11 Dual Nature of Radiation and Matter

MULTIPLE CHOICE QUESTIONS—I

Q11.1. A particle is dropped from a height H. The de-Broglie wavelength of the particle as a function of height is proportional to

(*a*) H (*b*) $H^{1/2}$ (*c*) H^0 (*d*) $H^{-1/2}$

Main concept used: de-Broglie wavelength $\lambda = \frac{h}{p}$

Ans. (*d*): Velocity of freeely falling body after falling from a height

$H = v = \sqrt{2gH}$

We know that de-Broglie wavelength $\lambda = \frac{h}{p}$

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2gH}}$$

h, *m*, and *g* are constants

$$\therefore \quad \frac{h}{m\sqrt{2g}} \text{ is canstant} \quad \Rightarrow \quad \lambda \propto \frac{1}{\sqrt{H}} \quad \Rightarrow \quad \lambda \propto H^{-1/2}$$

Q11.2. The wavelength of a photon needed to remove a proton from a nucleus which is bound to the nucleus with 1 MeV energy is nearly

(*a*) 1.2 nm (*b*) 1.2×10^{-3} nm (*c*) 1.2×10^{-6} nm (*d*) 1.2×10^{1} nm

Main concept used: Energy of photon = $\frac{hc}{\lambda}$

Ans. (*b*): Energy of the photon must be equal to the binding energy of proton

so Energy of photon = 1 MeV = $10^6 \times 1.6 \times 10^{-19}$ J

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-13}} = \frac{6.63 \times 3}{1.60} \times 10^{-26+13}$$

$$= \frac{19.89}{1.60} \times 10^{-13} = 12.4 \times 10^{-13} = 1.24 \times 10^{1} \times 10^{-13}$$

$$= 1.24 \times 10^{-9} \times 10^{-3} = \mathbf{1.24 \times 10^{-3}\ nm}$$

Q11.3. Consider a beam of electrons (each electron with energy E_0) incident on a metal surface kept in an evacuated chamber. Then

(*a*) no electrons will be emitted as only photons can emit the electrons.

(*b*) electrons can be emitted but all with an energy E_0.

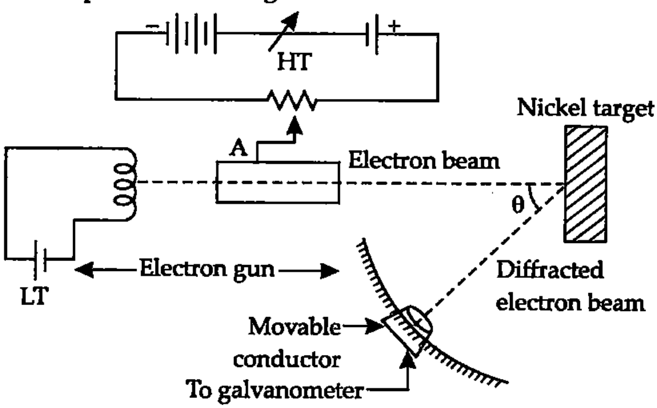
(*c*) electrons can be emitted with any energy with a maximum of $E_0 - \phi$ (ϕ is work function).
(*d*) electrons can be emitted with any energy with a maximum of E_0.

Main concept used: Elastic collision and work function.

Ans. (*d*): When a beam of electrons of energy E_0 of each electron incident on a metal surface kept in vacuum, then due to elastic collisions with electrons on surface, energy of incident electrons will be transferred to the emitted electrons. To emit the electrons below the surface a part of E_0 of incident electrons is consumed against work function so energy of emitted electrons becomes less than E_0.

So, the maximum energy of emitted electrons can be E_0.

Q11.4. Consider figure given below. Suppose the voltage applied to A is increased. The diffracted beam will have the maxima at a value of θ that
(*a*) will be larger than the earlier value.
(*b*) will be the same as the earlier value.
(*c*) will be less than the earlier value.
(*d*) will depend on the target.



Main concept used: Wave nature of cathode rays in Davisson-Germer experiment, $\lambda_d = \frac{12.27}{\sqrt{V}}$ Å, diffraction of waves. In interference of electrons from the crystalline layers

$$2d \sin \theta = \lambda$$

Ans. (*c*): In Davisson-Germer experiment, the de-Broglie wavelength of diffracted beam of electrons

$$\lambda_d = \frac{12.27}{\sqrt{V}} \text{ Å} \quad ...(i)$$

V is the applied voltage. If there is a maxima of the diffracted electrons at an angle θ, then

$$2d \sin \theta = \lambda \quad ...(ii)$$

From equation (i), as the applied voltage in this experiment increases, the wave length λ_d decreases in turn sin θ or θ decreases by relation (ii). Hence verifies the option (*c*).

Q11.5. A proton, a neutron, an electron and an α-particle have the same energy. Then their de-Broglie wavelengths compared as

(*a*) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$ (*b*) $\lambda_\alpha < \lambda_p = \lambda_n > \lambda_e$

(*c*) $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$ (*d*) $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$

Main concept used: $\lambda_d = \dfrac{h}{\sqrt{2mk}}$, $K = \dfrac{1}{2}mv^2$, $\lambda_d = \dfrac{h}{p}$

Ans. (*b*): de-Broglie wavelength $\lambda_d = \dfrac{h}{p}$

$$E_p = E_n = E_e = E_\alpha$$

$$\text{K.E.} = K = \frac{1}{2}mv^2$$

$$2K = mv^2$$

$$2Km = m^2v^2$$

$$2mK = p^2$$

$$\sqrt{2mK} = p$$

$$\therefore \quad \lambda_d = \frac{h}{p}$$

$$\lambda d = \frac{h}{\sqrt{2mK}}$$

or $$\lambda = \frac{h}{\sqrt{2mK}}$$ [as h and E (K.E.) is constt.]

$$\therefore \quad \lambda \propto \frac{1}{\sqrt{m}}$$

$$m_\alpha > m_p = m_n > m_e$$

$$\therefore \quad \lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$$

Q11.6. An electron is moving with an initial velocity $v = v_0\hat{i}$ and is in a magnetic field $B = B_0\hat{j}$. Then its de-Broglie wavelength

(*a*) remains constant. (*b*) increases with time.

(*c*) decreases with time. (*d*) increases and decreases periodically.

Main concept used: $F = q(v \times B)$ and $\lambda = \dfrac{h}{p}$

Ans. (*a*): Given $\vec{v} = v_0\hat{i}$ and $B = B_0\hat{j}$

Force on moving electron in perpendicular magnetic field B is

$$F = -e(\vec{v} \times \vec{B})$$

$$= -e\,[v_0 i \times B_0\hat{j}]$$

$$= -ev_0B_0\,\hat{i} \times \hat{j}$$

$$F = -ev_0B_0\hat{k}$$

So the force is perpendicular to v and B both as the force is perpendicular to the velocity. So will not change v or mv so the de-Broglie wavelength remains same.

Q11.7. An electron (mass m) with an initial velocity $v = v_0\hat{i}$ ($v_0 > 0$) is in an electric field $E = -E_0\hat{i}$ (E_0 = constant > 0). It's de-Broglie wavelength at time t is given by

(a) $\dfrac{\lambda_0}{\left[1+\dfrac{eE_0}{m}\dfrac{t}{v_0}\right]}$ (b) $\lambda_0\left[1+\dfrac{eE_0}{mv_0}t\right]$ (c) λ_0 (d) $\lambda_0 t$

Main concept used: $\lambda_0 = \dfrac{h}{mv}$, $F = qE$, $a_e = \dfrac{qE}{m}$

Ans. (a): Initial de-Broglie wavelength $\lambda_0 = \dfrac{h}{mv_0}$

Force on electron = $F = qE \Rightarrow F = (-e)(-E_0 i)$

$$ma = eE_0\hat{i}$$

$$a = \frac{eE_0}{m}\hat{i}$$

Velocity of electron after time t is $v = v_0 + at$

$$v = v_0 i + \frac{eE_0}{m} i \cdot t$$

$$v = \left[v_0 + \frac{eE_0 t}{m}\right]\hat{i}$$

$\therefore$ New de-Broglie wavelength $\lambda = \dfrac{h}{mv}$

$$\lambda = \frac{h}{m\left[v_0 + \dfrac{eE_0 t}{m}\right]i} = \frac{h}{mv_0\left[1+\dfrac{eE_0 t}{mv_0}\right]}$$

$$\lambda = \frac{\lambda_0}{\left[1+\dfrac{eE_0 t}{mv_0}\right]} \qquad \left(\because \frac{h}{mv_0} = \lambda_0 \text{ from eqn. I}\right)$$

Q11.8. An electron (mass m) with an initial velocity $v = v_0\hat{i}$ is in an electric field $E = E_0\hat{j}$. If $\lambda_0 = \dfrac{h}{mv_0}$, its de-Broglie wavelength at time t is given by

(a) λ_0 (b) $\lambda_0\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}$ (c) $\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}}$ (d) $\dfrac{\lambda_0}{\left[1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}\right]}$

Main concept used: $\lambda_0 = \dfrac{h}{mv}$, $F = qE$, $a_e = \dfrac{eE}{m}$

Ans. (*c*): Initial de-Broglie wavelength $\lambda_0 = \dfrac{h}{mv_0}$. Force on moving electron due to electric field E = $F = -eE = -eE_0\hat{j}$. Acceleration in electron due to force by electric field,

$$ma = -eE_0\hat{j}$$

$$a = \frac{-eE_0}{m}\hat{j}$$

Acceleration on electron acts in $-Y$ direction the initial velocity of electron along X-axis,

$$v_{x_0} = v_0\hat{i}$$

Initial velocity of electron in Y direction = 0

$\therefore$ $$v_{y_0} = 0$$

Velocity of electron after time t along X-axis

$$v_x = v_0\hat{i}$$

So velocity of electron after time *t* along Y-axis

$$v = u + at$$

$$v_y = 0 + \left(\frac{-eE_0}{m}\hat{j}\right)t$$

$$v_y = \frac{-eE_0}{m}\hat{j}\,t$$

Magnitude of velocity of electron after time *t* is

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\vec{v} = \sqrt{v_0^2 + \left(\frac{-eE_0\hat{j}t}{m}\right)^2}$$

$$|v| = v_0\sqrt{1 + e^2E_0^2t^2}$$

$\therefore$ $$\lambda' = \frac{h}{mv} = \frac{h}{mv_0\sqrt{1 + \dfrac{e^2E_0^2t^2}{m^2v_0^2}}}$$

$$\lambda' = \frac{l_0}{\sqrt{1 + \dfrac{e^2E_0^2t^2}{m^2v_0^2}}} \qquad \left(\because \lambda_0 = \frac{h}{mv_0}\right)$$

Hence, option (*c*) is verified.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q11.9. Relativistic corrections become necessary when the expression for the kinetic energy $\frac{1}{2}mv^2$, becomes comparable with mc^2, where m is the mass of the particle. At what de Broglie wavelength, will relativistic corrections becomes important for an electron?

(*a*) $\lambda = 10$ nm (*b*) $\lambda = 10^{-1}$ nm

(*c*) $\lambda = 10^{-4}$ nm (*d*) $\lambda = 10^{-6}$ nm

Main concept used: Relativistic correction become necessary when speed of particle is either more than speed of light or comparable to it.

Ans. (*c*) and (*d*): de-Broglie wavelength $\lambda = \frac{h}{mv}$

$$v = \frac{h}{m\lambda} \qquad \begin{pmatrix} h = 6.6 \times 10^{-34} \\ m = 9 \times 10^{-31}\,\text{kg} \end{pmatrix}$$

$$= \frac{6.6 \times 10^{-34}}{9 \times 10^{-31}\lambda} = \frac{6.6 \times 10^{-34+31}}{9\lambda} = \frac{0.73 \times 10^{-3}}{\lambda}$$

$$= \frac{7.3 \times 10^{-4}}{\lambda}$$

For option

(*a*) $\lambda = 10\text{ nm} = 10 \times 10^{-9}\text{ m} = 10^{-8}\text{ m}$

$\therefore \quad v = 7.3 \times 10^{-4} \times 10^{+8} = 7.3 \times 10^4 < 3 \times 10^8$ (Speed of light)

(*b*) $\lambda = 10^{-1}\text{ nm} = 10^{-1} \times 10^{-9}\text{ m} = 10^{-10}\text{ m}$

$\therefore \quad v = \frac{7.3 \times 10^{-4}}{10^{-10}} = 7.3 \times 10^{-4+10} = 7.3 \times 10^6 \approx 10^7 < 10^8$ (Speed of light)

(*c*) $\lambda = 10^{-4}\text{ nm} = 10^{-4} \times 10^{-9}\text{ m} = 10^{-13}\text{ m}$

$v = \frac{7.3 \times 10^{-4}}{10^{-13}} = 7.3 \times 10^{-4+13} = 7.3 \times 10^9 \approx 10^{10} > 10^8$ (Speed of light)

(*d*) $\lambda = 10^{-6}\text{ nm} = 10^{-15}\text{ m}$

$v = 7.3 \times 10^{-4} \times 10^{15} = 7.3 \times 10^{11} \approx 10^{12} > 10^8$ (Speed of light)

So the velocity of electron is more for option (*c*) and (*d*) where the relativistic correction become necessary although the speed of electron is 7.3×10^6 m/s is comparable with (c) speed of light, there must be relativistic correction. Options are (*c*) and (*d*).

Q11.10. Two particles A_1 and A_2 of masses m_1 and m_2 $(m_1 > m_2)$ have the same de-Broglie wavelength. Then

(*a*) their momenta are the same.

(*b*) their energies are the same.

(*c*) energy of A_1 is less than the energy of A_2.

(*d*) energy of A_1 is more than the energy of A_2.

Main concept used: $\lambda_d = \frac{h}{p}$

Ans. (*a*) and (*c*): $\lambda = \frac{h}{p}$

$$p = \frac{h}{\lambda} \quad \text{or} \quad p \propto \frac{1}{\lambda}$$

$$\frac{p_1}{p_2} = \frac{\lambda_2}{\lambda_1}$$

$\because$ $\lambda_1 = \lambda_2 = \lambda$ (given)

$\therefore$ $p_1 = p_2$ verifies Ans. (*a*)

$$E_a = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2\text{ m}}$$

$E \propto \frac{1}{m}$ [as $p_1 = p_2$ = const. (as proved above)]

$$\frac{E_1}{E_2} = \frac{m_2}{m_1}$$

$\frac{E_1}{E_2} < 1$ [$\because m_1 > m_2$ (given)]

$\therefore$ $E_2 > E_1$ verifies answer (*c*)

Q11.11. The de-Broglie wavelength of a photon is twice the de-Broglie wavelength of an electron. The speed of the electron $v_e = \frac{c}{100}$. Then

(*a*) $\frac{E_e}{E_p} = 10^{-4}$ (*b*) $\frac{E_e}{E_p} = 10^{-2}$ (*c*) $\frac{p_e}{m_e c} = 10^{-2}$ (*d*) $\frac{p_e}{m_e c} = 10^{-4}$

Main concept used: $\lambda_d = \frac{h}{p}$ and $p^2 = 2mK$

Ans. (*b*) and (*c*): de-Broglie wavelength $\lambda = \frac{h}{p}$

$$\therefore \quad \lambda_e = \frac{h}{m_e v_e} = \frac{h}{m_e \frac{c}{100}}$$

$$= \frac{100\,h}{m_e c} \quad \text{...(I)}$$

Now $\text{K.E.} = \frac{1}{2}mv^2 \Rightarrow E = \frac{m^2v^2}{2m}$ ...(II)

$\Rightarrow$ $m_e v_e = \sqrt{2m_e E_e}$

Now $\lambda_e = \frac{h}{m_e v_e} = \frac{h}{\sqrt{2m_e E_e}}$ [from (II)]

$\Rightarrow$ $$E_e = \frac{h^2}{2m_e\lambda_e^2} \quad ...(III)$$

For proton $\lambda_p = 2\lambda_e$ [given]

$\therefore$ $$E_p = \frac{hc}{\lambda_p} = \frac{hc}{2\lambda_e}$$

Now $$\frac{E_p}{E_e} = \frac{hc}{2\lambda_e} \times \frac{2m_e\lambda_e^2}{h^2} = \frac{\lambda_e m_e c}{h} \quad \text{[from (III)]}$$

$$= \frac{100\,h}{m_e c} \times \frac{m_e c}{h} = 100$$

$\Rightarrow$ $$\frac{E_e}{E_p} = 10^{-2}$$ [verifies Ans. (b)]

Now $$p_e = m_e v_e = m_e \times \frac{c}{100} \quad \left(\because v_e = \frac{c}{100}\right)$$

$\Rightarrow$ $$\frac{p_e}{m_e c} = \frac{1}{100} = 10^{-2}$$ [verifies Ans. (c)]

Q11.12. Photons absorbed in matter are converted to heat. A source emitting n photons per-second of frequency ν is used to convert 1 kg of ice at 0°C to water at 0°C. Then the time T taken for the conversion

(*a*) decreases with increasing n with ν fixed.
(*b*) decreases with n fixed ν increasing.
(*c*) remains constant with n and ν changing such that nν is a constant.
(*d*) increases when the product nν increases.

Main concept used: $E = nh\nu$ and $mL = E$ (Heat)

Ans. (*a*), (*b*) and (*c*): Heat energy required to convert 1 kg (1000 gm) of ice at 0°C to water at 0°C is

$E = mL$

$E' = nh\nu$

n = no. of photons incident per second.

Let time T is taken by radiation to melt the ice at 0°C. Then,

$$E = n\,h\nu\,t$$

$$mL = n\,h\nu\,t$$

$$t = \frac{mL}{nh\nu}$$

$\therefore$ $$t \propto \frac{1}{n} \quad \text{and} \quad T \propto \frac{1}{\nu}$$

or $$t \propto \frac{1}{n\nu}$$ [$\because m$, L and h are constants]

[verifies Ans. (*a*), (*b*), and (*c*)]

$\therefore$ if $n\nu$, increases then, T decreases so does not verify answer (*d*).

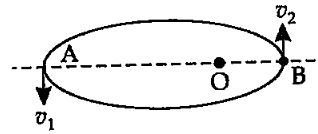
Q11.13. A particle moves in a closed orbit around the origin, due to a force which is directed towards the origin. The de-Broglie wavelength of the particle varies cyclically between two values λ_1 and λ_2 ($\lambda_1 > \lambda_2$). Which of the following statement are true?

(*a*) The particle could be moving in a circular orbit with origin as centre.

(*b*) The particle could be moving in an elliptical orbit with origin as its focus.

(*c*) When the de-Broglie wavelength is λ_1, the particle is nearer the origin than when its value is λ_2.

(*d*) When the de-Broglie wavelength is λ_2 the particle is nearer the origin than when its value is λ_1.



Ans. (*b*) and (*d*): As the de-Broglie wavelength of particle varies cyclically between two values λ_1 and λ_2. It is possible when the particle is moving in elliptical orbit with origin as one of its focus. Because if λ_1 and λ_2 are equal, their speed must be equal and particle must move in circular orbit. Hence it verifies answer (*b*).

Let v_1 and v_2 be the speeds of particle at A and B respectively and origin is at O. If λ_1 and λ_2 are the de-Broglie wavelengths associated with particle at A and B respectively, then

$$\lambda_1 = \frac{h}{mv_1} \quad \text{and} \quad \lambda_2 = \frac{h}{mv_2}$$

so $$\frac{\lambda_1}{\lambda_2} = \frac{v_2}{v_1} \qquad [\because \lambda_1 > \lambda_2] \quad \text{(Given)}$$

so $$v_1 < v_2 \qquad ...(I)$$

By the law of conservation of angular momentum the speed of the particle will be more when it is closer to focus. It is verified by I.

∴ So the object is close to B than A or the particle is nearer to the origin when wavelength is λ_2 than when wavelength is λ_1. Hence verifies answer (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q11.14. A proton and an α-particle are accelerated using the same potential difference. How are the de-Broglie wavelength. λ_p and λ_α related to each other?

Main concept used: As both proton and α-particles are accelerated at same potential so their K.Es. will be same *i.e.*, $K_1 = K_2 = K = qV$ by relation ($E = eV$) and

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Ans. As we know that both the particles are accelerated at the same potential difference so their K.Es. will be equal *i.e.,*

$$K_1 = K_2 = K = qV \qquad \left(\because \ V = \frac{W}{q}\right)$$

So $$\lambda = \frac{h}{\sqrt{2mK}} \quad \text{or} \quad \lambda_d = \frac{h}{\sqrt{2mqV}}$$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \frac{h}{\sqrt{2m_p q_p V_p}} \times \frac{\sqrt{2m_\alpha q_\alpha V_a}}{h}$$

$$m_\alpha = 4m_p; \quad q_\alpha = 2e; \quad q_p = e; \quad V_p = V_\alpha = V \text{ (P.D. applied)}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{2 \times 4\, m_p\, 2e\, V}{2m_p\, eV}}$$

$$\lambda_p = \sqrt{8}\, \lambda_\alpha$$

So, the de-Broglie wavelength of proton is $\sqrt{8}$ times of alpha (α) particle.

Q11.15. (*i*) In an explanation of photoelectric effect, we assume one photon of frequency ν collides with an electron and transfers its energy. This leads to the equation for maximum energy E_{max} of the emitted electron as $E_{max} = h\nu - \phi_0$ where ϕ_0 is the work function of the metal. If an electron absorbs two photons (each of frequency ν) what will be the maximum energy for the emitted electron?

(*ii*) Why is this fact (two photons absorption) not taken into consideration in our discussion of stopping potential ?

Ans. (*i*) Here, 2 photons transfer its energy to one electron as $E = h\nu$

$$\therefore \quad E_e = E_p$$
$$h\nu_e = 2h\nu$$
$$\therefore \quad \nu_e = 2\nu$$

Maximum energy of emitted electron is

$$E_{max} = h\nu_e - \phi_0 = h(2\nu) - \phi_0 = 2h\nu - \phi_0$$

(*ii*) The probability of absorbing 2 photons by electron is very low due to their mass difference. So possibilities of such emission of electrons is negligible.

Q11.16. There are materials which absorb photons of shorter wavelength and emit photons of longer wavelength. Can there be stable substances which absorb photons of longer wavelength and emit light of shorter wavelength?

Ans. We know that as the wavelength of photon increases, its frequency decreases or energy increases.

Case I: Photons of shorter wavelength *i.e.*, of larger energy emits the photons of smaller energy here some energy, is consumed against work function. So, it is possible by law of conservation of energy.

Case II: Photons of longer wavelength always emits photons of shorter wavelength and photons of smaller energy can never emits photons of larger energy as some part of energy ($h\nu$) is consumed in work function (ϕ) of metal. Otherwise it will discard the law of conservation of energy (universal law) so, it cannot be possible in stable materials.

Q11.17. Do all the electrons that absorb a photon come out as photoelectrons?

Ans. In photoelectric effect, we can observe that most of the electrons knocked by photons are scattered into the metal by absorbing photons.

So number of emitted electrons are very small than number of photons absorbed.

One photon cannot emit one electron generally. Some energy of photons is consumed against work function of metal.

So, not all the electrons that absorb a photon comes out from metal surface.

Q11.18. There are two sources of light each emitting with a power of 100 W. One emits X-rays of wavelength 1 nm and the other visible light at 500 nm. Find the ratio of number of photons of X-rays to the photons of visible light of the given wavelength.

Ans. Let E_x and E_v are the energies given by one photon in X-rays and visible rays then,

$$E_x = h\nu_x \quad \text{and} \quad E_v = h\nu_v$$

Let n_x and n_v are number of photons in X-rays and visible light to give equal energies as both sources (X-rays and visible) emitting same power 100 W each so

$$n_x h\nu_x = n_v h\nu_v$$

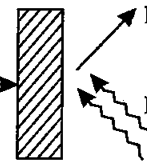
$$\frac{n_x}{n_v} = \frac{\nu_v}{\nu_x} = \frac{\frac{1}{\lambda_v}}{\frac{1}{\lambda_x}} = \frac{\lambda_x}{\lambda_v} \qquad \left(\because \nu = \frac{c}{\lambda} \text{ or } \nu = \frac{1}{\lambda}\right)$$

$$\frac{n_x}{n_v} = \frac{1\text{ nm}}{500\text{ nm}} \qquad \left(\begin{array}{l}\because \quad \lambda_x = 1\text{ nm} \\ \text{and } \lambda_v = 500\text{ nm}\end{array}\right) \quad \text{[Given]}$$

$$\therefore \qquad n_x : n_v = 1 : 500$$

SHORT ANSWER TYPE QUESTIONS

Q11.19. Consider figure for photo-emission. How would you reconcile with momentum conservation? Note light (Photons) have momentum in a different direction, than the emitted electrons.



Main concept used: Momentum is vector quantity.

Ans. When photons strike to metal surface, photons transfer their momentum to atoms of metal by decreasing its own speed upto zero. This momentum of photons transfer to nucleus and electrons of the metal.

The exited electrons emit approximately opposite to the direction of photons. But total momentum transferred by the photons will be equal to the momentum of all electrons and nucleus.

Q11.20. Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. Find the work function in eV.

Main concept used: $E_{max} = h\nu - \phi$

Ans. Let the maximum energies of emitted electrons are K_1 and K_2 when 600 nm and 400 nm visible light are used according to question

$$K_2 = 2K_1$$

$$K_{max} = h\nu - \phi = \frac{hc}{\lambda} - \phi$$

$$K_1 = \frac{hc}{\lambda_1} - \phi$$

$$K_2 = \frac{hc}{\lambda_2} - \phi = 2K_1$$

$$\frac{hc}{\lambda_2} - \phi = 2\left[\frac{hc}{\lambda_1} - \phi\right] = \frac{2hc}{\lambda_1} - 2\phi$$

$$\phi = hc\left[\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right] \qquad (\because \ hc = 1240 \text{ eV nm})$$

$$\therefore \qquad \phi = 1240\left[\frac{2}{600} - \frac{1}{400}\right]\text{eV} = \frac{1240}{200}\left[\frac{2}{3} - \frac{1}{2}\right] = 6.2\frac{(4-3)}{6}$$

Work function $\phi = \frac{6.2}{6} = 1.03$ eV.

Q11.21. Assuming an electron is confined to a 1 nm wide region, find the uncertainty in momentum using Heisenberg uncertainty principle. ($\Delta x \times \Delta p \cong h$). You can assume the uncertainty in position Δx as 1 nm. Assuming $p \approx \Delta p$, find the energy of the electron in electron volt.

Ans. As electron revolves in circular path so $\Delta r = 1 \text{ nm} = 10^{-9}$ m

$$\Delta x \times \Delta p \cong h \qquad \text{[Given]}$$

$$\Delta p = \frac{h}{\Delta x} = \frac{6.62 \times 10^{-34}}{2\pi\, \Delta r} \text{JS}$$

$$\therefore \quad \Delta p = \frac{6.62 \times 10^{-34}}{2 \times 3.14 \times 10^{-9}} \text{ kg m/s}$$

$$= \frac{331}{314} \times 10^{-34+9} \quad \text{or} \quad \Delta p = \frac{331}{314} \times 10^{-25}$$

$$\therefore \quad E = \frac{1}{2} mv^2 = \frac{m^2 v^2}{2m} = \frac{\Delta p^2}{2m}$$

$$= \frac{331}{314} \times \frac{331}{314} \times \frac{10^{-25} \times 10^{-25}}{2 \times 9.1 \times 10^{-31}} = \frac{331 \times 331 \times 10^{-50+31}}{314 \times 314 \times 18.2} \text{J}$$

$$= \frac{331 \times 331 \times 10^{-19} \times 1.6 \times 10^{-19}}{314 \times 314 \times 18.2} e$$

$$= \frac{331 \times 331 \times 16}{314 \times 314 \times 182} = 3.8 \times 10^{-2} \text{ eV}$$

Q11.22. Two monochromatic beams of light A and B of equal intensity I, hit a screen. The number of photons hitting the screen by beam A is twice that by beam B. Then, what inference can you make about their frequencies?

Main concept used: Intensity or Energy of photons $E = nh\nu$

Ans. $I_A = I_B$ [Given]

$$n_A h\nu_A = n_B h\nu_B$$

$\because$ $n_A = 2n_B$ [Given]

$$\therefore \quad 2n_B \nu_A = n_B \nu_B$$

$$2\nu_A = \nu_B$$

So the frequency of source B is twice the frequency of source A.

Q11.23. Two particles A and B of de-Broglie wavelengths λ_1 and λ_2 combine to form a particle C. The process conserves the momentum. Find the de-Broglie wavelength of the particle C (The motion is one dimensional).

Ans. de-Broglie wavelengths

$$\lambda = \frac{h}{p} \quad \text{or} \quad p = \frac{h}{\lambda} \Rightarrow p_1 = \frac{h}{\lambda_1}, \quad p_2 = \frac{h}{\lambda_2} \quad \text{and} \quad p_3 = \frac{h}{\lambda_3}$$

$$p_1 + p_2 = p_3$$

$$\frac{h}{\lambda_1} + \frac{h}{\lambda_2} = \frac{h}{\lambda_3} \qquad (\lambda_3 = \text{the wavelength of particle C})$$

$$\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$$

$$\frac{\lambda_2+\lambda_1}{\lambda_1\lambda_2} = \frac{1}{\lambda_3}$$

$$\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$$

Case I: When p_1 and p_2 are positive then $\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$

Case II: When p_1 and p_2 both are negative

$$\lambda_3 = \left|\frac{-\lambda_1\lambda_2}{\lambda_1+\lambda_2}\right| = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$$

Case III: $p_A > 0, p_B < 0$ or

$$\frac{h}{\lambda_3} = \frac{h}{\lambda_1} - \frac{h}{\lambda_2} \quad \text{or} \quad \frac{1}{\lambda_3} = \frac{\lambda_2-\lambda_1}{\lambda_1\lambda_2}$$

$$\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}$$

Case IV: $p_A < 0, p_B > 0$

$$\frac{h}{\lambda_3} = \frac{h}{\lambda_2} - \frac{h}{\lambda_1} = \frac{(\lambda_1-\lambda_2)h}{\lambda_1\lambda_2}$$

$\therefore$
$$\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}$$

Q11.24. A neutron beam of energy E scatters from atoms on a surface, with a spacing $d = 0.1$ nm. The first maxima of intensity in the reflected beam occur at $\theta = 30°$. What is the kinetic energy E of the beam in eV?

Main concept used: Bragg's law of diffraction $2d \sin\theta = n\lambda$ (maxima)

$$E = \frac{p^2}{2m}, \quad p = \frac{h}{\lambda}$$

Ans. By Bragg's law of diffraction, condition for nth maxima is

$$2d \sin\theta = n\lambda$$

$n = 1$ so $\quad \lambda = 2d \sin\theta \qquad$ [$\theta = 30°$ (Given)]

$= 2 \times 0.1 \times 10^{-9} \sin 30° \qquad$ ($\because \; d = 0.1$ nm)

$= 0.1 \times 10^{-9}$ m $= 10^{-10}$ m

$$p = \frac{h}{\lambda} = \frac{6.6\times10^{-34}}{10^{-10}} = 6.6\times10^{-24} \text{ kg m/s}$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$$

$$\therefore \quad E = \frac{6.6\times6.6\times10^{-2.4}\times10^{-24}}{2\times1.6\times10^{-27}} \text{ J}$$

$$= \frac{6.6 \times 6.6 \times 10^{-48}}{2 \times 1.6 \times 10^{-27} \times 1.6 \times 10^{-19}} \text{ eV} = \frac{66 \times 66 \times 10^{-48+46}}{2 \times 16 \times 16}$$

$$E = \frac{33 \times 33}{128} \times 10^{-2} = 8.5 \times 10^{-2} = \mathbf{0.085\ eV}$$

LONG ANSWER TYPE QUESTIONS

Q11.25. Consider a thin target [sq. of side 10^{-2} m and 10^{-3} m thickness] of sodium, which produces a photocurrent of 100 µA, when a light of intensity 100 W/m^2 (λ = 660 nm) falls on it. Find the probability that a photoelectron is produced when a photon strikes a sodium atom. (Take density of Na = 0.97 kg/m^3)

Ans. Area of square sheet $= (A) = 10^{-2} \times 10^{-2} = 10^{-4}\ m^2$

Thickness $(d) = 10^{-3}$ m

Current $(i) = 100\ \mu A = 10^{-4}$ A

Intensity $(I) = 100\ W/m^2$

Mass of target (m) = Vol. × density

m = Area of sheet × thickness × density

$= (10^{-4} \times 10^{-3}) \times 0.97 \text{ kg} = 0.97 \times 10^{-7}$ kg

$m = 0.97 \times 10^{-4}$ gm

$\therefore$ No. of Na Atoms in target $= \frac{6.023 \times 10^{23}}{23} \times 0.97 \times 10^{-4} = 0.254 \times 10^{19}$

Number of Na atoms in Na target $= 2.54 \times 10^{18}$ Atoms

Total energy falling per second on target $= nh\nu$

Intensity × Area $= n \times h \times \frac{c}{\lambda}$

$$I \times A = \frac{nhc}{\lambda}$$

$$n = \frac{IA\lambda}{hc} = \frac{100 \times 10^{-4} \times 660 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}} = \frac{1000 \times 660}{66 \times 3} \times 10^{-13-8+34}$$

$$= \frac{10000}{3} \times 10^{-21+34} = \frac{10}{3} \times 10^{3} \times 10^{13}$$

Number of photons (n) incident per second on Na-metal

$\therefore$ $n = 3.3 \times 10^{16}$

Let P is the probability of emission of photoelectrons per atom per photon.

Number of photoelectrons emitted per second

$N = P \cdot n.$ (No. of sodium atom)

$N = P \times 3.3 \times 10^{16} \times 2.54 \times 10^{18}$ ...(I)

$i = 100\ \mu A = 10^{-4} A = N e$

$$\Rightarrow \qquad N = \frac{i}{e}$$

$$\therefore \qquad P = \frac{i}{e \times 3.3 \times 10^{16} \times 2.54 \times 10^{18}}$$

$$= \frac{10^{-4}}{1.6 \times 10^{-19} \times 3.3 \times 10^{16} \times 2.54 \times 10^{18}}$$

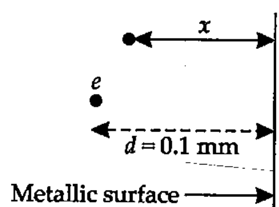
$$= \frac{10^{-4}}{1.6 \times 3.3 \times 2.54 \times 10^{-19+34}} = \frac{10^{-4-34+19}}{13.4} = 0.075 \times 10^{-19}$$

$$\mathbf{P = 7.5 \times 10^{-21}}$$

Q11.26. Consider an electron in front of a metallic surface at a distance d (treated as an infinite plane surface). Assume the force of attraction by the plate is given as $\frac{1}{4} \cdot \frac{q^2}{4\pi\varepsilon_0 d^2}$. Calculate work in taking the charge to an infinite distance from the plate. Taking d = 0.1 nm, find the work done in electron volts. (such a force law is not valid for $d < 0.1$ nm)

Main concept used: $W.D. = \int_0^{\alpha} F \cdot dr$

Ans. As per question $F = \frac{1}{4}\frac{q^2}{4\pi\varepsilon_0 d^2}$



Let at any instant electron is at distance x from the metal surface. Force of attraction between metal surface and electron is F

$$F = \frac{1}{4}\frac{q^2}{4\pi\varepsilon_0 x^2}$$

Work done by external agency in taking the electron from distance d to ∞ is $W.D. = \int_d^{\infty} F \cdot dx$

$$W.D. = \int_d^{\infty} \frac{q^2 dx}{4 \times 4\pi\varepsilon_0 x^2} = \frac{q^2}{4 \times 4\pi\varepsilon_0}\int_d^{\infty} x^{-2} dx = \frac{q^2}{4 \times 4\pi\varepsilon_0}\frac{x^{-1}}{-1}$$

$$W.D. = \frac{-q^2}{4 \cdot 4\pi\varepsilon_0}\left[\frac{1}{x}\right]_d^{\infty} = \frac{-q^2 k}{4}\left[\frac{1}{\infty} - \frac{1}{d}\right]$$

$$W.D. = \frac{+kq^2}{4d} \qquad [d = 0.1 \text{ nm} = 10^{-10} \text{ m}]$$

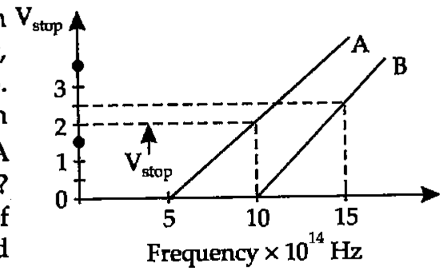
Work done is positive

$$\text{So} \quad W.D. = \frac{(1.6 \times 10^{-19})^2 \times 9 \times 10^9}{4 \times 10^{-10}} \text{J} = \frac{1.6 \times 9 \times 1.6 \times 10^{-38+9+10}}{4 \times 1.6 \times 10^{-19}} \text{eV}$$

$$= \frac{9 \times 1.6 \times 10^{-19+19}}{4} = 3.6 \text{ eV}$$

W.D. = **3.6 eV**

Q11.27. A student performs an experiment on photoelectric effect, using two materials A and B. A plot of V_{stop} versus ν is given in figure. (*i*) Which material A or B has higher work function? (*ii*) Given the electric charge of an electron = 1.6×10^{-19} C, find the value of *h* obtained from the experiment for both A and B. Comment on whether it is consistent with Einstein's theory.



Ans. (*i*) ∵ $\phi_0 = h\nu_0$

ν_0 = Threshold frequency

For metal A $\quad \nu_{0A} = 5 \times 10^4$ Hz

For metal B $\quad \nu_{0B} = 10 \times 10^4$ Hz

$$\frac{\phi_{0A}}{\phi_{0B}} = \frac{h\nu_{0A}}{h\nu_{0B}} = \frac{5 \times 10^{14}}{10 \times 10^{14}}$$

∴ $\quad \phi_{0B} = 2\phi_{0A}$

So work function of material B is twice of material A.

(*ii*) By the differentiation of potential $V = \frac{W}{Q}$

if W = E and Q = *e* (charge on an electron)

then $\quad V = \frac{E}{e} \quad$ or $\quad E = eV$

$h\nu = eV \qquad (\because E = h\nu)$

Differentiating both sides we get

$$h \,.\, d\nu = e dV$$

$$h = e \cdot \frac{dV}{d\nu}$$

For metal A, $h = \frac{1.6 \times 10^{-19}[2-0]}{(10-5) \times 10^{14}} = \frac{3.2}{5} \times 10^{-19-14}$

$= 0.64 \times 10^{-33}$ JS $= 6.4 \times 10^{-33}$ JS

$h = 6 \times 10^{-34}$ JS ...(I)

For metal B, $h = \frac{e \times (2.5-0)}{(15-10) \times 10^{14}} = \frac{1.6 \times 10^{-19} \times 2.5 \times 10^{-14}}{5}$

$= \frac{4.00}{5} \times 10^{-33} = 0.8 \times 10^{-33}$

$h = 8 \times 10^{-34}$ JS ...(II)

The value of planks constant (h) for both experimental graphs are not equal, so the experiment is not consistent with Einstein's theory.

But due to experimental limitation values are very near to 6.6×10^{-34} JS, so can be considered consistent with Einstein theory.

Q11.28. A particle A with mass m_A is moving with a velocity v and hits a particle B (mass m_B) at rest (one dimensional motion). Find the change in de-Broglie wavelength of particle A. Treat the collision as elastic.

Ans. As collision is elastic so law of conservation of momentum and Kinetic energy are obeyed.

$$m_A v + m_B(0) = m_A v_1 + m_B v_2$$

$$m_A(v - v_1) = m_B v_2 \quad \text{...(I)}$$

and

$$\frac{1}{2} m_A v^2 + \frac{1}{2} m_B (0)^2 = \frac{1}{2} m_A v_1^2 + \frac{1}{2} m_B v_2^2$$

$$m_A (v^2 - v_1^2) = m_B v_2^2$$

$$m_A (v - v_1)(v + v_1) = m_B v_2^2 \quad \text{...(II)}$$

Dividing (II) by (I) we get,

$$v + v_1 = v_2 \quad \text{...(III)}$$

$$v = v_2 - v_1$$

Put (III) in (I)

$$m_A v - m_A v_1 = m_B(v + v_1)$$

$$m_A v - m_A v_1 = m_B v + m_B v_1$$

$$(m_A - m_B)v = v_1(m_A + m_B)$$

$$v_1 = \frac{(m_A - m_B)}{(m_A + m_B)} v$$

From (II)

$$v_2 = v + \frac{(m_A - m_B)}{(m_A + m_B)} v = v\left[1 + \frac{(m_A - m_B)}{(m_A + m_B)}\right]$$

$$= \frac{(m_A + m_B + m_A - m_B)v}{(m_A + m_B)} = \frac{2m_A v}{m_A + m_B}$$

$$\lambda_{A\,\text{initial}} = \frac{h}{m_A v} \quad \text{and} \quad \lambda_{A\,\text{final}} = \frac{h}{m_A v}$$

$$\lambda_{A\,\text{final}} = \frac{h(m_A + m_B)}{m_A(m_A - m_B)v}$$

$$\Delta\lambda = \lambda_{A\,\text{final}} - \lambda_{A\,\text{initial}} = \frac{h}{m_A v}\left[\frac{m_A + m_B}{(m_A - m_B)} - 1\right]$$

$$= \frac{h}{m_A v}\left[\frac{m_A + m_B - (m_A - m_B)}{(m_A - m_B)}\right]$$

Change in de-Broglie wavelength $\Delta\lambda = \dfrac{2m_B h}{m_A (m_A - m_B) v}$

Q11.29. Consider a 20 W bulb emitting light of wavelength 5000 Å and shining on a metal surface kept at a distance 2 m. Assume that the metal surface has work function of 2eV and that each atom on the metal surface can be treated as a circular disk of radius 1.5 Å.

(*i*) Estimate number of photons emitted by the bulb per second. (assume no other losses)

(*ii*) Will there be photoelectric emission?

(*iii*) How much time would be required by atomic disk to receive energy equal to work function 2eV?

(*iv*) How many photons would atomic disk receive within time duration calculated in (*iii*) above ?

(*v*) Can you explain how photoelectric effect was observed instantaneously?

Ans. (*i*) $P = 20$ W, $\lambda = 5000$ Å $= 5000 \times 10^{-10}$ m, $d = 2$ m, $\phi = 2$ eV, $r = 1.5$ Å $= 1.5 \times 10^{-10}$ m (Atomic radius)

Let number of photons emitted by bulb per second is n_1 then Power P is

$$P = n_1 h\nu \quad \text{or} \quad P = n_1 \frac{c}{\lambda}$$

$$\therefore \quad n_1 = \frac{P\lambda}{hc} = \frac{20 \times 5000 \times 10^{-10}}{6.62 \times 10^{-34} \times 3 \times 10^8}$$

$$n_1 = \frac{100000 \times 10^{-10-8+34}}{19.86} \approx \frac{100000}{20} \times 10^{-18+34}$$

$$n_1 = 5 \times 10^{16+3} = 5 \times 10^{19} \text{ no. of photons per sec.}$$

Number of photons emitted by bulb per second

$$n_1 = 5 \times 10^{19}$$

(*ii*) Energy of incident photon $E = h\nu = \dfrac{hc}{\lambda}$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}} \text{ J} = \frac{19.86 \times 10^{-34+10+8}}{5000 \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= 2.48 \text{ eV}$$

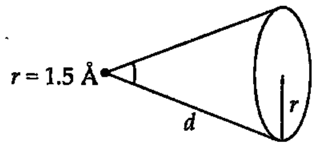
Energy of photon $= h\nu = \dfrac{hc}{\lambda} = \dfrac{(6.62 \times 10^{-34})(3 \times 10^8)}{5000 \times 10^{-10}}$ J

$$= \frac{6.62 \times 3 \times 10^{-24+8}}{5000 \times 1.6 \times 10^{-19}} \text{eV} = \frac{19.86 \times 10^{-16+19}}{8000}$$

$$\text{Energy of photon} = \frac{20 \times 10^3}{8 \times 10^3} = \frac{5}{2} \text{eV} = 2.5 \text{ eV}$$

As the energy of an incident photon is more than 2 eV *i.e.*, the work function of metal surface hence the photoelectric emission takes place.

(*iii*) Let Δt be the time spent in getting the energy ϕ (work function of metal)



Energy received by atomic disk in Δt time $\quad E = P \times A \cdot \Delta t$

$$E = P \times \pi r^2 \cdot \Delta t$$

energy transferred by bulb in full solid angle $4\pi d^2$ to atoms = $4\pi d^2 \phi$

$$\therefore \quad P \times \pi r^2 \Delta t = 4\pi d^2 \phi$$

$$\Delta t = \frac{4d^2\phi}{Pr^2} = \frac{4 \times 2 \times 2 \times 2 \times 1.6 \times 10^{-19}}{20 \times 1.5 \times 1.5 \times 10^{-10} \times 10^{-10}} \text{ sec}$$

$$\Delta t = \frac{12.8 \times 10^{-19+20}}{5 \times 2.25} = \frac{128}{12.25} = 11.4 \text{ sec}$$

(*iv*) Number of photons received by one atomic disk in time Δt is

$$N = \frac{n_1 \pi r^2 \Delta t}{4\pi d^2} = \frac{n_1 r^2 \Delta t}{4d^2} = \frac{5 \times 10^{19} \times 1.5 \times 1.5 \times 10^{-20} \times 11.4}{4 \times 2 \times 2}$$

[n_1 from part (*i*) and Δt from part (*iii*)]

$$N = \frac{12.25 \times 11.4 \times 10^{-1}}{8 \times 2 \times 2} \approx 0.80 \cong 1 \text{ photon per atom}$$

N = 1 photon per atom.

(*v*) Time of emission of electrons is 11.4 sec. So the photoelectric emission is not instantaneous in the problem. It takes about 11.4 sec.

In photoelectric emission there is a collision between incident photon and free electron and nucleus, which lasts for very-very short interval of time (10^{-9} sec) hence we say photoelectric emission is instantaneous.

❑❑❑