6 Electromagnetic Induction

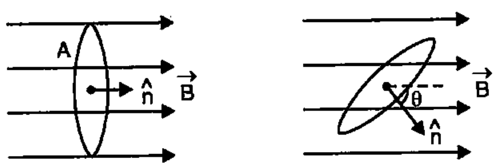
Lesson at a Glance

• Magnetic Flux

It is the total number of magnetic field lines passing through a given area. Mathematically it is given by

$$Q = \int \vec{B} . \vec{ds}$$
$$= B . A \cos \theta$$

where ds is elementary area and B is the intensity of magnetic field and θ is the angle between magnetic field and normal drawn to the surface.



(a) $\phi = BA \cos 0° = BA$ (b) $\phi = BA \cos \theta$

Fig. 6.1

- The unit of magnetic flux is weber (Wb) or Tm^{-1}. Its C.G.S. unit is maxwell

$$1 Wb = 10^8 \text{ max}$$

- To calculate magnetic flux the normal is always drawn outward.

• Electromagnetic Induction

According to Faraday where there is a change in magnetic flux linked with a circuit an induced emf is developed. The induced emf developed is directly proportional to the negative rate of change in magnetic flux.

$$\varepsilon = -\frac{d\phi}{dt}$$

This is called Faraday's law of electromagnetic induction. Negative sign shows that the induced emf opposes the cause of production.

Motional EMF

Let a metallic rod of length l is moved in uniform magnetic field of intensity B downward with velocity v.

The free electrons present in the metal rod experience a magnetic force $F_B = evB$ and move toward one end of the rod due to which one end of the rod becomes negative and the other end relatively positive. Correspondingly an electric field also developed directed from positive end to the negative end. The electrons present in the rod also experience an electric force in the direction opposite to the direction of magnetic force. In equilibrium

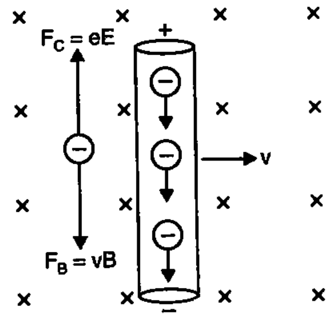


Fig. 6.2

$$eE = evB$$

or $$\frac{\varepsilon}{l} = vB \qquad (\because \ \varepsilon = El)$$

or induced emf developed due to motion of rod in magnetic field called motional emf,

$$\boxed{\varepsilon = Blv}$$

• Direction of Induced EMF

Direction of induced emf is determined by Fleming's right hand rule according to which when we open our right hand keeping thumb, for finger and middle finger mutually perpendicular to each other having direction of field along the fore finger, direction of motion along the thumb, the direction of induced current will be along the middle finger (Fig. 6.3).

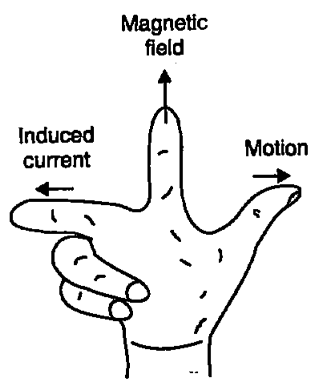


Fig. 6.3

• Lenz's Rule

It states that when there is a change in magnetic flux linked with a circuit, the induced emf is developed in the circuit which opposes the causes of its production.

• Self Induction

When there is a change in current in a circuit, the magnetic field associated with current also changes and correspondingly the flux linked with the circuit changes due to which an induced emf is developed in the circuit. This phenomenon of producing the induced emf due to change in current in the same circuit is called **self induction.**

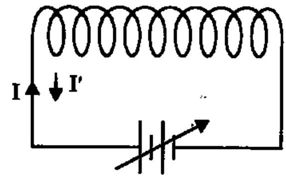


Fig. 6.4

In self induction the change in magnetic flux is directly proportional to the change in current.

$$d\phi \propto dI$$

or

$$d\phi = LdI$$

where L is the coefficient of self induction. Its S.I. unit is henery.

• Coefficient of Self Induction of a Solenoid

$$L = \frac{N^2\mu_0\pi R^2}{l}$$

• Coefficient of Self Induction of Straight Wire

$$L = -\frac{1}{2}\mu_0 r$$

• Coefficient of Self induction due to a Circular Loop

$$L = \frac{1}{2}N\mu_0\pi R$$

• Coefficient of Mutual Induction

When two circuits are placed very close to each other and current changes in one circuit, the magnetic field associated with this current changes and the flux linked with the other circuit also changes due to which an induced emf is developed. *The phenomenon of producing induced emf in a circuit due to change in current in near the circuit is called mutual induction.*

In mutual induction change in magnetic flux linked with the circuit is directly proportional to the change in current in nearby circuit.

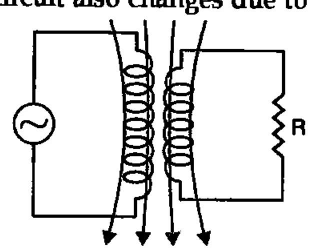


Fig. 6.5

$$d\phi \propto dI$$

$$d\theta = MdI$$

Where M is the coefficient of mutual induction. Its unit is henery.

Applying the Faraday's law, induced emf,

$$\varepsilon = -\frac{d\phi}{dt}$$

or $$\boxed{\varepsilon = -M\frac{dI}{dt}}$$

or $$M = -\frac{\varepsilon}{dI/dt}$$

If $$-\frac{dI}{dt} = 1\ AS^{-1}$$

then $$M = \varepsilon$$

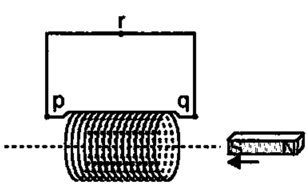
Thus, *coefficient of mutual induction is defined as the induced emf produced in a circuit due to change in current in nearby circuit at the rate of one ampere per second.*

• Coefficient of Mutual Induction between Two Solenoids

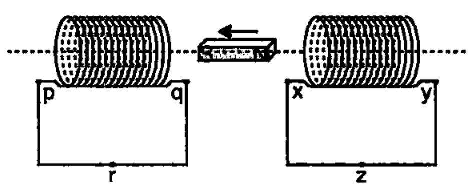
$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

TEXTBOOK QUESTIONS SOLVED

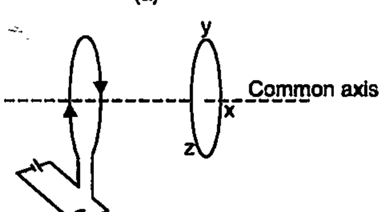
6.1. *Predict the direction of induced current in the situations described by the following figs. (a) to (f).*



(a)

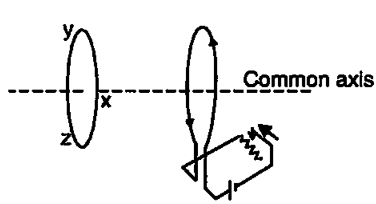


(b)



(Tapping key just closed)

(c)



Rheostat setting being changed

(d)

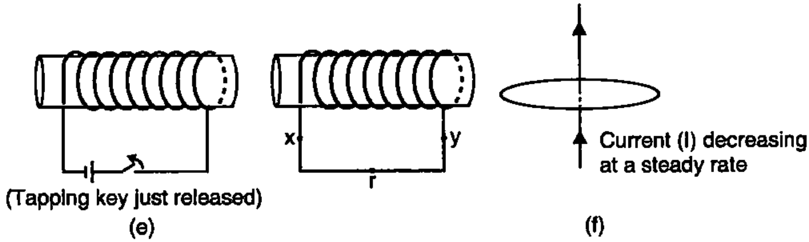


Fig. 6.6

Sol. (*a*) South pole develops at *q*, the induced current should flow clockwise. Therefore, induced current in the coil flows from *qr* to *pq*.

(*b*) Coil *pq* in this case would develop *S*-pole at *q* and coil *XY* would also develop *S* pole at X. Therefore, induced current in coil *pq* will be from *q* to *p* and induced current in the coil *XY* will be from *Y* to *X*.

(*c*) Induced current in the right loop will be along XYZ.

(*d*) Induced current in the left loop will be along ZYX as seen from front.

(*e*) Induced current in the right coil is from *X* to *Y*.

(*f*) Since magnetic lines of force lie in the plane of the loop, no current is induced.

6.2. *Use Lenz's law to determine the direction of induced current in the situation described by Fig.:*

(a) A wire of irregular shape turning into a circular shape;

(b) A circular loop being deformed into a narrow straight wire.

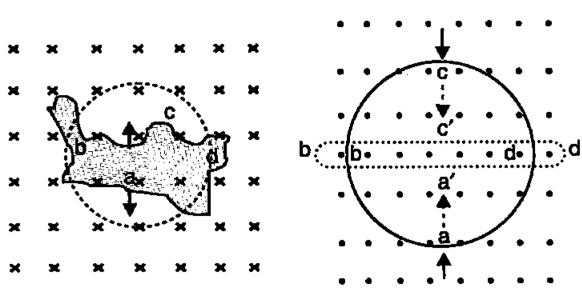


Fig. 6.7

Sol. (*a*) When a wire of irregular shape turns into a circular loop, area of the loop tends to increase. Therefore, magnetic flux linked with the loop increases. According to Lenz's law, the direction of induced current must oppose the magnetic field, for which induced current should flow along *adcba*.

(*b*) In this case, the magnetic flux tends to decrease. Therefore, induced current must support the magnetic field, for which induced current should flow along *adcba*.

6.3. *A long solenoid with 15 turns per cm has a small loop of area 2.0 cm² placed inside normal to the axis of the solenoid. If the current carried by the solenoid changes steadily from 2A to 4A in 0.1 s, what is the induced voltage in the loop while the current is changing?*

Sol. n = 15 turns/cm = 1500 turns/m; A = 2 cm^2 = 2 × 10^{-4} m^2;

I_1 = 2A, I_2 = 4A; Δt = 0.1s

The magnetic field associated with current I_1, $B_1 = \mu_0 n I_1$

The magnetic field associated with current I_2, $B_2 = \mu_0 n I_2$

The change in the flux, $\Delta\phi = (B_2 - B_1) A$

$= 4\pi \times 10^{-7} \times 1500 \times (4-2) \times 2 \times 10^{-4}$

$= 7.6 \times 10^{-7}$ weber

The induced EMF, $|E| = \frac{\Delta\phi}{\Delta t} = \frac{7.6\times10^{-7}}{0.1} = 7.6 \times 10^{-6}$ V

6.4. *A rectangular loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 tesla directed normal to the loop. What is the voltage developed across the cut if velocity of loop is 1 cm s^{-1} in a direction normal to the (i) longer side (ii) shorter side of the loop? For how long does the induced voltage last in each case?*

Sol. (*i*) Given,

Length of loop, l = 8 cm = 8 × 10^{-2} m

Breadth of loop, b = 2 cm = 2 × 10^{-2} m

Strength of magnetic field,

B = 0.3 T

Velocity of loop v = 1 cm/sec = 10^{-2} m/sec

Let the field be perpendicular to the plane of the paper directed inwards.

The magnitude of induced emf,

$\varepsilon = Blv$

$= 0.3 \times 8 \times 10^{-2} \times 10^{-2}$

$= 2.4 \times 10^{-4}$ V

Time for which induced e.m.f. will last is equal to the time taken by the coil to move outside the field is

$$l = \frac{\text{distance travelled}}{\text{velocity}} = \frac{2 \times 10^{-2}}{10^{-2}\,\text{m}} = 2 \text{ sec.}$$

(*ii*) The conductor is moving outside the field normal to the shorter side.

$$b = 2 \times 10^{-2} \text{ m}$$

$\therefore$ The magnitude of induced emf is

$$\varepsilon = B.b.v$$

$$= 0.3 \times 2 \times 10^{-2} \times 10^{-2}$$

$$= 0.6 \times 10^{-4} \text{ V}$$

Time, $$t = \frac{\text{distance travelled}}{\text{velocity}} = \frac{8 \times 10^{-2}}{10^{-2}} = 8 \text{ sec.}$$

6.5. *A 1.0 m long conducting rod rotates with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the e.m.f. developed between the centre and the ring.*

Sol. Here, $l = 1$ m. $\omega = 400\ s^{-1}$, $B = 0.5$ T, $e = ?$

Note that linear velocity of one end of rod is zero and linear velocity of other end is ($l\,\omega$). Average linear velocity

$$v = \frac{0 + l\omega}{2} = l\,\omega/2. \qquad (\because \quad v = r\omega)$$

$$\therefore \quad e = Blv = Bl\frac{(l\,\omega)}{2} = \frac{Bl^2\omega}{2} = \frac{0.5 \times 1^2 \times 400}{2} = 100 \text{ V}$$

6.6. *A circular coil of radius 8.0 cm and 20 turns rotates about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of magnitude 3×10^{-2} T. Obtain the maximum and average e.m.f. induced in the coil. If the coil forms a closed loop of resistance 10 Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?*

Sol. Flux through each turn,

$$\boxed{\phi = \vec{B} \cdot \vec{A}} = BA \cos \theta$$

or $$\phi = B\,\pi r^2 \cos(\omega t)$$

For N turns, $\boxed{\phi_T = NB\,\pi r^2 \cos(\omega t)}$

The induced e.m.f, $|\varepsilon| = \dfrac{d\phi_T}{dt}$

$$= \frac{d\left[NB\,\pi r^2 \cos(\omega t)\right]}{dt}$$

or, $\boxed{|\varepsilon| = NB\,\pi r^2\,\omega \sin(\omega t)}$

The maximum e.m.f,

$$\varepsilon_0 = NB\,\pi r^2\,\omega$$
$$= 20 \times 50 \times \pi \times 64 \times 10^{-4} \times 3.0 \times 10^{-2}$$
$$= 0.603 \text{ V}$$

The average e.m.f, over a cycle = 0

The maximum current,

$$I_0 = \frac{\varepsilon}{R} = \frac{0.603}{10} = 0.0603 \text{ A}$$

Power loss, $P = \dfrac{1}{2}E_0 I_0 = \dfrac{1}{2} \times 0.603 \times 0.0603 = 0.018 \text{ W}$

The induced current causes a restoring torque in the coil. An external source is responsible for the supply of energy for this torque. So we can say that source of this power is the external rotor.

6.7. *A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 ms^{-1} at right angles to the horizontal component of the earth's magnetic field 0.30 × 10^{-4} Wb m^{-2}.*

(a) What is the instantaneous value of the e.m.f. induced in the wire?

(b) What is the direction of the e.m.f.?

(c) Which end of the wire is at higher electrical potential?

Sol. Here, $l = 10$ m, $v = 5.0$ ms^{-1}, $B = 0.30 \times 10^{-4}$ T

(*a*) $\boxed{e = Blv} = 0.30 \times 10^{-4} \times 10 \times 5.0 = 1.5 \times 10^{-3}$ V.

(*b*) According to Fleming's right hand rule, the direction of induced e.m.f. is from west to east.

(*c*) West end of the wire must be at higher electric potential.

6.8. *Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average e.m.f. of 200 V induced, give an estimate of the self-inductance of the circuit?*

Sol. Here, $\frac{dI}{dt} = \frac{(I_2 - I_1)}{t} = \frac{0{\cdot}0 - 5{\cdot}0}{0{\cdot}1} = -50\ As^{-1}$

$e = 200$ V, L =?

As $\boxed{|e| = L\left|\frac{dI}{dt}\right|}$ $\therefore$ $L = \frac{|e|}{|dI/dt|} = \frac{200}{50} = 4$H.

6.9. *A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change in flux linkage with the other coil?*

Sol. Given, $M = 1.5$ H

$I = 20$ A

$t = 0.5$ s

Using formula, $\phi = MI$

$= 1.5 \times 20$

$Q = 30$ H.

6.10. *A jet plane is travelling towards west at the speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of 5×10^{-4}T and the dip angle is 30°.*

Sol. Here, $v = 1800\ \text{km h}^{-1} = \frac{1800 \times 1000}{60 \times 60} = 500$ m/s

Earth's field, $B = 5.0 \times 10^{-4}$ T

Angle of dip, $\delta = 30°$

Length of wing $= 25$ m

The vertical component (B_v) of earth's field is normal to both wings and the direction of motion.

$\therefore$ $\boxed{B_v = B \sin \delta}$

$= 5.0 \times 10^{-4} \sin 30°$

$B_v = 5 \times 10^{-4} \times \frac{1}{2} = 2.5 \times 10^{-4}$ T

Induced e.m.f. produced

$\boxed{B_v = B_v vl} = 2.5 \times 10^{-4} \times 500 \times 25$

$= 3.125$ V.

6.11. *Suppose the loop in Exercise 4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of 0.02 T s^{-1}. If the cut is joined and the loop has a resistance of 1.6 Ω, how much power is dissipated by the loop as heat? What is the source of this power?*

Sol. Using formula, $|\varepsilon| = A\dfrac{dB}{dt}$ $\quad E = \dfrac{d\theta}{dt}$ $\quad (\because \ \theta = B.A.)$

or, $|\varepsilon| = 8 \times 10^{-2} \times 2 \times 10^{-2} \times 0.02$ V

$= 3.2 \times 10^{-5}$ V

$$(I_0) \text{ Induced current} = \frac{|\varepsilon|}{R} = \frac{3.2\times10^{-5}}{1.6}\text{A} = 2\times10^{-5}\text{A}$$

$$\text{Power loss} = I_0^2 R = (2 \times 10^{-5})^2 \times 1.6 \text{ W}$$

$$= 6.4 \times 10^{-10} \text{ A.}$$

6.12. *A square loop of side 12 cm with its sides parallel to X and Y-axes is moved with a velocity of 8 cm s^{-1} in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of 10^{-3} T cm^{-1} along the negative x-direction (i.e., it increases by 10^{-3} T cm^{-1} as one moves in negative x-direction), and it is decreasing in time at the rate of 10^{-3} Ts^{-1}. Determine the direction and magnitude of the induced current in the loop if its resistance is 4.5 m Ω.*

Sol. $A = (12 \times 10^{-2}) = 144 \times 10^{-4}\ m^2$

$v = 8$ cm/s $= 8 \times 10^{-2}$ m/s

$$\frac{dB}{dt} = 10^{-3} \text{ T/sec}$$

$$\frac{dB}{dx} = 10^{-3} \text{ T/cm} = 10^{-1} \text{ T/m}$$

Induced e.m.f. due to change of magnetic field B with time t

$$\varepsilon_1 = \frac{dQ}{dt} = \frac{dB.A}{dt} = \frac{A.dB}{dt} = 144 \times 10^{-4} \times 10^{-3}$$

$$\varepsilon_1 = 144 \times 10^{-7} \text{ V} \qquad ...(I)$$

Induced e.m.f. due to change of magnetic field B. with distance (x)

$$\varepsilon_2 = \frac{dQ}{dt} = \frac{d}{dt}BA = A \cdot \frac{dB}{dt}$$

$$= A \cdot \frac{dB}{dx} \cdot \frac{dx}{dt} = 144 \times 10^{-4} \times 10^{-1} \times 8 \times 10^{-2}$$

$$\varepsilon_2 = 1152 \times 10^{-7}$$

Total e.m.f. $= \varepsilon_1 + \varepsilon_2 = 144 \times 10^{-7} + 1152 \times 10^{-7}$

$$= 1296 \times 10^{-7} \text{ V}$$

$$\varepsilon = 129.6 \times 10^{-6} \text{ V}$$

$$R = 4.5 \text{ milli ohm}$$

Induced current $= \frac{\varepsilon}{R} = \frac{129.6 \times 10^{-6}}{4.5 \times 10^{-3}} \cong 2.9 \times 10^{-2}$ A.

6.13. *It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area 2 cm² with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction. The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is 7.5 mC. The combined resistance of the coil and the galvanometer is 0.50 Ω. Estimate the field strength of magnet.*

Sol. $\phi_1 = B.A. \qquad \phi_2 = 0 \quad \text{(Outside)}$

$$E = -N \cdot \frac{d\phi}{dt}$$

$$I.R = -N \frac{(\phi_2 - \phi_1)}{(t_2 - t_1)}$$

$$\frac{q}{t} R = -N \frac{(0 - B.A)}{t}$$

$$qR = +NBA$$

$$B = \frac{qR}{NA} = \frac{7.5 \times 10^{-3} \times .50}{25 \times 2 \times 10^{-4}} = \frac{75 \times 25 \times 10^{-3+4}}{25 \times 2 \times 1000}$$

$$= -75 \times 10^{1-3} = 0.75 \text{ Weber/m}^2.$$

6.14. *Figure shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, B = 0.50 T, resistance of the closed loop containing the rod = 9.0 mΩ. Assume the field to be uniform.*

(a) Suppose K is open and the rod is moved with a speed of 12 cm s⁻¹ in the direction shown. Give the polarity and magnitude of the induced emf.

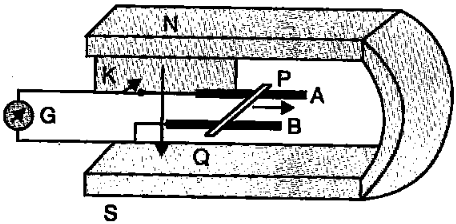


Fig. 6.8

(b) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed ?

(c) With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod. Explain.

(d) What is the retarding force on the rod when K is closed?

(e) How much power is required (by an external agent) to keep the rod moving at the same speed (= 12 cm s^{-1}) when K is closed? How much power is required when K is open?

(f) How much power is dissipated as heat in the closed circuit? What is the source of this power ?

(g) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

Sol. (*a*) The magnitude of the induced emf is given by

$$\varepsilon = B\,v\,l = 0.50 \times 0.12 \times 0.15 \text{ volt}$$
$$= 9 \times 10^{-3} \text{ volt} = 9 \text{ mV}$$

P is positive end and Q is negative end.

(*b*) Yes. When *k* is closed, the excess charge is maintained by the continuous flow of current.

(*c*) Magnetic force [$F_m = - e\,(v + B)$] is cancelled by the electric force [$F_e = eE$] set up due to the excess charge of opposite signs at the ends of the rod.

(*d*) Induced current, $I = \dfrac{e}{R} = \dfrac{9 \times 10^{-3}}{9 \times 10^{-3}} = 1\text{A}$

Retarding force on the rod

$$F = B\,I\,l = 0.5 \times 1 \times 0.15 = 7.5 \times 10^{-2} \text{ N}$$

(*e*) Power expended by an external agent against the above retarding force to keep the rod moving uniformly at 12 cm/s

$P = F . v$

$P = 75 \times 10^{-3} \times 12 \times 10^{-2}$ W $= 9.0 \times 10^{-3}$ W

(*f*) Power dissipated as heat $= I^2 R = 1^2 (9 \times 10^{-3})$

$= 9 \times 10^{-3}$ W

Source of this power external agent which keeps rod in motion, against magnetic retarding force.

(*g*) When the permanent magnet is rotated to a vertical position, the field becomes parallel to rails. The motion of rod will not cut across the lines of magnetic field and hence no e.m.f. is induced.

6.15. *An air-cored solenoid with length 30 cm, area of cross-section 25 cm² and number of turns 500, carries a current 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.*

Sol. Given, $l = 0.30$ m, $A = 25 \times 10^{-4}$ m²

$N = 500$, $I = 2.5$ A

Initial magnetic flux, $\boxed{\phi_1 = NBA} = N\left(\dfrac{\mu_0 NI}{l}\right)A$

$$\phi_1 = \frac{\mu_0 N^2 IA}{l}$$

or, $\phi_1 = 4 \times \dfrac{22}{7} \times 10^{-7} \times 500 \times 500 \times 2.5 \times 25 \times 10^{-4} \times \dfrac{1}{0.30}$

or, $\phi_1 = 6.55 \times 10^{-3}$ Wb

Final magnetic flux, $\phi_2 = 0$

Change of flux, $\Delta \phi_B = 0 - 6.55 \times 10^{-3} = - 6.55 \times 10^{-3}$ Wb

Corresponding time interval, $\Delta t = 10^{-3}$ s

Average e.m.f. induced across the open switch

$$= -\frac{\Delta\phi_B}{\Delta t} = -\frac{-6.55 \times 10^{-3}}{10^{-3}} = 6.55 \text{ V}.$$

6.16. (*a*) *Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in Fig.*

(*b*) *Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, v = 10 m/s; calculate the induced emf in the*

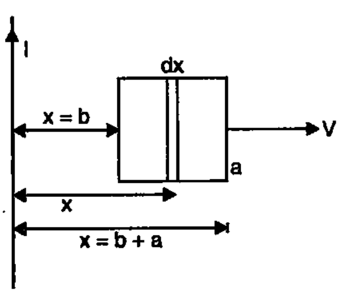


Fig. 6.9

loop at the instant when $x = 0.2m$. Take $a = 0.1$ m and assume that the loop has a large resistance.

Sol. Consider a strip of width dx (of the square loop) at a distance x from the wire carrying current.

Magnetic field due to current carrying wire at a distance x from the wire is $B = \frac{\mu_0 I}{2\pi x}$

Small amount of magnetic flux associated with the strip

$$d\phi = B\ dA = \frac{\mu_0 I}{2\pi x}(a\,dx)$$

Magnetic flux linked with the square loop

$$\phi = \frac{\mu_0 Ia}{2\pi}\int_{x=b}^{x=a+b}\frac{dx}{x} = \frac{\mu_0 Ia}{2\pi}[\log_e x]_{x=b}^{x=a+b}$$

$$\phi = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a+b}{b}\right) = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a}{b}+1\right)$$

As $\phi = MI$

$$\therefore \quad MI = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a}{b}+1\right)$$

$$M = \frac{\mu_0 a}{2\pi}\log_e\left(\frac{a}{b}+1\right)$$

(*a*) Induced *emf* in the loop

$$e = B\,l\,v = \left(\frac{\mu_0 I}{2\pi x}\right)l\,v = \frac{4\pi\times10^{-7}\times50}{2\pi\times0.2}\times0.1\times10$$

$$= 5\times10^{-5}\text{ volt}$$

❑❑❑