. Given that: $\begin{bmatrix}1 & -3\\ 2 & 4\end{bmatrix} = \begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 1\\ 2 & 4\end{bmatrix}$

Using $C_2 \to C_2 - 2C_1$, we get

$$\begin{bmatrix}1 & -5\\ 2 & 0\end{bmatrix} = \begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & -5\\ 2 & 0\end{bmatrix}$$

Hence, the correct option is (*d*).

Q67. On using elementary row operation $R_1 \to R_1 - 3R_2$ in the following matrix equation

$$\begin{bmatrix}4 & 2\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix},$$ we have:

(*a*) $\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & -7\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

(*b*) $\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}-1 & -3\\1 & 1\end{bmatrix}$

(*c*) $\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\1 & -7\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

(*d*) $\begin{bmatrix}4 & 2\\-5 & -7\end{bmatrix} = \begin{bmatrix}1 & 2\\-3 & -3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

Sol. We have, $\begin{bmatrix}4 & 2\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

Using elementary row transformation $R_1 \to R_1 - 3R_2$,

$$\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & -7\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$$

Hence, the correct option is (*a*).

Fill in the Blanks in Each of the Exercises 68-81.

Q68. .... matrix is both symmetric and skew symmetric matrix.

Sol. Null matrix *i.e.* $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ or $\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$ is both symmetric and skew symmetric matrix.

Q69. Sum of two skew symmetric matrices is always .... matrix.

Sol. Let A and B be any two matrices

$\therefore$ For skew symmetric matrices

$$A = -A' \quad ...(i)$$

and $$B = -B' \quad ...(ii)$$

Adding (*i*) and (*ii*) we get

$$A + B = -A' - B'$$

$\Rightarrow$ $A + B = -(A' + B')$, so $A + B$ is skew symmetric matrix.

Hence, the sum of two skew symmetric matrices is always **skew symmetric matrix.**

Q70. The negative of a matrix is obtained by multiplying it by ....

Sol. Let A be a matrix

$\therefore \quad -A = -1 \, . \, A$

Hence, negative of a matrix is obtained by multiplying it by – 1.

Q71. The protuct of any matrix by the scalar .... is the null matrix.

Sol. Let A be any matrix

$\therefore \quad 0 \, . \, A = A \, . \, 0 = 0$

Hence, the product of any matrix by the scalar **0** is the null matrix.

Q72. A matrix which is not a square matrix is called a .... matrix.

Sol. A matrix which is not a square matrix is called a **rectangular matrix.**

Q73. Matrix multiplication is .... over addition.

Sol. Matrix multiplication is **distributive** over addition. Let A, B and C be any matrices.

So, (*i*) $A(B+C) = AB + AC$

(*ii*) $(A+B)C = AC + BC$

Q74. If A is a symmetric matrix, then A^3 is a .... matrix.

Sol. Let A be a symmetric matrix

$\therefore \quad A' = A$

$(A^3)' = (A')^3 = A^3 \quad [\because (A')^k = (A^k)']$

Hence, if A is a symmetric matrix, then A^3 is a **symmetric matrix.**

Q75. If A is a skew symmetric matrix, then A^2 is a ....

Sol. If A is a skew symmetric matrix,

$\therefore \quad A' = -A$

$(A^2)' = (A')^2$

$= (-A)^2$

$= A^2$

Hence, A^2 is a **symmetric matrix.**

Q76. If A and B are square matrices of the same order then

(*i*) $(AB)' =$ ....

(*ii*) $(kA)' =$ .... (*k* is any scalar quantity)

(*iii*) $[k(A-B)]' =$ ....

Sol. (*i*) $(AB)' = B'A'$

(*ii*) $(kA)' = k \, . \, A'$

(*iii*) $[k(A-B)]' = k(A-B)' = k(A'-B')$

Q77. If A is a skew symmetric, then kA is a .... (k is any scalar)

Sol. If A is a skew symmetric matrix

$$\therefore \quad A' = -A$$

$$(kA)' = kA' = k(-A) = -kA$$

Hence, kA is a **skew symmetric** matrix.

Q78. If A and B are symmetric matrices, then

(*i*) AB – BA is a ....

(*ii*) BA – 2AB is a ....

Sol. (*i*) Let

$$P = (AB - BA)$$
$$P' = (AB - BA)'$$
$$= (AB)' - (BA)'$$
$$= B'A' - A'B' \qquad [\because (AB)' = B'A']$$
$$= BA - AB \qquad [\because A' = A \text{ and } B' = B]$$
$$= -(AB - BA)$$
$$= -P$$

Hence, (AB – BA) is a **skew symmetric matrix.**

(*ii*) Let

$$Q = (BA - 2AB)$$
$$Q' = (BA - 2AB)'$$
$$= (BA)' - (2AB)'$$
$$= A'B' - 2(AB)' \qquad [\because (kA)' = kA']$$
$$= A'B' - 2B'A'$$
$$= AB - 2BA \qquad [\because A' = A \text{ and } B' = B]$$
$$= -(2BA - AB)$$

Hence, (BA – 2AB) is neither a symmetric nor a skew symmetric matrix.

Q79. If A is a symmetric matrix, then B′AB is ....

Sol. If A is a symmetric matrix

$$\therefore \quad A' = A$$

Let

$$P = B'AB$$
$$P' = (B'AB)'$$
$$= B'A'(B')' \qquad [\because (AB)' = B'A']$$
$$= B'AB \qquad [\because A' = A \text{ and } (B')' = B]$$
$$\therefore \quad P' = P$$

So, P is a symmetric matrix.

Hence, B′AB is a symmetric matrix.

Q80. If A and B are symmetric matrices of same order, then AB is symmetric if and only if ....

Sol. Given that

$$A' = A$$

and

$$B' = B$$

Let $P = AB$

$$P' = (AB)'$$
$$= B'A'$$
$$P' = BA \qquad [\because A' = A \text{ and } B' = B]$$
$$= P$$

Hence, AB is symmetric if and only if **AB = BA**.

Q81. In applying one or more row operations while finding A^{-1} by elementary row operations, we obtain all zeros in one or more, then A^{-1} ....

Sol. A^{-1} does not exist if we apply one or more row operations while finding A^{-1} by elementary row operations, obtain all zeroes in one or more rows.

State (Exercises 82 to 101) which of the following statements are True or False

Q82. A matrix denotes a number.

Sol. False.

A matrix is an array of elements, numbers or functions having rows and columns.

Q83. Matrices of any order can be added.

Sol. False.

The matrices having same order can only be added.

Q84. Two matrices are equal if they have same number of rows and same number of columns.

Sol. False.

The two matrices are said to be equal if their corresponding elements are same.

Q85. Matrices of different orders can not be subtracted.

Sol. True.

For addition and subtraction, the order of the two matrices should be same.

Q86. Matrix addition is associative as well as commutative.

Sol. True.

If A, B and C are the matrices of addition then

$$A + (B + C) = (A + B) + C \qquad \text{(associative)}$$
$$A + B = B + A \qquad \text{(commutative)}$$

Q87. Matrix multiplication is commutative.

Sol. False.

Since $AB \neq BA$ if AB and BA are well defined.

Q88. A square matrix where every element is unity is called an identity matrix.

Sol. False.

Since, in identity matrix all the elements of principal diagonal are unity rest are zero.

e.g., $$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Q89. If A and B are two square matrices of the same order, then $A + B = B + A$.

Sol. True.

If A and B are square matrices then their addition is commutative *i.e.*, $A + B = B + A$.

Q90. If A and B are two matrices of the same order, then $A - B = B - A$.

Sol. False.

Since subtraction of any two matrices of the same order is not commutative *i.e.*, $A - B \neq B - A$.

Q91. If matrix $AB = O$, then $A = O$ or $B = O$ or both A and B are null matrices.

Sol. False.

Since for any two non-zero matrices A and B, we may get $AB = 0$.

Q92. Transpose of a column matrix is a column matrix.

Sol. False.

Transpose of a column matrix is a row matrix.

e.g., $A = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}_{3\times 1}$ $\quad \therefore A' = \begin{bmatrix} 2 & 3 & 5 \end{bmatrix}_{1\times 3}$

Q93. If A and B are two square matrices of the same order, then $AB = BA$.

Sol. False.

For two square matrices A and B, $AB = BA$ is not always true.

Q94. If each of the three matrices of the same order are symmetric, then their sum is a symmetric matrix.

Sol. True.

Let A, B and C be three matrices of the same order.

Given that $A' = A$, $B' = B$ and $C' = C$

Let $P = A + B + C$

$\Rightarrow$ $$\begin{aligned} P' &= (A + B + C)' \\ &= A' + B' + C' \\ &= A + B + C \\ &= P \end{aligned}$$

So, $A + B + C$ is also a symmetric matrix.

Q95. If A and B are any two matrices of the same order, then $(AB)' = A'B'$.

Sol. False.

Since $(AB)' = B'A'$.

Q96. If $(AB)' = B'A'$, where A and B are not square matrices, then number of rows in A is equal to number of columns in B and number of columns in A is equal to number of rows in B.

Sol. True.

Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{p \times q}$

AB is defined when $n = P$

$\therefore$ Order of AB $= m \times q$

$\Rightarrow$ Order of $(AB)' = q \times m$

Order of B' is $q \times p$ and order of A' is $n \times m$

$\therefore$ $B'A'$ is defined when $P = n$

and the order of $B'A'$ is $q \times m$

Hence, order of $(AB)'$ = Order of $B'A'$ *i.e.,* $q \times m$.

Q97. If A, B and C are square matrices of same order, then AB = AC always implies that B = C.

Sol. False.

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 0 \\ 3 & 4 \end{bmatrix}$

$$\therefore \quad AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Here $AB = AC = 0$ but $B \neq C$.

Q98. AA' is always a symmetric matrix of any matrix A.

Sol. True.

Let $P = AA'$

$P' = (AA')'$

$= (A')' . A'$ $\quad [(AB)' = B'A']$

$= AA'$

$= P$

So, P is symmetric matrix.

Hence, AA' is always a symmetric matrix.

Q99. If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ then AB and BA are defined and equal.

Sol. False.

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$$

Since AB is defined

$$\therefore \quad AB = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+12-2 & 6+15-1 \\ 2+16+4 & 3+20+2 \end{bmatrix} = \begin{bmatrix} 14 & 20 \\ 22 & 25 \end{bmatrix}$$

BA is also defined.

$$\therefore \quad BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+12 & -2+6 \\ 8+5 & 12+20 & -4+10 \\ 4+1 & 6+4 & -2+2 \end{bmatrix} = \begin{bmatrix} 7 & 18 & 4 \\ 13 & 32 & 6 \\ 5 & 10 & 0 \end{bmatrix}$$

So $\quad AB \neq BA$

Q100. If A is a skew symmetric matrix, then A^2 is a symmetric matrix.

Sol. True.

$$(A^2)' = (A')^2$$
$$= [-A]^2 \qquad [\because A' = -A]$$
$$= A^2$$

So, A^2 is a symmetric matrix.

Q101. $(AB)^{-1} = A^{-1}B^{-1}$ where A and B are invertible matrices satisfying commutative property with respective to multiplication.

Sol. True.

If A and B are invertible matrices of the same order

$\therefore \quad (AB)^{-1} = (BA)^{-1} \qquad [\because AB = BA]$

But $\quad (AB)^{-1} = A^{-1}B^{-1} \qquad$ [Given]

$\therefore \quad (BA)^{-1} = B^{-1}A^{-1}$

So $\quad A^{-1}B^{-1} = B^{-1}A^{-1}$

$\therefore$ A and B satisfy commutative property w.r.t. multiplication.

□□□