# 7 Alternating Current

## MULTIPLE CHOICE QUESTIONS—I

**Q7.1.** If the rms current in 50 Hz a.c. circuit is 5 A, the value of current 1/300 seconds after its value becomes zero is

(*a*) $5\sqrt{2}$ A (*b*) $5\sqrt{\frac{3}{2}}$ A (*c*) $\frac{5}{6}$ A (*d*) $\frac{5}{\sqrt{2}}$ A

**Main concept used:** Relations $I = I_0 \sin \omega t$, $\omega = 2\pi\nu$, $I_{rms} = I = I_0/\sqrt{2}$

**Ans.** (*b*): $I = 5$ A, $\nu = 50$ Hz, $t = \frac{1}{300}$ s

$$I_{rms} = I = \sqrt{2} I_0 = 5\sqrt{2} \text{ A}$$

$$I = I_0 \sin \omega t$$

at $t = \frac{1}{300}$ sec

$$I = 5\sqrt{2} \sin 2\pi\nu . t = 5\sqrt{2} \sin 2\pi \times 50 \times \frac{1}{300}$$

$$= 5\sqrt{2} \sin \frac{\pi}{3} = 5\sqrt{2} \sin 60°$$

$$= 5\sqrt{2}\,\frac{\sqrt{3}}{2} = 5\sqrt{\frac{3}{2}} \text{ A}$$

**Q7.2.** An alternating current generator has an internal resistance $R_g$ and an internal reactance $X_g$. It is used to supply power to a passive load consisting of a resistance $R_g$ and reactance $X_L$. For maximum power to be delivered from the generator to load the value of $X_L$ is equal to

(*a*) zero (*b*) $X_g$ (*c*) $-X_g$ (*d*) $R_g$

**Main concept used:** To delivering maximum power from generator its total reactance (internal and external) must be zero and resistance must be equal.

**Ans.** (*c*): As internal resistance of generator is already equal to external resistance $R_g$. So to deliver maximum power to make reactance equal to zero, the reactance in external circuit will be $-X_g$.

**Q7.3.** When a voltage measuring device is connected to AC mains, the meter shows the steady input voltage of 220 V. This means

(*a*) input voltage cannot be AC voltage, but a DC voltage.

(*b*) maximum input voltage is 220 V.

(*c*) the meter reads not $v$ but $\langle v^2 \rangle$ and is calibrated to read $\sqrt{\langle v^2 \rangle}$.

(*d*) the pointer of the meter stuck by some mechanical defect.

**Main concept used:** Voltmeter in AC reads r.m.s. value of voltage

$$\boxed{I_{rms} = \sqrt{2}\, I_0} \quad V_{rms} = \sqrt{2}\, V_0$$

**Ans.** (*c*): The voltmeter in AC circuit reads value $[\langle v^2 \rangle]$ and meter is calibrated to r.m.s. value $\langle v^2 \rangle$ which is multiplied by $\sqrt{2}$ to get $V_{rms}$.

**Q7.4.** To reduce the resonant frequency in an LCR series circuit with a generator

(*a*) the generator frequency should be reduced.
(*b*) another capacitor should be added in parallel to the first.
(*c*) the iron core of the inductor should be removed.
(*d*) dielectric in the capacitor should be removed.

**Main concept used:** Resonant frequency.

**Ans.** (*b*): The resonant frequency of LCR series circuit is

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

So to reduce resonant frequency $\nu_0$, we have either to increase L or to increase C.

To increase capacitance another capacitor must be connected in parallel with the first.

**Q7.5.** Which of the following combinations should be selected for better turning of LCR circuit used for communication?

(*a*) $R = 20\ \Omega, L = 1.5$ H, $C = 35\ \mu F$ (*b*) $R = 25\ \Omega, L = 2.5$ H, $C = 45\ \mu F$
(*c*) $R = 15\ \Omega, L = 3.5$ H, $C = 30\ \mu F$ (*d*) $R = 25\ \Omega, L = 1.5$ H, $C = 45\ \mu F$

**Main concept used:** Quality factor of LCR circuit must be as high as possible.

**Ans.** (*c*): We know that for communication, quality factor must be higher and quality factor is Q.

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

So for higher Q, L must be large, and R and C must be of smaller value.

This condition is satisfied in (*c*) part.

**Q7.6.** An inductor of reactance 1 Ω and resistor of 2 Ω are connected in series to the terminals of 6 V (rms) AC source. The power dissipated in the circuit is

(*a*) 8 W (*b*) 12 W (*c*) 14.4 W (*d*) 18 W

**Main concept used:** Impedance, average power, $I_{rms} = \frac{I}{\sqrt{2}}$ and $\cos\phi = \frac{R}{Z}$ of AC.

**Ans.** (*c*): We know that average power dissipated in L, R series circuit with AC source.

$$P_{av} = E_{rms} I_{rms} \cos\phi$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{E_{rms}}{Z} = \frac{6}{\sqrt{5}}$$

$$Z = \sqrt{X_R^2 + X_L^2}$$

$$Z = \sqrt{2^2 + 1} = \sqrt{5}$$

$$\cos\phi = \frac{R}{Z} = \frac{2}{\sqrt{5}}$$

$$P_{av} = 6 \times \frac{6}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} = \frac{72}{5} = 14.4 \text{ Watt}$$

**Q7.7.** The output of a stepdown transformer is measured to be 24 V, when connected to a 12 W light bulb. The value of the peak current is

(*a*) $\frac{1}{\sqrt{2}}$ A (*b*) $\sqrt{2}$ A (*c*) 2 A (*d*) $2\sqrt{2}$ A

**Main concept used:** $V_s I_s = V_p I_p$

**Ans.** (*a*)

$$V_s = 24 \text{ V}$$

$$P_s = 12 \text{ W}$$

$$I_s V_s = 12$$

$$I_s = \frac{12}{V_s} = \frac{12}{24} = 0.5 \text{ Amp}$$

$$I_0 = I_s\sqrt{2} = 0.5\sqrt{2}$$

$$I_0 = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}} \text{ Amp}$$

## MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

**Q7.8.** As the frequency of an AC circuit increases, the current first increases and then decreases. What combination of circuit elements is most likely to comprise the circuit?

(*a*) Inductor and capacitor (*b*) Resistor and inductor
(*c*) Resistor and capacitor (*d*) Resistor, inductor and capacitor

**Main concept used:** Dependence of reactance on frequency.

**Ans.** (*a*) (*d*): In a circuit $I_{max}$ will be when reactance is minimum. On increasing frequency in LCR circuit the reactance ($X_L = 2\pi\nu L$) due to inductance will increase and due to capacitance $\frac{1}{2\pi\nu C}$ it will decrease. So on increasing frequency $X_L$ will be positive and $X_C$ will be negative for minimum reactance $X_L - X_C$ must be zero. As reactance or impedance (Z) due to LCR series AC circuit is $Z = \sqrt{R^2 + (X_L \sim X_C)^2}$, Z will be minimum when $X_L \sim X_C = 0$ or $X_L = X_C$. So for given condition, $X_L$ and $X_C$ must be in circuit.

**Q7.9.** In an alternating current circuit, consisting of elements in series, the current increases on increasing frequency of supply. Which of the following elements are likely to constitute the circuit?

(*a*) Only resistor
(*b*) Resistor and an inductor
(*c*) Resistor and a capacitor
(*d*) Only a capacitor

**Main concept used:** Reactance decreases and the current increases on increasing ν.

**Ans.** (*c*) and (*d*) Here in question on increasing ν the current also increases. So reactance of circuit decreases as reactance or resistance does not depend on frequency and $X_L = 2\pi\nu L$ and $X_C = \frac{1}{2\pi\nu C}$. So on increasing frequency, $X_C$ decreases to increase current capacitors are in *c* and *d*.

**Q7.10.** Electrical energy is transmitted over large distances at high alternating voltages. Which of the following statements is (are) correct?

(*a*) For a given power level, there is a lower current.
(*b*) Lower current implies less power loss.
(*c*) Transmission lines can be made thinner.
(*d*) It is easy to reduce the voltage at the receiving end using step-down transformers.

**Main concept used:** Power loss = $I^2R$, $V_1I_1 = V_2I_2$ (Transformers)

**Ans.** (*a*) (*b*) and (*d*): As we know that Power = $I_{rms}^2 R$

So to decrease power loss $I_{rms}$ and R must be lower for a constant power supply. To decrease $I_{rms}$, $V_{rms}$ must be increased by step up transformer to get same power in step up transformer

Output power = Input power

$$V_sI_s = V_pI_p$$

$\because$ $V_s > V_p$ so $I_p \gg I_s$ so loss of power during transmission become lower.

We can again reduce voltage by using step down transformer.

**Q7.11.** For an LCR circuit, the power transferred from the driving source to the driven oscillator is $P = I^2Z \cos\phi$.

(*a*) Here, the power factor $\cos\phi \geq 0$, $P \geq 0$.
(*b*) The driving force can give no energy to the oscillator ($P = 0$) in some cases.
(*c*) The driving force cannot syphon out ($P < 0$), the energy out of oscillator.
(*d*) The driving force can take away energy out of the oscillator.

**Main concept used:** $P = I^2Z \cos\phi$

$$\cos\phi = \frac{R}{Z}$$

**Ans.** (*a*) (*b*) and (*c*): As per question $P = I^2Z \cos\phi$

$$\text{Power factor } \cos\phi = \frac{R}{Z}$$

as $R > 0$ and $Z > 0$

So $\cos\phi = \frac{R}{Z}$ is positive $\Rightarrow P > 0$

**Q7.12.** When an ac voltage of 220 Volt is applied to the capacitor C,

(*a*) the maximum voltage between plates is 220 V.

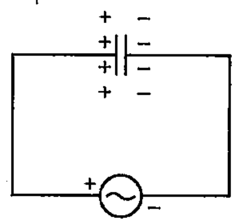
(*b*) the current is in phase with the applied voltage.

(*c*) the charge on the plates is in phase with the applied voltage.

(*d*) power delivered to the capacitor is zero.

**Main concept used:** Power factor for C is zero $\because$ $\phi = 90°$.

**Ans.** (*c*) (*d*) When capacitor is connected to ac supply the plate of capacitor will be at higher potential which is connected to positive terminal than the plate connected to the negative terminal.




Power applied to circuit is

$$P_{av} = V_{rms} I_{rms} \cos\phi$$

$$\phi = 90° \text{ for pure capacitor circuit}$$

$$\therefore \quad P_{av} = 0 \text{ as } \cos 90° = 0$$

**Q7.13.** The line that draws power supply to your house from street has

(*a*) zero average current.

(*b*) 220 V average voltage.

(*c*) voltage and current out of phase by 90°.

(*d*) voltage and current possibly differing in phase $\phi$ such that $|\phi| < \frac{\pi}{2}$.

**Main concept used:** Power factor $\cos\phi = \frac{R}{Z}$

**Ans.** (*a*) (*d*): AC supply are used in houses.

So average current over a cycle of AC is zero. In household circuit L and C are connected, so R and Z cannot be equal.

So, power factor= $\cos\phi = \frac{R}{Z} \neq 0$

as $\phi \neq \frac{\pi}{2} \quad \Rightarrow \quad \phi < \frac{\pi}{2}$

i.e. phase angle between voltage and current lies between 0 and $\frac{\pi}{2}$.

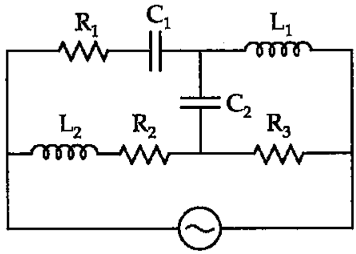
## VERY SHORT ANSWER TYPE QUESTIONS

**Q7.14.** If a L-C circuit is considered analogous to a harmonically oscillating spring block system, which energy of the L-C circuit would be analogous to the potential energy and which one analogous to kinetic energy?

**Main concept used:** P.E. is related to electrostatic energy and K.E. to magnetic energy.

**Ans.** An L-C circuit is analogous to a harmonically oscillating spring block system. The electrostatic energy due to charging of capacitor $\frac{1}{2}CV^2$ is analogous to potential energy and energy due to motion of charge particle i.e., magnetic energy $\frac{1}{2}LI^2$ is analogous to kinetic energy.

**Q7.15.** Draw the effective equivalent circuit of a circuit shown in figure, at very high frequencies and find the effective impedance.



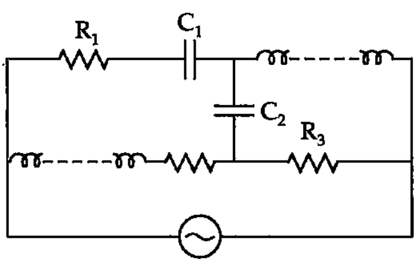

**Main concept used:** Reactance $X_L$ and $X_C$ at high frequencies.

**Ans.** ∵ We know that reactance due to inductance $X_L = 2\pi\nu$. As frequency is high so $X_L$ will be high or L can be considered as open circuit for high frequency of ac.

In similar way, $X_C = \dfrac{1}{2\pi\nu C}$.

At high frequency, $X_C$ will be low.

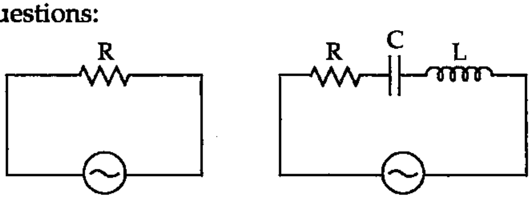
So reactance of capacitance can be considered negligible and capacitors can be considered as closed circuit.




Here the above figure is equivalent circuit to given circuit.

Total impedance = R = $R_1 + R_3$

**Q7.16.** Study the circuits (*a*) and (*b*) shown in figure and answer the following questions:




(*a*) (*b*)

(*a*) Under which conditions would the rms currents in two circuits be the same?

(*b*) Can the rms current in circuit (*b*) be larger than that in (*a*)?

**Ans.** $(I_{rms})$ in (*a*) $= \dfrac{V_{rms}}{R}$

$$(I_{rms}) \text{ in } b = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

(*a*) $(I_{rms})_a = (I_{rms})_b$

$$\frac{V_{rms}}{R} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$\therefore$ $R = \sqrt{R^2 + (X_L - X_C)^2}$

Squaring both sides

$$R^2 = R^2 + (X_L - X_C)^2$$

or $(X_L - X_C)^2 = 0$

$$X_L = X_C$$

So $I_{rms}$ in circuits (*a*) and (*b*) will be equal if $X_L = X_C$.

(*b*) For $(I_{rms})_b > (I_{rms})_a$

$$\frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} > \frac{V_{rms}}{R}$$

as $V_{rms} = V$ so,

$$\sqrt{R^2 + (X_L - X_C)^2} < R$$

Squaring both sides

$$R^2 + (X_L - X_C)^2 < R^2$$

$$(X_L - X_C)^2 < 0$$

Square of any number can never be negative. Reactance of $X_L$ and $X_C$ cannot be negative.

So the rms current in circuit (*b*) cannot be larger than that in (*a*).

**Q7.17.** Can the instantaneous power output of an ac source ever be negative? Can the average power output be negative?

**Ans.** Let the applied e.m.f. = $E = E_0 \sin(\omega t)$

$$I = I_0 \sin(\omega t - \phi)$$

Instantaneous power output of ac source

$$P = EI$$

$$= E_0 \sin \omega t . I_0 \sin(\omega t - \phi)$$

$$= E_0 I_0 \sin \omega t [\sin \omega t \cos \phi - \cos \omega t \sin \phi]$$

$$= E_0 I_0 [\sin^2 \omega t \cos \phi - \sin \omega t \cos \omega t \sin \phi]$$

$$= E_0 I_0 \left[\frac{(1 - \cos 2\omega t)}{2} \cos \phi - \frac{1}{2} \sin 2\omega t \sin \phi\right]$$

$$= \frac{E_0 I_0}{2} [\cos \phi - \cos 2\omega t \cos \phi - \sin 2\omega t \sin \phi]$$

$$= \frac{E_0 I_0}{2} [\cos \phi - (\cos 2\omega t \cos \phi + \sin 2\omega t \sin \phi)]$$

$$P = \frac{E_0 I_0}{2} [\cos \phi - \cos(2\omega t - \phi)]$$

Taken phase angle $\phi$, ±ve.

Instantaneous power $P = \frac{E_0 I_0}{2}[\cos\phi - \cos(2\omega t \pm \phi)]$

as $\cos\phi = \frac{R}{Z}$, R and Z can never be negative and value of $\cos\theta$ ($\theta = 2\omega t \pm \phi$) can vary from (1 to 0 to − 1) in any case P can never be negative.

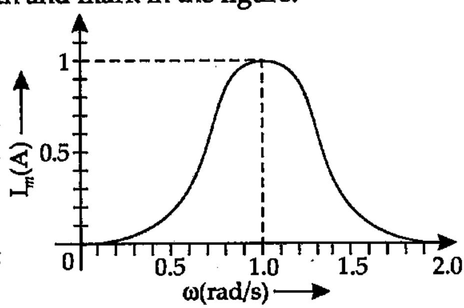
We know that average power of LCR series ac circuit is

$$P_{av} = \frac{E_0 I_0}{2}\cos\phi$$

again as $\cos\phi = \frac{R}{Z}$ is always positive, because R and Z, the reactances are always positive.

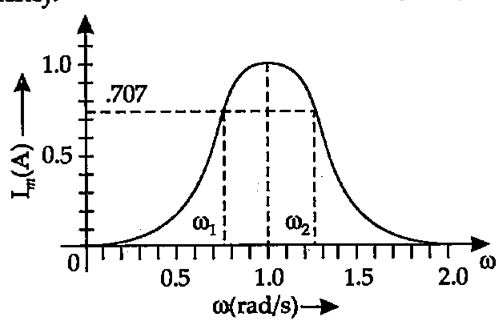
So $P_{av}$ can never be negative.

**Q7.18.** In series LCR circuit, the plot of $I_{max}$, versus $\omega$ is shown in figure. Calculate the bandwidth and mark in the figure.




**Ans.** We know that bandwidth $= (\omega_2 - \omega_1)$.

Where $\omega_1$ and $\omega_2$ are two frequencies where the current amplitude of LCR circuit becomes $\frac{1}{\sqrt{2}}$ times (i.e., $I_{rms}$) the value of current is maximum at resonant frequency.

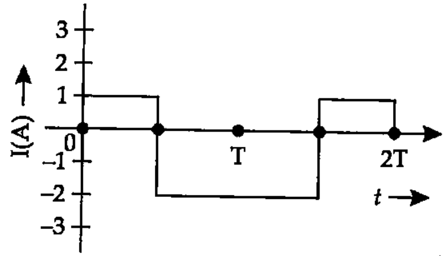



$$\boxed{I = \frac{E_0}{\sqrt{2}}} = \frac{1}{\sqrt{2}} = 0.707 \text{ Amp}$$

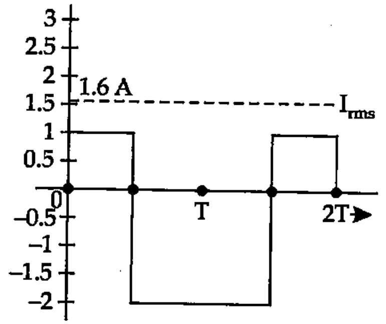
From graph $\omega_1$ and $\omega_2$ at 0.707 Amp current is 0.8 and 1.2 rad/sec.

So Bandwidth $\omega_2 - \omega_1 = 1.2 - 0.8 = 0.4$ rad/sec.

**Q7.19.** The alternating current in the circuit is described by the graph as shown in the figure. Show rms current in this graph.




**Ans.** Graph of rms current is shown below by dotted line




$$I_{rms} = \sqrt{\frac{1^2 + 2^2}{2}} = \sqrt{\frac{5}{2}} \cong 1.6 \text{ A}$$

**Q7.20.** How does the sign of the phase angle $\phi$, by which the supply voltage leads the current in an LCR series circuit, change as the supply frequency is gradually increased from very low to high values.

**Ans.**The phase angle ($\phi$) by which voltage leads the current in LCR series circuit where $XL > X_C$.

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{2\pi\nu L - \frac{1}{2\pi\nu C}}{R}$$

If $\nu$ is small $X_C > X_L$ so $\left[2\pi\nu L - \frac{1}{2\pi\nu C}\right]$ is negative, so $\tan\phi < 0$.

For $\nu$ is large, $X_L > X_C$

So $X_L - X_C$ is positive or $\tan\phi > 0$

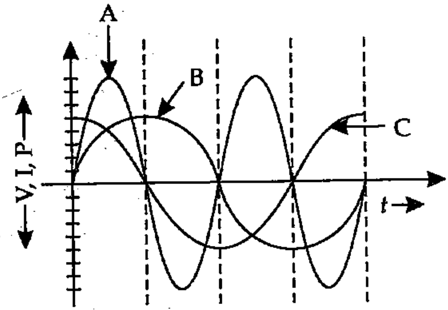
for $X_L = X_C$ *i.e.,* at resonant frequency

$X_L - X_C = 0$ so $\tan\phi = 0$.

So phase angle in series LCR ac circuit will change from a negative to zero and then zero to positive value.

## SHORT ANSWER TYPE QUESTIONS

**Q7.21.** A device X is connected to an AC source. The variation of voltage, current and power in one complete cycle is shown in figure.




(*a*) Which curve shows the power consumption over a full cycle?

(*b*) What is the average power consumption over a cycle?

(*c*) Identify the device X.

**Ans.** (*a*) We know that P = VI, the amplitude of power will be equal to multiplication of V and I so it must be maximum amplitude as compared to V and L. So curve A shows the power consumption over a cycle. The power of a circuit will become zero, when either V = 0 or I = 0 or both become zero which is clear from graph 'A is zero (on dotted lines) at these position.

(*b*) The graph of power is represented by A. The graph 'A' is symmetric with X-axis, *i.e.* area of graph on positive and negative side are equal. So net area of power graph is zero.
So power consumption in circuit is zero, which tallies with that average power of A.C. circuit over a cycle is zero.

(*c*) As the average power of device is zero so power factor $\cos\phi = 0$ i.e., $\phi = 90°$. Change in phase between current and voltage is in either inductor or capacitor. So device may be either capacitor or inductor or the combination of both inductor and capacitor.

**Q7.22.** Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current?

**Ans.** For direct current 'One Ampere' is equal to one coulomb charge flowing per sec.

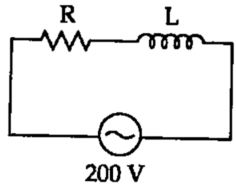
In an AC, current changes it's direction with time equal to the half of time period of AC. So the charge vibrate to and fro with frequency equal to the frequency of AC. So net force acting on the charge is zero.

So the AC current in ampere must be defined in terms of some property which is independent of direction of charge or current.

Joule's heating effect is such a property and hence Joule's heating law is used to define one ampere current in AC. According to Joule's heating effect, one ampere current in AC is the current which can produce same quantity of heat per second as the direct current can produce in one ohm resistance.

**Q7.23.** A coil of 0.01 H inductance and 1 Ω resistance is connected to 200 V, 50 Hz AC supply. Find the impedance of circuit and time lag between maximum alternating voltage and current.

**Ans.** $R = 1\ \Omega$, $L = 0.01$ H, $V = 200$ V, $\nu = 50$ Hz.
Impedance of the circuit




$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi\nu L)^2}$$
$$= \sqrt{(1)^2 + (2 \times 3.14 \times 50 \times 0.01)^2}$$
$$= \sqrt{1 + 9.86} = \sqrt{10.86}$$
$$= 3.3\ \Omega.$$

$\therefore$ For phase angle $\phi$, $\tan\phi = \frac{Z}{R}$

$$\tan\phi = \frac{X_L}{R} = \frac{2\pi\nu L}{R} = \frac{2 \times 3.14 \times 50 \times 0.01}{1}$$
$$\tan\phi = 3.14$$
$$\phi = \tan^{-1} 3.14 = 72°$$
$$\text{Phase difference } \phi = \frac{72 \times \pi}{180°} \text{ rad}$$
$$\phi = 1.20 \text{ radian}$$

Time lag between alternating voltage and current

$$\boxed{\phi = \omega t}$$

$$t = \frac{\phi}{\omega} = \frac{72 \times \pi}{180 \times 2 \times \pi \times 50} = \frac{1}{250} \text{ sec}$$

**Q7.24.** A 60 W load is connected to the secondary of a transformer whose primary draws line voltage. If a current of 0.54 A flows in the load, what is the current in primary coil? Comment on the type of transformer being used.

**Ans.** $P_s = 60$ W, $I_s = 0.54$ A, $I_p = ?$

$$P_s = V_s I_s$$
$$60 = V_s \times 0.54$$
$$\frac{60}{0.54} = V_s$$
$$V_s = 111.10 \text{ Volt}$$

In multiple of 11

$$V_s \cong 110 \text{ Volt}$$

$$\text{Ratio factor of transformer} = \frac{\text{output voltage}}{\text{input voltage}}$$

or $$r = \frac{V_s}{V_p} \Rightarrow r = \frac{110 \text{ Volt}}{220 \text{ Volt}} = \frac{1}{2}$$

or $r < 1$, so transformer is step down transformer.

In transformer, output power = input power

$$I_s V_s = I_p V_p$$

$$I_p = \frac{I_s V_s}{V_p} = \frac{0.54 \times 110}{220} = \textbf{0.27 Ampere}$$

**Q7.25.** Explain why the reactance provided by a capacitor to an alternating current decreases with increasing frequency?

**Ans.** When AC current is applied across a capacitor plate, the plates of the capacitor get charge and discharge alternately. Current through the capacitor is a result of this charging and discharging by AC voltage or current.

A capacitor does not allow a direct current (having zero frequency) to pass through it. But as frequency of current increases capacitor will pass more current through it as on increasing frequency, the charging and discharging happens at fast rate. It implies that the reactance offered by capacitor decreases on increasing frequency.

So the reactance of capacitor can be written as $\boxed{X_C = \frac{1}{\omega C}}$.

**Q7.26.** Explain why the reactance offered by an inductor increases with increasing frequency of an AC voltage?

**Ans.** According to Lenz's law, when current in an inductor change, the direction of induced e.m.f. will oppose the change in current in inductor. The magnetic flux will be in opposite direction with the magnetic flux produced by changing e.m.f. or current in coil and vice-versa.

Since the induced e.m.f. is directly proportional to the rate of change of current, so an inductor produces greater reactance to flow of current through it.

If direct current passes through an inductor the reactance produced by inductor is zero. So on applying an AC current its reactance increases with increasing the frequency of AC.

So the reactance is directly proportional to frequency or reactance of inductor is $X_L = 2\pi\nu L = \omega L$.

## LONG ANSWER TYPE QUESTIONS

**Q7.27.** An electrical device draws 2 kW power from AC mains [voltage 223 V (rms) = $\sqrt{50,000}$ V]. The current differs (lags) in phase by $\phi$ $\left(\tan\phi = \frac{-3}{4}\right)$ as compared to the voltage. Find (*i*) R (*ii*) $(X_C - X_L)$ and (*iii*) $I_M$. Another device has twice the values for R, $X_C$ and $X_L$. How are the answers affected?

**Ans.** $P = 2000$ W  Current lags the voltage so

$V^2 = 50,000$ V  $X_C > X_L$

$$\tan\phi = \frac{-3}{4}$$

$$I_m = I_0$$

$$P = \frac{V^2}{Z}$$

$$2000 = \frac{50{,}000}{Z}$$

$$Z = \frac{50{,}000}{2000} = 25\ \Omega$$

$$\boxed{Z = \sqrt{R^2 + (X_C - X_L)^2}}$$

$$R^2 + (X_C - X_L)^2 = 25^2$$

$$R^2 + (X_C - X_L)^2 = 625 \qquad \text{...(I)}$$

$$\tan\phi = \frac{-3}{4}$$

$$\frac{X_C - X_L}{R} = \frac{-3}{4}$$

$$X_C - X_L = \frac{-3}{4}R$$

$$(X_C - X_L)^2 = \frac{9}{16}R^2 \qquad \text{...(II)}$$

Put the value of $(X_C - X_L)^2$ in (I)

$$R^2 + \frac{9}{16}R^2 = 625$$

$$\frac{25}{16}R^2 = 625$$

$$R^2 = 16 \times 25$$

$$R = 4 \times 5 = 20\ \Omega$$

$$\frac{X_C - X_L}{R} = \frac{-3}{4}$$

$$X_C - X_L = \frac{-3}{4} \times 20$$

$$X_C - X_L = -15\ \Omega$$

$$I_{rms} = \frac{V}{Z} = \frac{223}{25} \cong 9\ A$$

$$I_0 = \sqrt{2}\, I_{rms} = \sqrt{2} \times 9$$

$$I_0 = 12.6\ A$$

As (*i*) R, $X_C$, $X_L$ all are doubled $\tan\phi = \dfrac{X_C - X_L}{R}$ will remain same.

(*ii*) Z will become double then $I = \frac{V}{Z}$ become half as value of V does not change. (*iii*) As I become half P = VI will become again half as voltage remains same.

**Q7.28.** 1 MW power is to be delivered from a power station to a town 10 km away. One uses a pair of Cu wires of radius 0.5 cm for this purpose. Calculate the fraction of ohmic losses to power transmitted if.

(*i*) Power is transmitted at 220 V. Comment on the feasibility of doing this.

(*ii*) A step-up transformer is used to boost the voltage to 11000 Volt, power transmitted, then a step-down transformer is used to bring voltage to 220 V. ($\rho_{Cu} = 1.7 \times 10^{-8}\ \Omega\text{-m}$)

**Ans.** (*i*) When power is transmitted at 220 V.

Power lost in transmission $\boxed{P = I^2R}$

$$\boxed{P = VI}$$

$$P = 1\text{ MW} = 10^6\text{ W}$$

$$V = 220\text{ Volt}$$

$$\therefore \quad I = \frac{P}{V} = \frac{1000000}{220}$$

$$I = \frac{50000}{11}\text{ Amp}$$

$$\boxed{R = \rho\frac{l}{A}} \Rightarrow R = \frac{\rho l}{\pi r^2}$$

$$l = 10\text{ km} \times 2 = 20{,}000\text{ m}$$

$$\therefore \quad A = \pi r^2$$

$$r = 0.5\text{ cm} = 0.5 \times 10^{-2} = 5 \times 10^{-3}\text{ m}$$

$$\rho_{Cu} = 1.7 \times 10^{-8}\ \Omega\text{-m}$$

$$R = \frac{\rho l}{A}$$

$$\therefore \quad R = \frac{1.7 \times 10^{-8} \times 20{,}000}{3.14 \times 5 \times 10^{-3} \times 5 \times 10^{-3}}$$

$$= \frac{170 \times 20000 \times 10^{-8+6}}{314 \times 5 \times 5} = \frac{170 \times 20000}{314 \times 25 \times 100}\ \Omega$$

$$R = \frac{170 \times 800}{314 \times 100} = \frac{170 \times 4}{157} = \frac{680}{157} \cong 4\ \Omega$$

$$\therefore \quad \text{Power loss} = I^2R$$

$$= \frac{50000}{11} \times \frac{50000}{11} \times 4 = \frac{100 \times 10^8}{121} = 8.26 \times 10^7$$

Power loss in heating = **82.6 MW**

as 82.6 MW > 1 MW

So this method cannot be used to transmit the power.

(*ii*) When power is transmitted at 11000 V

$$P = 10^6 \text{ W}$$
$$VI = 1000000$$
$$11000\, I = 1000000$$
$$I = \frac{1000000}{11000} = \frac{1000}{11}$$

$\therefore$ $R_{Cu} = 4\,\Omega$ [as already calculated in part (*i*)]

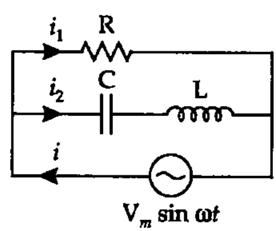
$\therefore$ Power loss = $P = I^2R$

$$P = \frac{1000}{11} \times \frac{1000}{11} \times 4 = \frac{4000}{121} \times 10^4$$
$$P = 3.3 \times 10^4 \text{ Watt}$$

Fractional power loss = $\dfrac{3.3 \times 10^4}{10^6} = \dfrac{3.3}{1000} = 0.033$

Power loss in % = 3.3%

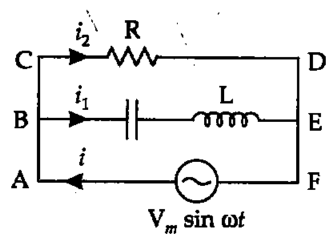
**Q7.29.** Consider the LCR circuit shown in figure. Find the net current *i* and the phase of *i*. Show that $i = \dfrac{V}{Z}$. Find the impedance Z for this circuit.




**Main concept used:** Kirchhoff's law.

**Ans.** Total current *i* from the source $V_m \sin \omega t$ is divided at B in two parts, $i_1$ through capacitor and inductor and part $i_2$ through resistance.

Potential across R = Potential of source




P.D. across R = $V_m \sin \omega t$

$$i_2 R = V_m \sin \omega t$$

$$\boxed{i_2 = \frac{V_m \sin \omega t}{R}} \quad \text{...I}$$

$q_1$ is charge on the capacitor at any time *t*, then for series combination of C, L. applying Kirchhoff's voltage law in loop ABEFA.

$$V_C + V_L = V_m \sin \omega t$$
$$\frac{q_1}{C} + L\frac{di_1}{dt} = V_m \sin \omega t$$
$$\frac{q_1}{C} + L\frac{d^2q_1}{dt^2} = V_m \sin \omega t \quad \text{...II}$$

Let
$$q_1 = q_m \sin(\omega t + \phi) \quad \text{...III}$$
$$i_1 = \frac{dq_1}{dt} = q_m \omega \cos(\omega t + \phi) \quad \text{...IV}$$

$$\frac{d^2 q_1}{dt^2} = -q_m \omega^2 \sin(\omega t + \phi) \quad ...V$$

Substitute the values of equations III and V in equation II

$$\frac{q_m \sin(\omega t + \phi)}{C} - L q_m \omega^2 \sin(\omega t + \phi) = V_m \sin \omega t$$

$$q_m \sin(\omega t + \phi)\left[\frac{1}{C} - L\omega^2\right] = V_m \sin \omega t$$

at $\phi = 0$,

$$q_m \sin \omega t \left[\frac{1}{C} - L\omega^2\right] = V_m \sin \omega t$$

$$q_m \left[\frac{1}{C} - L\omega^2\right] \sin \omega t = V_m \sin \omega t$$

$$q_m \left[\frac{1}{C} - L\omega^2\right] = V_m$$

$$q_m = \frac{V_m}{\omega\left[\frac{1}{C\omega} - L\omega\right]} \quad ...(VI)$$

Applying Kirchhoff's junction rule at junction B, $i = i_2 + i_1$ using relation I, IV

$$i = \frac{V_m \sin \omega t}{R} + q_m \omega \cos(\omega t + \phi)$$

Now using relation VI for $q_m$ and at $\phi = 0$

$$i = \left[\frac{V_m \sin \omega t}{R} + \frac{V_m \omega \cos \omega t}{\omega\left[\frac{1}{\omega C} - \omega L\right]}\right]$$

$$i = \frac{V_m}{R} \sin \omega t + \frac{V_m}{\left(\frac{1}{\omega C} - \omega L\right)} \cos \omega t$$

Let

$$A = \frac{V_m}{R} = C \cos \phi \quad ...(VII)$$

$$B = \frac{V_m}{\frac{1}{\omega C} - \omega L} = C \sin \phi \quad ...(VIII)$$

$$i = C \cos \phi \sin \omega t + C \sin \phi . \cos \omega t$$

$$= C[\cos \phi \sin \omega t + \sin \phi \cos \omega t]$$

$$i = C \sin(\omega t + \phi)$$

Squaring and adding (VII), (VIII)

$$A^2 + B^2 = C^2 \cos^2 \phi + C^2 \sin^2 \phi$$

$$= C^2[\cos^2 \phi + \sin^2 \phi)$$

$$A^2 + B^2 = C^2$$

or

$$C = \sqrt{A^2 + B^2}$$

$$\phi = \tan^{-1}\frac{B}{A} = \tan^{-1}\frac{\dfrac{V_m}{\frac{1}{\omega C} - \omega L}}{\dfrac{V_m}{R}}$$

$\therefore$
$$\tan\phi = \frac{R}{\left(\frac{1}{\omega C} - \omega L\right)}$$

$\because$
$$C^2 = A^2 + B^2 = \frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\frac{1}{\omega C} - \omega L\right)^2}$$

$$C = \left[\frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\frac{1}{\omega C} - \omega L\right)^2}\right]^{\frac{1}{2}}$$

$\therefore$
$$i = \left[\frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\frac{1}{\omega C} - \omega L\right)^2}\right]^{\frac{1}{2}} \sin(\omega t + \phi)$$

$$i = V_m\left[\frac{1}{R^2} + \frac{1}{\left(\frac{1}{\omega C} - \omega L\right)^2}\right]^{\frac{1}{2}} \sin(\omega t + \phi) \quad ...(IX)$$

and
$$\phi = \tan^{-1}\frac{R}{\left(\frac{1}{\omega C} - \omega L\right)}$$

$\because$
$$I = \frac{V}{R} \quad \text{or} \quad i = \frac{V}{Z}$$

for ac
$$i = \frac{V}{Z}\sin(\omega t + \phi) \quad ...(X)$$

Comparing (IX) and (X)

So
$$\frac{1}{Z} = \left[\frac{1}{R^2} + \frac{1}{\left(\frac{1}{\omega C} - \omega L\right)^2}\right]^{\frac{1}{2}}$$

This is the impedance Z for the circuit.

**Q7.30.** For LCR circuit driven at frequency $\omega$, the equation reads,

$$L\frac{di}{dt} + Ri + \frac{q}{C} = V_i = V_m \sin \omega t.$$

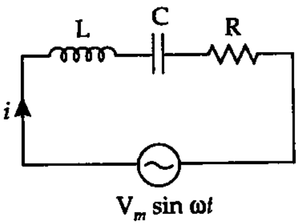
(*a*) Multiply the equation by $i$ and simplify where possible.

(*b*) Interpret each term physically.

(*c*) Cast the equation in the form of conservation energy statement.

(*d*) Integrate the equation over one cycle to find that the phase difference between V and $i$ must be acute.

**Main concept used:** (*i*) Voltage Kirchhoff's law (*ii*) Loss of energy through resistor to know net loss of energy.




**Ans.** Consider L-C-R series circuit with AC supply

$$V = V_m \sin \omega t$$

Applying voltage Kirchhoff's law over the circuit

$$\therefore \quad V_L + V_C + V_R = V_m \sin \omega t$$

$$L\frac{di}{dt} + \frac{q}{C} + iR = V_m \sin \omega t$$

(*a*) Multiply the above equation by $i$ on both the sides

$$Li\frac{di}{dt} + \frac{iq}{C} + i^2R = V_m\, i \sin \omega t$$

Multiply above equation by $\frac{1}{2}$ on both sides

$$\frac{1}{2}Li\frac{di}{dt} + i\frac{q}{2C} + \frac{i^2R}{2} = \frac{1}{2}V_m i \sin \omega t \qquad \left(\because i = \frac{dq}{dt}\right)$$

$$\frac{d\left(\frac{1}{2}Li^2\right)}{dt} + \frac{1}{2C}\frac{dq^2}{dt} + \frac{i^2R}{2} = \frac{1}{2}V_m i \sin \omega t \qquad \text{...(I)}$$

(*b*) (*i*) $\frac{d\left(\frac{1}{2}Li^2\right)}{dt}$ represents the rate of change of potential energy in inductance L.

(*ii*) $\boxed{\frac{d}{dt}\frac{q^2}{2C}}$ represents energy stored in $dt$ time in the capacitor.

(*iii*) $i^2R$ represents Joules heating loss.

(*iv*) $\frac{1}{2}V_m i \sin \omega t$ is the rate at which driving force pours in energy. It goes into ohmic loss and increase of stored energy in capacitor and inductor.

(*c*) Here equation (I) is in the form of conservation of energy statement.

(*d*) Integrating eqn. (I) both sides with respect to $dt$ over a cycle we get

$$\int_0^T \frac{d\left(-\ i\right)}{dt}dt + \int_0^T \frac{dq}{dt\,2C}dt + \int_0^T \frac{-}{2}i \quad dt = \frac{1}{2}\int_0^T V_m i \sin \omega t\, dt$$

$$\int_0^T \frac{d}{dt}\left[\frac{1}{2}Li^2 + \frac{q^2}{2C}\right]dt + \frac{1}{2}\int_0^T i^2 R\, dt = \frac{1}{2}\int_0^T Vi\, dt \qquad [\because V = V_m \sin \omega t]$$

$$0 + \frac{1}{2}i^2 RT = \frac{1}{2}\int_0^T Vi\, dt$$

$$i^2 RT = \int_0^T Vi\, dt$$

as $i^2$ RT is +ve [$\because$ $i^2$, R and T can never be negative]

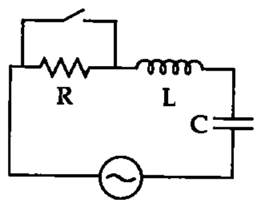
So, $\int_0^T Vi\, dt$ is positive which is only possible if phase difference $\phi$ is constant and the angle is acute.

**Q7.31.** In the LCR circuit shown in the figure below, the AC driving voltage is $V = V_m \sin \omega t$.

(*a*) Write down the equation of motion for $q(t)$.

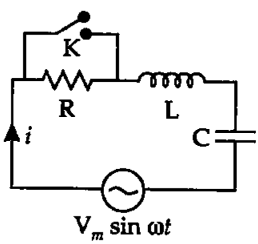
(*b*) at $t = t_0$, the voltage source stop and R is short circuited. Now write down how much energy is stored in each of L and C.

(*c*) Describe subsequent motion of charges.




**Main concept used:** Kirchhoff's voltage law, and get relations after differentiating either one or two with respect to time.

**Ans.** (*a*) Consider series LCR circuit and tapping key K to short circuit R. Let $i$ be the current in circuit. Then by Kirchhoff's Voltage Law, when key K is open,




$$V_R + V_L + V_C = V_m \sin \omega t$$

$$iR + L\frac{di(t)}{dt} + \frac{q(t)}{C} - V_m \sin \omega t = 0 \qquad [\because i(t) = i = I_m \sin(\omega t + \phi)]$$

$\Rightarrow$ As charge $q(t)$ changes in circuit with time in AC,

then $$i = \frac{dq(t)}{dt}$$

$$\frac{di}{dt} = \frac{d^2q(t)}{dt^2} \quad \text{(differentiating again)}$$

$$R\frac{dq(t)}{dt} + L\frac{d^2q(t)}{dt^2} + \frac{q(t)}{C} = V_m \sin \omega t$$

$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{q(t)}{C} = V_m \sin \omega t$$

This is the equation for variation of motion of charge with respect to time.

(*b*) Let time dependent charge in circuit is at phase angle with voltage then $q = q_m \cos(\omega t + \phi)$

$$i = \frac{dq}{dt} = \omega q_m \sin(\omega t + \phi) \quad ...(I)$$

$$i_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}} \quad ...(II)$$

$$\tan\phi = \frac{(X_C - X_L)}{R}$$

At $t = t_0$, R is short circuited, then energy stored in L and C, when K is closed will be,

$$U_L = \frac{1}{2}Li^2 \quad ...(III)$$

At $t = t_0$,

$$i = i_m \sin(\omega t_0 + \phi) \quad ...(IV)$$

From (II), $$i = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}} \sin(\omega t_0 + \phi) \quad ...(V)$$

So, from (III),

$$\therefore \quad U_L = \frac{1}{2}L\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \sin^2(\omega t_0 + \phi)$$

$$U_C = \frac{q^2}{2C} = \frac{1}{2C}\left[q_m^2 \cos^2(\omega t_0 + \phi)\right]$$

Comparing (IV) and (I) $\quad i_m = q_m \omega$

$$\therefore \quad q_m = \frac{i_m}{\omega}$$

$$\therefore \quad U_C = \frac{1}{2C} \cdot \frac{i_m^2}{\omega^2} \cos^2(\omega t_0 + \phi) = \frac{i_m^2}{2C\omega^2} \cos^2(\omega t_0 + \phi)$$

Using equation (II)

$$U_C = \frac{}{2C\ \ ^2}\left[\frac{}{\left(\sqrt{R\ \ + (X_C - X_L)}\right)^2}\right] \cos(\omega\ \ + \phi)$$

(from IV and V)

(*c*) When R is short circuited, the circuit becomes L-C oscillator. The capacitor will go on discharging and all energy will transfer to L, and back and forth. Hence there is oscillation of energy from electrostatic to magnetic and vice versa.

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