9 Ray Optics and Optical Instruments

Lesson at a Glance

The direction of propagation of light energy is called ray and the study of the behaviour of light ray is called ray optics or geometrical optics.

• Reflection of Light

When light falls on a smooth and rigid surface and comes back to the same medium, the phenomenon is called *reflection of light*. In reflection, the angle of incidence and the angle of reflection are always equal. The image formation is the consequence of reflection of light. If surface is not smooth and rigid, the reflected rays will not be parallel and image will not formed. This reflection is called irregular or diffused reflection. [Fig. 9.1]

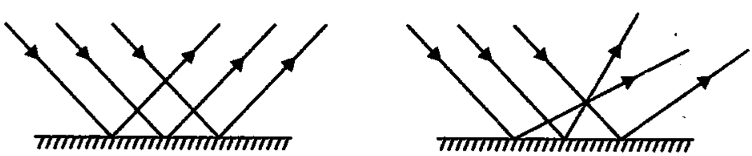

Fig. 9.1

• Reflection in Spherical Mirrors

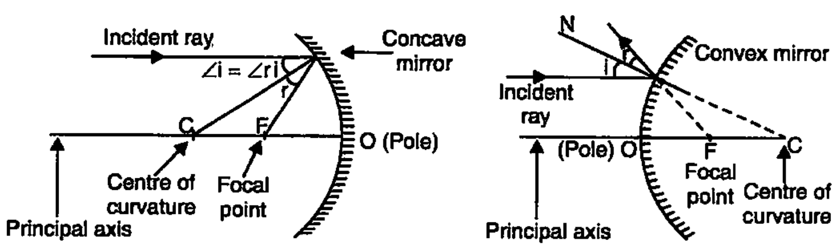

Fig. 9.2

• Relation between Focal Length and Radius of Curvature

$$R = 2f$$

• Images in Mirrors

If rays emanating from a point actually meet at another point after reflection the images of the point is formed at that point. It is called *real* image. If the rays do not actually meet, they appear to meet at the point of image formulation, it is called virtual image.

• Sign Convention

- All distances are measured from the pole.
- Distances measured in the direction of incident ray are taken as positive.
- Distance measured in the opposite direction of incident ray are taken as negative.

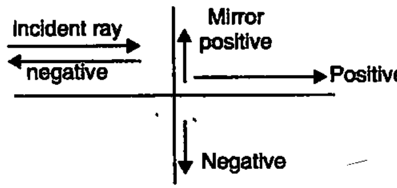

Fig. 9.3

- Distance above the principal axis are positive axis are positive and below the principal axis are negative. [Fig. 9.3]

• Mirror Formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

- Mirror formula will remain same for convex mirror. Only there will be a difference in sign convention.

• Magnification

It is the ratio of height of the image to the height of the object.

$$m = \frac{v - f}{f}$$

• Refraction of Light

When light passes from one medium to another its speed changes and for oblique incidence it deviates from its path and the phenomenon is called refraction of light.

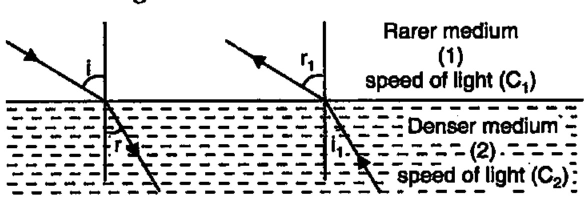

Fig. 9.4

$$n_{12} = \frac{C_1}{C_2} = \frac{\sin i}{\sin r} \quad ...(i)$$

The refractive index of medium 1 with respect to medium 2,

$$n_{21} = \frac{C_2}{C_1} = \frac{\sin i_1}{\sin r_1} \quad ...(ii)$$

From Eqs. (*i*) and (*ii*)

$$n_{21} = \frac{1}{n_{21}}$$

• Critical Angle and Total Internal Reflection

When light passes obliquely, from denser medium to rarer medium the angle of incidence remains smaller than angle of refraction.

- The angle incidence for which angle of refraction becomes 90° is called critical angle.

  In Fig. 9.5

$$n_{12} = \frac{\sin i_c}{\sin 90°} = \sin i_c$$

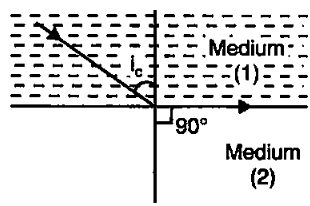

Fig. 9.5

or critical angle, $i_c = \sin^{-1}(n_{12})$

or $$i_c = \sin^{-1}\left(\frac{1}{n_{21}}\right)$$

- When Light passes from denser to rarer medium and angle of incidence becomes greater than the critical angle, it comes back to the same medium and the phenomenon is called Total Internal Reflection.

• Refraction through Lenses

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}.}$$ This is called lens formula.

• Power of the Lens

$$P = \frac{1}{f(m)}$$

• Refraction in Prism

When light passes through a prism refraction takes place at both the surfaces of the prism.

• Dispersion

The splitting of white light into constituent colours is called Dispersion. A prism causes deviation as well as dispersion If δ_v, δ_r and δ_y are the deviations caused by prism in violet, red and yellow colours, the angular dispersion

$\phi = \delta_v - \delta_r = (\mu_v - \mu_r)$ A for small angled prism.

• Dispersion Power

The ratio of (angular) dispersion to the deviation of the mean ray (yellow) is called the dispersion power of the prism. It is denoted by ω.

$$\omega = \frac{\theta}{\delta} = \frac{\delta_v - \delta_R}{\delta}$$

where δ is the deviation of the mean ray; δ_v and δ_R are the deviations of the violet and the red rays respectively.

• Human Eye

Power of accommodation: The ability of human eye to adjust its focus depending upon the distance of the object is known as the power of accommodation.

Astigmatism: It occurs when cornea is not spherical in shape. Due to this defect, the person is unable to focus on both the horizontal as well as vertical lines. This defect can be corrected by the use of cylindrical lens.

• Microscope

- Magnifying power of a compound microscope (*M*) = magnification by the objective × magnifying power of the eyepiece

i.e.,

$$(M)\ mp = \frac{v_1}{u_1} \times \left(1 - \frac{v_2}{f_e}\right)$$

where, u_1 = distance of the object from the objective

v_1 = distance (from the objective) of the image formed by the objective

f_e = focal length of the eyepiece

v_2 = distance of the final image from the eyepiece.

• Telescope

A telescope consists of two convergent lenses called the objective and the eyepiece. The objective has much longer focal length than the eyepiece.

(*i*) The magnifying power, *M*, of refracting telescope is given by

$$M = \frac{f_0}{f_e}$$

and $L = (f_0 + f_e)$; L = length of the telescope.

(*ii*) For the final image is formed at the least distance of distant vision,

$$M = \frac{f_0}{f_e}\left(1 + \frac{F_e}{D}\right)$$

(*iii*) The resolving power of a telescope is given by

$$\theta = \frac{1.22\,\lambda}{d}$$

where, λ = wavelength of light

d = diameter of the objective of the telescope

θ = angle subtended by the point object on the objective.

TEXTBOOK QUESTIONS SOLVED

9.1. *A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?*

Sol. $u = -27$ cm, $f = \frac{R}{2} = \frac{-36}{2} = -18$ cm

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{18} - \frac{1}{-27}$$

or $$\frac{1}{v} = \frac{1}{27} - \frac{1}{18}$$

On simplification, $v = -54$ cm

The negative sign indicates that the image is formed in front of the mirror. Thus, the screen should be placed at a distance of 54 cm in front of the mirror.

Now, $$m = \frac{I}{O} = -\frac{v}{u}$$

$\therefore$ $$\frac{I}{2.5} = \frac{-54}{-27} \text{ or } I = -5 \text{ cm}$$

As size of image is (-)ive. So image is inverted and real.

When the candle is moved closer to mirror, the screen would have to be moved farther and farther. However, when the candle is closer than 18 cm from the mirror, the image would be virtual and therefore cannot be collected on the screen.

9.2. *A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.*

Sol. Given, $u = -12$ cm, $f = +15$ cm

$O = 4.5$ cm

As, $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} = \frac{9}{60}$$

$$v = 60/9 = 6.7 \text{ cm}$$

$\therefore$ Image formed at 6.7 cm at the back of the mirror.

As, $$m = \frac{I}{O} = -\frac{v}{u}$$

$\therefore$ $$\frac{I}{4.5} = -\frac{6.7}{-12} \text{ or } I = \frac{6.7 \times 4.5}{12} = 2.5 \text{ cm.}$$

$\therefore$ Image is erect, and of course virtual.

As needle is moved farther from the mirror, image moves away from the mirror (upto *F*) and goes on decreasing is size.

9.3. *A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the mircoscope have to be moved to focus on the needle agains?*

Sol. *Case I:* Given, real depth = 12.5 cm; apparent depth = 9.4 cm

As, $$\boxed{\mu = \frac{\text{real depth}}{\text{apparent depth}}}$$

or $$\mu = \frac{12.5}{9.4} = 1.33$$

Case II: μ = 1.63, real depth = 12.5 cm

$$\text{apparent depth} = \frac{\text{real depth}}{\mu}$$

or $$\text{A.D.} = \frac{12.5}{1.63} = 7.67 \text{ cm}$$

∴ Distance through which microscope has to be moved downward.

$$= (9.4 - 7.67) \text{ cm} = 1.73 \text{ cm.}$$

9.4. *Figures 9.6 (a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. 9.6 (c)].*

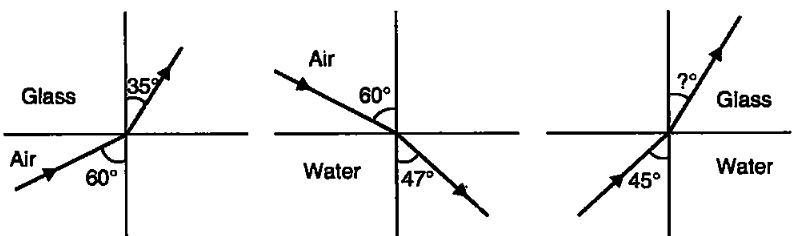

Fig. 9.6

Sol. In Fig. 9.6 (*a*)

$$i = 60°, \quad r = 35°$$

$$^{a}\mu_{g} = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 35°} = \frac{0.8660}{0.5736} = 1.51$$

In Fig. 9.6 (*b*) $i = 60°, \quad r = 41°$

$$^{a}\mu_{w} = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 41°} = 1.32$$

In Fig. 9.6 (*c*) $i = 45°$

$$^{w}\mu_g = \frac{^{a}\mu_g}{^{a}\mu_w} = \frac{\sin i}{\sin r}$$

$$\frac{1.51}{1.32} = \frac{\sin 45°}{\sin r} = \frac{0.7071}{\sin r}$$

$$\therefore \quad \sin r = \frac{1.32 \times 0.7071}{1.51} = 0.6181$$

$$r = 38.2°.$$

9.5. *A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)*

Sol. Let O be the bulb at the bottom of the tank.

$$OA = 80 \text{ cm}$$

The rays of light emitted from O will be refracted into air only if the angle of incidence is less than critical angle i_c. When the angle of incidence is equal to critical angle i_c, the light will not be refracted into air. Instead, it will graze the air-water interface. Thus, the light will appear to come out of a cone having vertex angle $2i_c$. [If the angle of incidence exceeds the critical of angle, then the rays of light will be totally reflected.]

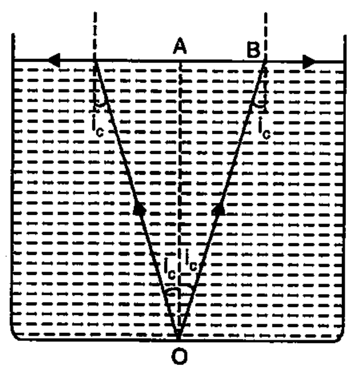

Fig. 9.7

We know that

$$\sin i_c = \frac{1}{_a\mu_w}$$

or $$i_c = \sin^{-1}\left(\frac{1}{_a\mu_w}\right)$$

$$= \sin^{-1}\left(\frac{1}{1.33}\right)$$

$$= \sin^{-1}(0.752) = 48.76°$$

Radius of patch of light = *AB*

Now, $$\tan i_c = \frac{AB}{OA} \text{ or } AB = OA \tan i_c$$

∴ Radius of patch of light

$$= 80 \tan 48.76° = 80 \times 1.14 \text{ cm} = 91.2 \text{ cm}$$

Area of patch of light = $\pi(91.2)^2$ cm^2 = 26140.5 cm^2 = 2.61 m^2.

9.6. *A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.*

Sol. $A = 60°, \delta m = 40°$

$$_a\mu_g = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin A/2}$$

or $$_a\mu_g = \frac{\sin 50°}{\sin 30°} = \frac{0.766}{0.54} = 1.532$$

After the prism is placed in water,

$$_w\mu_g = \frac{\sin\left(\frac{A+\delta'_m}{2}\right)}{\sin A/2}$$

or, $$\frac{^a\mu_g}{^a\mu_2} = \frac{\sin\left(\frac{60+\delta'_m}{2}\right)}{\sin 30°}$$

∴ $$\sin\left(30°+\frac{\delta'_m}{2}\right) = \frac{1}{2}\times\frac{1.532}{1.33} = 0.5759$$

or, $$30° + \frac{\delta'_m}{2} = 35°10'$$

on simplification, $\delta'_m = 10°20'$.

9.7. *Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?*

Sol. Given, $\mu = 1.55$, $R_1 = R$ and $R_2 = -R$, $f = 20$ cm.

Since, $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \qquad \frac{1}{20} = (1.55 - 1)\left(\frac{1}{R}+\frac{1}{R}\right) = \frac{1.10}{R}$$

Hence, $R = 20 \times 1.1 = 22$ cm.

9.8. *A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of a focal length 20 cm, and (b) a concave lens of focal length 16 cm?*

Sol. Here, the point P on the right of the lens acts as a virtual object,

$$u = 12 \text{ cm}$$

(a) $f = 20$ cm

Since, $$\frac{1}{v} = \frac{1}{f}+\frac{1}{u}$$

$$\therefore \qquad \frac{1}{v} = \frac{1}{20}+\frac{1}{12} = \frac{3+5}{60} = \frac{8}{60}$$

or, $v = 60/8 = 7.5$ cm.

Image is at 7.5 cm to the right of the lens, where the beam converges.

(b) $f = -16$ cm, $u = 12$ cm,

$$\therefore \qquad \frac{1}{v} = \frac{1}{f}+\frac{1}{u} = -\frac{1}{16}+\frac{1}{12} = \frac{-3+4}{48} = \frac{1}{48}$$

$$v = 48 \text{ cm}$$

Hence the image is at 48 cm to the right of the lens, where the beam would converge.

9.9. *An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?*

Sol. $O = 3.0$ cm, $u = -14$ cm, $f = -21$ cm

Since, $$\frac{1}{v} = \frac{1}{f}+\frac{1}{u} = -\frac{1}{21}-\frac{1}{14} = -\frac{35}{14\times 21}$$

or, $v = -8.4$ cm.

The image is located 8.4 cm from the lens on the same side as the object.

As, $$m = \frac{I}{O} = \frac{v}{u}$$

$$\therefore \qquad I = \frac{v}{u}\times O = \frac{-8.4}{-14}\times 3 = 1.8 \text{ cm}$$

As size of image is +ve. So, image is errect and virtual of smallar size.

As the object is moved away from the lens, the virtual image moves towards the focus of the lens but never beyond. The image progressively diminishes in size.

9.10. *What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.*

Sol. Here, $f_1 = 30$ cm, $f_2 = -20$ cm

Since, $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\therefore \quad \frac{1}{f} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = \frac{-1}{60}$$

$$f = -60 \text{ cm}$$

$\therefore$ The combination of lenses behaves as a concave lens. The system is not converging.

9.11. *A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm), and (b) at infinity? What is the magnifying power of the microscope in each case?*

Sol. $f_o = 2$ cm, $f_e = 6.25$ cm, $u_o = ?$

(*a*) For eyepiece, $v_e = -25$ cm, $f_e = 6.25$ cm, $u_e = ?$

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

or $$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$\therefore \quad \frac{1}{u_e} = \frac{1}{-25} - \frac{1}{6.25} = -\frac{1}{5}$$

or, $u_e = -5$ cm

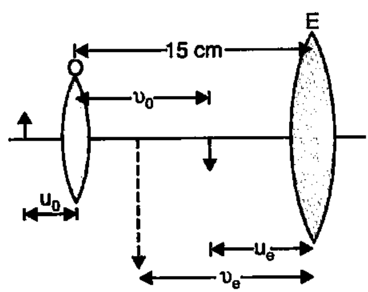

Fig. 9.8

Now, $v_o = 15 - |u_e| = 10$ cm

For objective, $$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \text{ or } \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0}$$

or, $\frac{1}{u_0} = \frac{1}{10} - \frac{1}{2} = -\frac{2}{5}$ or $u_0 = \frac{-5}{2}$ cm = –2.5 cm

$$M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

Magnifying power, $M = \frac{10}{-2.5}\left(1 + \frac{25}{6.25}\right) = -20$

(*b*) The final image will be formed at infinity only if the image formed by the objective is in the focal plane of the eyepiece.

$\therefore \quad |u_e| = f_e = 6.25$ cm;

$|v_0| = (15 - 6.25)$ cm = 8.75 cm

Now, $\frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{8.75} - \frac{1}{2} = \frac{-6.75}{8.75 \times 2}$

$\therefore \quad u_0 = -\frac{8.75 \times 2}{6.75}$ cm or $u_0 = -2.59$ cm

Magnifying power, $M = \frac{8.75}{-2.59} \times \frac{25}{6.25} = -13.5.$

9.12. *A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eye piece of focal length 2.5 cm can bring an object placed 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope?*

Sol. Here, $u_0 = -0.9$ cm, $f_o = 0.8$ cm

As, $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$

$\therefore \quad \frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{0.8} - \frac{1}{0.9} = \frac{1}{7.2}$

or, $v_o = 7.2$ cm

Now for the eyepiece, we have

$f_e = 2.5$ cm, $v_e = -D$, $u_e = ?$

$\therefore \quad \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{2.5} = -\frac{11}{25}$

or, $u_e = -\frac{25}{11} = -2.27$ cm

Separation between the two lenses

$$= v_0 + |u_e| = 7.2 + 2.27 = 9.47 \text{ cm}$$

Magnifying power, $M = M_0 \times M_e$

$$M = \frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right) = \frac{7.2}{0.9}\left(1+\frac{25}{25}\right)$$

$$= 8 \times 11 = 32.$$

9.13. *A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?*

Sol. Here, $f_0 = 144$ cm, $f_e = 6.0$ cm

As, $\boxed{m = \frac{-f_0}{f_e}} = \frac{-144}{6.0} = -24$

and $\boxed{L = f_0 + f_e} = 144 + 6.0 = 150$ cm.

9.14. *(a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of telescope ?*

(b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^6 m and radius of lunar orbit is 3.8×10^8 m.

Sol. Given, $f_o = 15$ m, $f_e = 1.0$ cm $= 10^{-2}$ m

(*a*) Angular magnification $= \frac{f_o}{f_e} = \frac{15}{10^{-2}} = 1500$

(*b*) If d is diameter of the image, then angle subtended by diameter of moon

$$= \frac{3.48 \times 10^6}{3.8 \times 10^8}$$

and, angle subtended by image $= \frac{d}{f_o} = \frac{d}{15}$

$$\therefore \quad \frac{d}{15} = \frac{3.48 \times 10^6}{3.8 \times 10^8} \quad \text{or,} \quad d = \frac{3.48 \times 15 \times 10^{-2}}{3.8}$$

$$= 13.73 \times 10^{-2} \text{ m} = 13.73 \text{ cm}.$$

9.15. *Use the mirror equation to deduce that:*

(a) an object placed between f and 2f of a concave mirror produces a real image beyond 2f.

(b) a convex mirror always produces a virtual image independent of the location of the object.

(c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

[Note: *This exercise helps you deduce algebraically properties of images that one obtains from explicit ray diagrams.*]

Sol. (*a*) The mirror formula is

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\therefore \quad \frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

Now for a concave mirror, $f < 0$ and for an object on the left $u < 0$.

$$2f < u < f$$

or, $$\frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$$

or, $$-\frac{1}{2f} < -\frac{1}{u} < -\frac{1}{f}$$

or, $$\frac{1}{f}-\frac{1}{2f} < \frac{1}{f}-\frac{1}{u} < \frac{1}{f}-\frac{1}{f}$$

or, $$\frac{1}{2f} < \frac{1}{v} < 0$$

This implies that $v < 0$ so that real image is formed on left. Also the above inequality implies that

$$2f > v$$

or, $$|2f| > |v| \qquad [\because 2f \text{ and } v \text{ are } -ve]$$

i.e., real image is formed beyond $2f$.

(*b*) Now, for convex mirror, $f > 0$ and for an object of left, $u < 0$.

From mirror formula

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

$$\Rightarrow \qquad \frac{1}{v} > 0 \text{ or } v > 0$$

This shows that whatever be the value of u, a convex mirror form a virtual image on the right.

(c) For convex mirror $f > 0$ and for an object on left $u < 0$, so from mirror formula

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \qquad [\because v \text{ is +ive and } u \text{ is –ive}]$$

$$\Rightarrow \qquad \frac{1}{v} > \frac{1}{f} \text{ or } v < f \quad (\therefore -\frac{1}{u} \text{ is a '+ve' quantity})$$

This shows that the image is located between the pole and the focus of the mirror. Also from the mirror formula

$$\frac{1}{v} > -\frac{1}{u} \qquad \left(\because \frac{1}{f} > 0\right)$$

Multiply v to both side

$$\therefore \qquad \frac{v}{v} > -\frac{u}{v} \qquad [\because v \text{ is +ive}]$$

$$1 > m \qquad (\because u < 0)$$

$\therefore$ Magnitude of magnification, $m = \dfrac{v}{|u|} < 1.$

So the image is diminished in size.

(d) From the mirror formula, for a concave mirror, $f < 0$ and for an object located between the pole and focus of a concave mirror,

$$f < u < 0$$

$$\therefore \qquad \frac{1}{f} > \frac{1}{u} \quad \text{or,} \quad \frac{1}{f} - \frac{1}{u} > 0$$

$$\text{or,} \qquad \frac{1}{v} > 0 \text{ or } v > 0 \qquad \left(\because \frac{1}{v} = \frac{1}{f} - \frac{1}{u}\right)$$

i.e., a virtual image is formed on the right.

$$\text{Also,} \qquad \frac{1}{v} < \frac{1}{|u|} \quad \text{or} \quad v > |u|$$

$$\therefore \qquad |m| = \frac{v}{|u|} > 1.$$

9.16. *A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?*

Sol. Image of pin appear through glass slab

$$\mu = \frac{\text{real depth}}{\text{apparent depth}}$$

$$\text{apparent depth} = \frac{\text{real depth (thickness of glass slab)}}{\mu}$$

$$= \frac{15}{1.5} = 10 \text{ cm}$$

∴ Image left up by = 15 − 10 = 5 cm.

Location of slab will not affect the answer is any way.

9.17. *(a) Figure shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflection inside the pipe take place, as shown in the figure.*

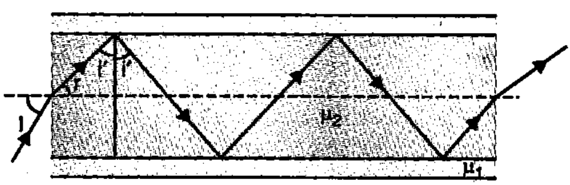

Fig. 9.9

(b) What is the answer if there is no outer covering of the pipe?

Sol. (*a*) $\mu_2 = 1.68, \mu_1 = 1.44$

$$\mu = \frac{\mu_2}{\mu_1} = \frac{i}{\sin i_c}$$

∴ Critical angle i_c' is given by

$$\sin i_c' = \frac{\mu_1}{\mu_2} = \frac{1.44}{1.68} = 0.8571$$

or, $i_c' = 59°$

Total internal reflection will occur if $i' > i_c'$ *i.e.*, if $i' > 59°$ or when $r > r_{max}$, where

$r_{max} = 90° - 59° = 31°$. By Snell's law

$$\frac{\sin i_{max}}{\sin r_{max}} = 1.68$$

or, $$\sin i_{max} = 1.68 \times \sin r_{max} = 1.68 \times \sin 31°$$

$$= 1.68 \times 0.5150 = 0.8662$$

∴ $$i_{max} = 60°.$$

So, all incident rays which make angles in the range $0 < i < 60°$ with the axis of the pipe will suffer total internal reflection in the pipe.

For the finite length of the pipe, the lower limit on i is determined by the ratio of the diameter to the length of the pipe.

(*b*) If there is no outer covering of the pipe

$$\mu_2 = 1.68, \mu_1 = 1$$

$$\sin i_c' = \frac{\mu_1}{\mu_2} = \frac{1}{1.68} = 0.5952$$

or, $$i_c' = 36.5°$$

Now for $i = 36.5°$, $r = 36.5°$ and $i' = 90 - 36.5° = 53.5°$ which is greater than i_c'. Thus all incident rays of angle in the range $0 < i < 90°$ will suffer total internal reflection.

9.18. *Answer the following questions:*

(*a*) *You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.*

(*b*) *A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?*

(*c*) *A diver under water, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is?*

(*d*) *Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?*

(*e*) *The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?*

Sol. (*a*) Rays converging to a point 'behind' a plane or convex mirror are reflected to a point in front of the mirror on a screen. In other words, a plane or convex mirror can produce a real image if the object is virtual.

(*b*) When the reflected or refracted rays are divergent, the image is virtual. The divergent rays can be converged on to a screen with the help of a suitable converging lens. The convex lens of the eye performs this function precisely. In this case, the 'virtual image' serves as the 'virtual object' for the lens to produce a real image. It may be noted here that the screen is not located at the position of the virtual image. There is no contradiction.

(*c*) Taller. When the object is in rarer medium and the observer is in denser medium, then the "apparent depth" is greater than "real depth". In figure, *AB* represents a fisherman standing on the bank of the lake. The rays of light from the head (*A*) of the fisherman suffer refraction and bend towards the normals. The refracted rays appear to come from *A*′. The fisherman appears as *A*′*B* (>*AB*) *i.e.*, taller than what he actually is.

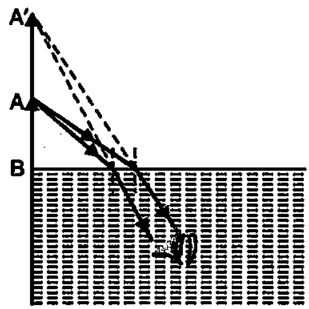

Fig. 9.10

(*d*) The apparent depth of oblique viewing decreases from its value for near-normal viewing.

(*e*) Refractive index of diamond is nearly 2.42. It is much larger than that of ordinary glass (nearly 1.5). The critical angle for diamond is nearly 24°. This is much less than that of glass. A skilled diamond cutter exploits the large range of angles of incidence (in the diamond), 24° to 90°, to ensure that light entering the diamond is totally reflected from many faces before getting out. This produces brilliance *i.e.*, sparkling effect in the diamond.

9.19. *The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?*

Sol. For a real image (on wall), minimum distance between the object and image should be $4f$.

$$\boxed{u = v = uf}$$

$\therefore \quad 4f = 3$ m

$\therefore \quad f = \frac{3}{4}$ m $= 0.75$ m.

9.20. *A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.*

Sol. Here, distance between object and screen $D = 90$ cm

Distance between two locations of convex lens $d = 20$ cm

Since, $$f = \frac{D^2 - d^2}{4D}$$

$$\therefore \quad f = \frac{(90)^2 - (20)^2}{4 \times 90} = \frac{(90+20)(90-20)}{360} = \frac{110 \times 7}{36}$$

or, $f = 21.4$ cm.

9.21. *(a) Determine the ¢effective focal length¢ of the combination of the two lenses having focal lenghs 30 cm and – 20 cm; if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?*

(b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40 cm. Determine the magnification produced by the two-lens system, and the size of the image.

Sol. (*a*) (*i*) Here, $f_1 = 30$ cm, $f_2 = -20$ cm, $d = 8.0$ cm

Let a parallel beam be incident on the convex lens first. If second lens were absent, then

$\therefore \quad u_1 = \infty$ and $f_1 = 30$ cm

As $$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1} \quad \therefore \quad \frac{1}{v_1} - \frac{1}{\infty} = \frac{1}{30}$$

or, $v_1 = 30$ cm

This image would now act as a virtual object for second lens.

$\therefore \quad u_2 = +(30 - 8) = +22$ cm

$f_2 = -20$ cm

Since, $$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} \quad \therefore \quad \frac{1}{v_2} = \frac{1}{-20} + \frac{1}{22}$$

$$= \frac{-11+10}{220} = \frac{-1}{220}$$

$$v_2 = -220 \text{ cm.}$$

∴ Parallel incident beam would appear to diverge from a point 220 – 4 = 216 cm from the centre of the two lens system.

(*ii*) Assume that a parallel beam of light from the left is incident first on the concave lens.

∴ $u_1 = -\infty, f_1 = -20$ cm

As $\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$

∴ $\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{-20} + \frac{1}{-\infty} = -\frac{1}{20}$

$v_1 = -20$ cm

This image acts as a real object for the second lens

$u_2 = -(20 + 8) = -28$ cm, $f_2 = 30$ cm

Since, $\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$

∴ $\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = \frac{1}{30} - \frac{1}{28} = \frac{14-15}{420}$

$v_2 = -420$ cm

∴ The parallel beam appears to diverge from a point 420 – 4 = 416 cm, on the left of the centre of the two lens system.

We finally conclude that the answer depends on the side of the lens system where the parallel beam is incident. Therefore, the notion of effective focal length does not seem to be meaningful here.

(*b*) For convex lens

$u = -40$ cm, $f = 30$ cm, $O = 1.5$ cm

Using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

We get, $\frac{1}{v} - \frac{1}{-40} = \frac{1}{30}$ or, $\frac{1}{v} = \frac{1}{30} - \frac{1}{40} = \frac{1}{120}$

$v = 120$ cm (for real object)

From relation,

$$m = -\frac{v}{u}, \text{ we get}$$

$$m = -\frac{120}{-40} = +3$$

The image formed by the convex lens becomes object for concave lens at a distance of 120 – 8 = 112 cm on the other side.

For concave lens, $f = -20$ cm, $u = +112$ cm (on the other side)

$$v = ?$$

Using lens formula, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Now, $$\frac{1}{v} - \frac{1}{112} = \frac{1}{-20}$$

or, $$\frac{1}{v} = -\frac{1}{20} + \frac{1}{112} = -\frac{23}{560}$$

$$v = -\frac{560}{23} \text{ cm (for virtual object)}$$

From relation, $m = \frac{v}{u}$, we get

$$m = -\frac{560/23}{-112} = -\frac{560}{23} \times \frac{1}{112} = -\frac{5}{23}$$

Net magnification $= 3 \times \left(\frac{-5}{23}\right) = -\frac{15}{23} = 0.652$ (negative due to virtual image)

As $$m = \frac{I}{0}$$

$I = m \times 0 = 0.652 \times 1.5 = 0.98$ cm (size of final image).

9.22. *At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.*

Sol. The refracted ray in the prism is incident on the second face at critical angle i_c.

Now, $60° + 90° - r + 90° - i_c = 180°$

$$r_1 + r_2 = \angle A$$

or, $r = 60° - i_c$

Now, $\sin i_c = \dfrac{1}{\mu} = \dfrac{1}{1.524}$

or, $i_c = \sin^{-1}\left(\dfrac{1}{1.524}\right)$

or, $i_c \simeq 41°$

$\therefore$ $r = 60° - 41° = 19°$

Using Snell's law,

$\sin i = \sin 19° \times 1.524$
$= 0.4962$
$i = \sin^{-1}(0.4962)$
$= 29.75°.$

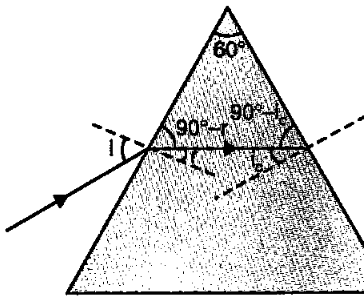

Fig. 9.11

9.23. *You are given prisms made of crown glass and flint glass with a wide variety of angles. Suggest a combination of prisms which will*

(i) deviate a pencil of white light without much dispersion.

(ii) disperse (and displace) a pencil of white light without much deviation.

Sol. (*i*) For no dispersion, angular dispersion produced by two prisms should be zero.

Angular dispersion by crown glass prism + angular dispersion by flint glass prism = 0

i.e., $(\mu_b - \mu_r)A + (\mu_b' - \mu_r')A' = 0.$

Since $(\mu_b' - \mu_r')$ for flint glass is more than that for crown glass, therefore, $A' < A$ *i.e.*, flint glass prism of smaller angle has to be suitably combined with crown glass prism of larger angle.

(*ii*) For almost no deviation, $(\mu_y - 1)A' + (\mu_y' - 1)A' = 0$

Taking crown glass prism of certain angle, we go on increasing angle of flint glass prism till this condition is met. In the final combination however, angle of flint glass prism will be smaller than the angle of crown glass prism as μ_y' for flint glass is more than μ_y for crown glass.

9.24. *For a normal eye, the far point is at infinity and the near point of distinct vision is about 25 cm in front of the eye. The cornea of the eye provides a converging power of about 40 dioptres, and the least* *converging power of the eye-lens behind the cornea is about 20 dioptres. From this rough data estimate the range of accommodation (i.e., the range of converging power of the eye-lens) of a normal eye.*

Sol. To see objects at infinity, the eye uses its least converging power = 40 + 20 = 60 dioptres.

This gives the rough idea of the distance between the retina and cornea eye-lens.

To focus at the near point ($u = -25$ cm)

$$v = \text{focal length of eye-lens} = \frac{100}{P} = \frac{10}{6} = \frac{5}{3} \text{ cm}$$

$$\therefore \quad v = -\frac{5}{3} \text{ cm} \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{3}{5} + \frac{1}{25} = \frac{16}{25}$$

$f = \frac{25}{16}$, corresponding to a converging power,

$$P = \frac{100}{\frac{25}{16}} = 64 \text{ dioptre.}$$

The power of the eye-lens = 64 − 40 = 24 dioptre.

The range of accommodation of the eye-lens is roughly 20 to 24 dioptre.

9.25. *Does short-sightedness (myopia) or longsightedness (hypermetropia) imply necessarily that the eye has partially lost its ability of accommodation? If not, what might cause these defects of vision?*

Sol. No, a person may have normal ability of accommodation and yet he may be myopic or hypermetropic.

In fact, myopia arises when length of eye ball (from front to back) gets elongated and hypermetropic arises when length of eye ball gets shortened.

However, when eye ball has normal length, but the eye-lens losses partially its power of accommodation, the defect is called presbiopia.

9.26. *A myopic person has been using spectacles of power −1.0 dioptre for distant vision. During old age, he also needs to use separate reading glass of power +2.0 dioptres. Explain what may have happened.*

Sol. As the person is using spectacles of power −1.0 dioptre (*i.e.*, focal length − 100 cm), the far point of the person is at 100 cm. Near point of the eye might have been normal (*i.e.*, 25 cm).

The objects at infinity produce virtual images at 100 cm (using spectacles). To see objects between 25 cm to 100 cm, the person uses the ability of accommodation of his eye-lens. This ability is partially lost in old age. The near point of the eye may recede to 50 cm. He has, therefore, to use glasses of suitable power for reading.

Here, $u = -25$ cm, $v = -50$ cm

Since, $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = -\frac{1}{50} + \frac{1}{25} \quad \text{or} \quad f = 50 \text{ cm}$$

As $$P = \frac{100}{f} = \frac{100}{50} = +2 \text{ dioptre.}$$

9.27. *A person looking at a person wearing a shirt with a pattern comprising vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?*

Sol. This defect is called Astigmatism. It arises because curvature of the cornea plus eye-lens refracting system is not the same in different planes. As vertical lines are seen distinctly, the curvature in the vertical plane is enough, but in the horizontal plane, curvature is insufficient.

This defect is removed by using a cylindrical lens with its axis along the vertical.

9.28. *A man with normal near point (25 cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5 cm.*

(a) What are the closest and farthest distance at which he should keep the lens from the page so that he can read the book, when viewing through the magnifying glass?

(b) What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope?

Sol. (*a*) For the closest distance

$$v = -25 \text{ cm}, f = 5 \text{ cm}$$

As $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{5} = \frac{-1-5}{25} = \frac{-6}{25}$$

$$u = -\frac{25}{6} = -4.2 \text{ cm}$$

This is the closest distance at which the man can read the book.

For the farthest image

$$v = \infty, f = 5 \text{ cm}$$

Since, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

or, $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{\infty} - \frac{1}{5} = -\frac{1}{5}$

or, $u = -5$ cm

This is the farthest distance at which the man can read the book.

(*b*) Maximum angular magnification is

$$\frac{D}{U_{min}} = \frac{25}{25/6} = 6$$

Minimum angular magnification is

$$\frac{D}{U_{max}} = \frac{25}{5} = 5.$$

9.29. *A card sheet divided into squares each of size 1 mm² is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 10 cm) held close to the eye.*

(a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?

(b) What is the angular magnification (magnifying power) of the lens?

(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

Sol. (*a*) $u = -9$ cm, $f = 10$ cm

Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$

or, $\frac{1}{v} = \frac{1}{10} + \frac{1}{-9} = \frac{-9+10}{-90} = -\frac{1}{90}$

or, $v = -90$ cm

Magnification $= \frac{v}{u} = \frac{-90}{-9} = 10$

As magnification is 10 therefore, the area of the each square in the virtual image is $(10 \times 10 \times 1)$ mm^2 *i.e.*, 100 mm^2 = 1 cm^2

(*b*) Angular magnification $= \frac{D}{u} = \frac{-25}{-9} = 2.78$

(*c*) No. Magnification of an image by a lens and angular magnification (or magnifying power) of an optical instrument are two separate things. The latter is the ratio of the angular size of the object (which is equal to the angular size of the image even if the image is magnified) to the angular size of the object if placed at the near point (25 cm). Thus magnification magnitude is $\left|\frac{v}{u}\right|$ and magnifying power is $\frac{25}{|u|}$.

Only when the image is located at the near point ($|v| = 25$ cm), are the two quantities equal.

9.30. *(a) At what distance should the lens be held from the figure in Question 9.29 in order to view the squares distinctly with the maximum possible magnifying power?*

(b) What is the magnification in this case?

(c) Is the magnification equal to the magnifying power in this case? Explain.

Sol. (*a*) Maximum magnifying power is obtained when the image is at the near point (25 cm). Thus,

$$v = -25 \text{ cm}, f = +10 \text{ cm}, u = ?$$

As, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\therefore$ $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{10} = \frac{-2-5}{50} = \frac{-7}{50}$

or, $u = -\frac{50}{7} = -7.14$ cm

So the lens should be held 7.14 cm away from the figure.

(*b*) Magnitude of magnification is

$$m = \frac{v}{u} = \frac{25}{50/7} = 3.5$$

(c) Magnifying power, $M = \frac{D}{u} = \frac{25}{50/7} = 3.5$

Yes, the magnifying power is equal to the magnitude of magnification because image is formed at the least distance of distinct vision.

9.31. *What should be the distance between the object (in Question 9.30) and the magnifying glass if the virtual image of each square in the figure is to have an area of 6.25 mm². Would you be able to see the squares distinctly with your eyes very close to the magnifier?*

Note: Questions 9.29 to 9.31 will help you clearly understand the difference between magnification in absolute size and the angular magnification (or magnifying power of an instrument).

Sol. Here, magnification in area = 6.25

∴ Linear magnification

$$m = \sqrt{6.25} = 2.5$$

As $m = \frac{v}{u}$ or $v = mu = 2.5\,u$

Using formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ ∴ $\frac{1}{2.5\,u} - \frac{1}{u} = \frac{1}{10}$

$$\frac{1-2.5}{2.5\,u} = \frac{1}{10}$$

$$2.5\,u = -15 \quad \text{or} \quad u = -6 \text{ cm}$$

∴ $v = 2.5\,u = 2.5\,(-6) = -15$ cm

As the virtual image is at 15 cm; whereas distance of distinct vision is 25 cm, therefore, the image cannot be seen distinctly by the eye.

9.32. *Answer the following questions:*

(a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?

(b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?

(c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?

(*d*) *Why must both the objective and eyepiece of a compound microscope have short focal lengths?*

(*e*) *When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?*

Sol. (*a*) Even though the absolute image size is bigger than object size, the angular size of the image is equal to the angular size of the object. The magnifier helps in the following way: without it the object would be placed no closer than 25 cm; with it, the object can be placed much closer. The closer object has larger angular size than the same object at 25 cm. It is in this sense that angular magnification is achieved.

(*b*) Yes, the angular magnification decreases a little because the angle subtended at the eye by the image is then slightly less than the angle subtended by the image at the lens. The angle subtended by the object at the eye is also less than that subtended by the object at the lens. However, this decrease is very small as compared to the case of image.

Also, when the eye is separated from the lens, the angles subtended at the eye by the object and its image are not equal.

It may further be noted that if the image is a very large distance away, then the effect, on magnification, of moving the eye shall be negligible.

(*c*) First, grinding lenses of very small focal lengths is not easy. More important, if you decrease focal length, aberrations (both spherical and chromatic) become more pronounced. So, in practice, you can't get a magnifying power of more than 3 or so with a simple convex lens. However, using an aberration-corrected lens system, one can increase this limit by a factor of 10 or so.

(*d*) Angular magnification of eyepiece is $\frac{25}{f_e}+1$ (f_e in cm) which increase if f_e is smaller. Further, magnification of the objective is given by

$$\frac{v_0}{|u_0|} = \frac{1}{\frac{|u_0|}{f}-1}$$

which is large when $|u_0|$ is slightly greater than f_0. Now the microscope is used for viewing very close objects. So $|u_0|$ is small, and so is f_0.

(*e*) The image of the objective lens in the eyepiece is known as the 'eye-ring'. All the rays from the object refracted by the objective go through the eye-ring. Therefore, it is an ideal position for our eyes for viewing. If we place our eyes too close to the eyepiece, we shall not collect much of the light and also reduce our field of view. If we position our eyes on the eye-ring and the area of the pupil of our eye is greater or equal to the area of the eye-ring, our eyes will collect all the light refracted by the objective. The precise location of the eye-ring naturally depends on the separation between the objective and the eyepiece and the focal length of the eyepiece. When we view through a microscope by placing our eyes on one end, the ideal distance between the eye and the eyepiece is usually built in the design of the instrument.

9.33. *An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope?*

Sol. In normal adjustment, image is formed at least distance of distinct vision,

$$d = 25 \text{ cm}$$

Angular magnification of eyepiece $= \left(1+\frac{D}{f_e}\right)$

$$= \left(1+\frac{25}{5}\right) = 6$$

Since the total magnification is 30, magnification of objective lens,

$$m = \frac{30}{6} = 5$$

$\therefore$ $$m = -\frac{v_0}{u_0} = 5 \text{ or, } v_0 = -5u_0$$

As $$\frac{1}{v_0}-\frac{1}{u_0} = \frac{1}{f_0}$$

$\therefore$ $$\frac{1}{-5u_0}-\frac{1}{u_0} = \frac{1}{1.25}$$

$$-\frac{6}{5u_0} = \frac{1}{1.25}$$

$$u_0 = -\frac{6\times 1.25}{5} = -1.5 \text{ cm}$$

i.e., object should be held at 1.5 cm in front of objective lens.

As $\qquad v_0 = -5\, u_0$

$\therefore \qquad v_0 = -5(-1.5) = 7.5$ cm

From $\qquad \frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{5} = -\frac{6}{25}$$

$$u_e = -\frac{25}{6} = -4.17 \text{ cm}$$

$\therefore$ Separation between the objective lens and eyepiece

$$= |u_e| + |v_0| = 4.17 + 7.5$$

$$= 11.67 \text{ cm.}$$

9.34. *A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when*

(a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?

(b) the final image is formed at the least distance of distinct vision (25 cm)?

Sol. Given, $f_0 = 140$ cm and $f_e = 5$ cm

(*a*) Magnifying power $= \frac{f_0}{f_e} = -\frac{140}{5} = -28$

[For normal or final image at infinity]

(*b*) Magnifying power $= -\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$

[For final image at least distinct of vision]

$$= -\frac{140}{5}\left(1+\frac{5}{25}\right) = -33.6.$$

9.35. *(a) For the telescope describe in Question 9.34 (a), what is the separation between the objective lens and the eyepiece?*

(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

(c) *What is the height of the final image of the tower if it is formed at 25 cm?*

Sol. (a) Separation between objective and eye-lens

$$L = f_o + f_e$$

$$= 140 + 5 = 145 \text{ cm}.$$

(b) Angle subtended by 100 m tall tower at 3 km,

$$\theta = \frac{100}{3 \times 1000} = \frac{1}{30} \text{ radian}$$

If y be the height of the image formed by the objective,

then, $$\theta = \frac{y}{140}$$

Now, $$\frac{1}{30} = \frac{y}{140} \text{ or } y = 4.7 \text{ cm}.$$

(c) Magnification produced by eyepiece

$$= 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$$

Height of final image $= 4.7 \times 6 = 28.2$ cm.

9.36. *A Cassegrain telescope uses two mirrors as shown in figure below. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be?*

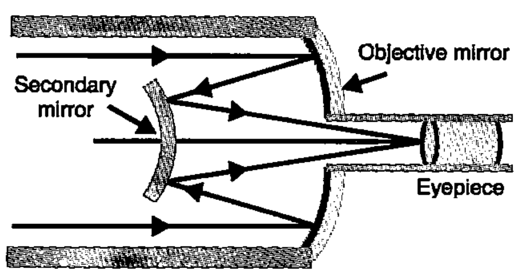

Fig. 9.12 *Schematic diagram of a reflecting telescope (Cassegrain)*

Sol. Here, radius of curvature of objective mirror

$$R_1 = 220 \text{ mm}$$

radius of curvature of secondary mirror

$$R_2 = 140 \text{ mm}; f_2 = \frac{R_2}{2} = \frac{140}{2} = 70 \text{ mm}$$

Distance between the two mirror, $d = 20$ mm

When object is at infinity, parallel rays falling on objective mirror, on reflection, would collect at its focus at

$$f_1 = \frac{R_1}{2} = \frac{220}{2} = 110 \text{ mm.}$$

Instead, they fall on secondary mirror at 20 mm from objective mirror.

$\therefore$ For secondary mirror, $u = f_1 - d = 110 - 20 = 90$ mm

Using formula, $\frac{1}{v}+\frac{1}{u} = \frac{1}{f_2}$

$$\frac{1}{v} = \frac{1}{f_2}-\frac{1}{u} = \frac{1}{70}-\frac{1}{90} = \frac{9-7}{630} = \frac{2}{630}$$

$$v = \frac{630}{2} = 315 \text{ mm}$$

$= 31.5$ cm to the right of secondary mirror.

9.37. *Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in figure A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?*

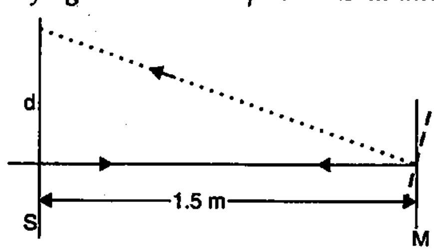

Fig. 9.13

Sol. We know that if the mirror is turned by an angle θ, the reflected ray turns by an angle 2θ. The current in the coil has produced a deflection of 3.5° in the mirror, therefore, the reflected ray will be deflected by an angle $2 \times 3.5° = 7°$.

From the figure, we have

$$\frac{d}{1.5} = \tan 7° \qquad \left[\tan\theta = \frac{\text{perpendicular}}{\text{base}}\right]$$

or, $d = 1.5 \tan 7°$

$= 1.5 \times 0.123 = 0.184 \text{ m} = 18.4 \text{ cm.}$

9.38. *The following figure shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?*

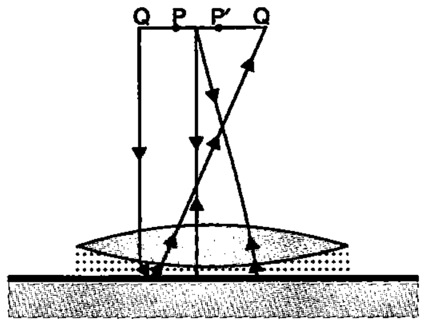

Fig. 9.14

Sol. Consider that the focal length of convex lens of glass

$$= f_1 = 30 \text{ cm}$$

and focal length of plane concave lens of liquid = f_2 combined focal length, F = 45.0 cm

As $$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$$

∴ $$\frac{1}{f_2} = \frac{1}{F} - \frac{1}{f_1} = \frac{1}{45} - \frac{1}{30} = -\frac{1}{90}$$

$$f_2 = -90 \text{ cm}$$

For glass lens, let $R_1 = R$, $R_2 = -R$

Since, $$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

∴ $$\frac{1}{30} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right) = \frac{1}{2} \times \frac{2}{R} = \frac{1}{R}$$

Thus, $R = 30$ cm

For liquid lens,

$$R_1 = -R = -30.0 \text{ cm}$$
$$R_2 = \infty$$

$$\frac{1}{f_2} = (\mu_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (\mu_l - 1)\left(\frac{1}{-30} - \frac{1}{\infty}\right)$$

or, $$\frac{1}{-90} = (\mu_l - 1) \times \frac{1}{-30}$$

or, $$(\mu_l - 1) = \frac{30}{90} = \frac{1}{3}$$

$$\mu_l = 1+\frac{1}{3} = \frac{4}{3}.$$

9.39. *A convex lens of focal length 20 cm, has a point object placed on its principal axis at a distance of 40 cm from it.A plane mirror is placed 30 cm behind the convex lens. Locate the position of image formed by this combination.*

Sol.

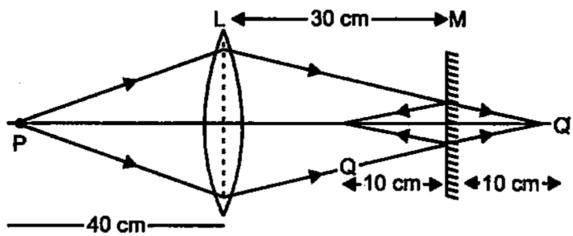

Fig. 9.15

Let us consider for lens only

$$u = -40 \text{ cm} \quad \text{and} \quad f = +20 \text{ cm}$$

∴ From lens formula, we get

$$\frac{1}{v} - \frac{1}{(-40)} = \frac{1}{20}$$

∴ $$v = +40 \text{ cm}$$

If the mirror was not there the image would have been at Q' i.e. 40 cm away from the lens or 40 – 30 = 10 cm from the mirror. This image will become the vertical object for the mirror which gives the real image at Q at the same distance *i.e.* 10 cm.

9.40. *A convex lens and a convex mirror of radius of curvature 20 cm are placed co-axially with the convex mirror placed at a distance of 30 cm from the lens. For a point object, at a distance of 20 cm from the lens, the final image due to this combination concides with the object itself. What is the focal length of the convex lens?*

Sol.

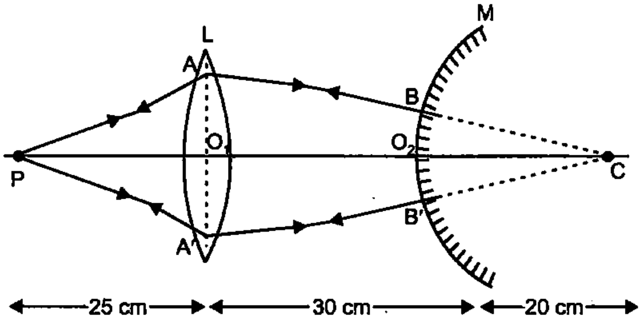

Fig. 9.16

Here $O_2C = 20$ cm (the radius of curvature of the mirror)

$u = -25$ cm

$v = +(30 + 20) = +50$ cm

If f be the focal length of the lens, then

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{+50} - \frac{1}{(-25)} = \frac{1}{50} + \frac{1}{25}$$

$$\frac{1}{f} = \frac{3}{50}$$

$$\therefore \quad f = \frac{50}{3} = 16.67 \text{ cm}$$

9.41. *A convex lens of focal length 20 cm, is placed co-axially with a convex mirror of radius of convature 20 cm. The two are kept 15 cm apart from each other. A point object is placed 60 cm in front of the convex lens. Find the position of the image formed by this combinations.*

Sol.

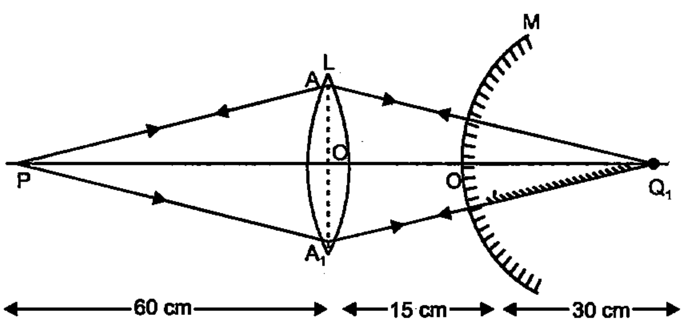

Fig. 9.17

For the convex lens, we have

$$u_1 = -60 \text{ cm} \quad \text{and} \quad f = +20 \text{ cm}$$

∴ Using lens formula, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{+20} = \frac{1}{v_1} - \frac{1}{-60}$$

∴ $$\frac{1}{v_1} = \frac{1}{20} - \frac{1}{60}$$

∴ $$v_1 = 30 \text{ cm}$$

Had there been only the lens L, the image of P would have been formed at Q_1 which acts as a virtual object for the convex mirror.

∴ $$OQ_1 = (30 - 15) = 15 \text{ cm}$$

Here for the convex mirror

$$u_2 = +15 \text{ cm and } R = +20 \text{ cm}$$

∴ From mirror formula, we get

$$\frac{1}{v_2} + \frac{1}{u_2} = \frac{2}{R}$$

$$\frac{1}{v_2} + \frac{1}{15} = \frac{2}{20}$$

$$\frac{1}{v_2} = \frac{1}{10} - \frac{1}{15}$$

∴ $$v_2 = +30 \text{ cm}$$

9.42. *A convex lens of focal length 20 cm and a concave mirror of focal length 10 cm, are placed co-axially 50 cm apart from each other. An incident beam parallel to its principal axis, is incident on the convex lens. Locate the position of the final image formed due to this combination.*

Sol.

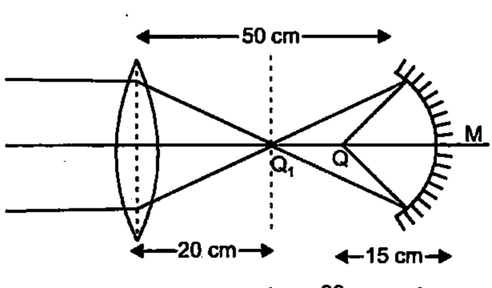

Fig. 9.18

Here $u = -30$ cm and $f = 10$ cm

Hence using mirror formula

$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{-10}$$

$$\frac{1}{v} = \frac{1}{30} - \frac{1}{10}$$

$$v = -15$$

Therefore lens-mirror combination forms a real image Q at a distance of 15 cm from the mirror.

9.43. *A point object is placed 60 cm in front of a convex lens of focal length 30 cm. A plane mirror is placed 10 cm behind the convex lens. Where is the image formed by this system?*

Sol.

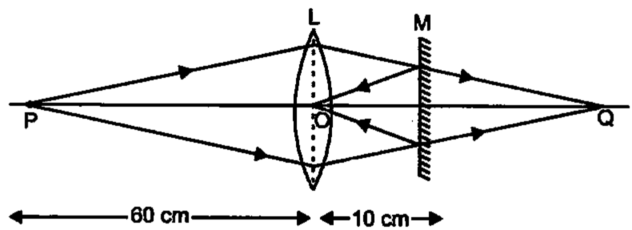

Fig. 9.19

Here $u = -60$ cm

$f = 30$ cm

$\therefore$ From lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{30} = \frac{1}{v} - \frac{1}{-60}$$

$$\frac{1}{v} = \frac{1}{30} - \frac{1}{60}$$

$$\frac{1}{v} = \frac{1}{60}$$

$\therefore$ $v = 60$ cm

The real image of the object will be formed at a distance of 60 cm i.e. on radius of convature which has become the virtual object for plane mirror. So the real image will form at the optical centre.

9.44. *A convex lens of focal length 15 cm, and a concave mirror of radius of convature 20 cm are placed coaxially 10 cm apart. An object is placed in front of the convex lens so that there is no parallex between the object and its image formed by the combination. Find the position of the object.*

Sol.

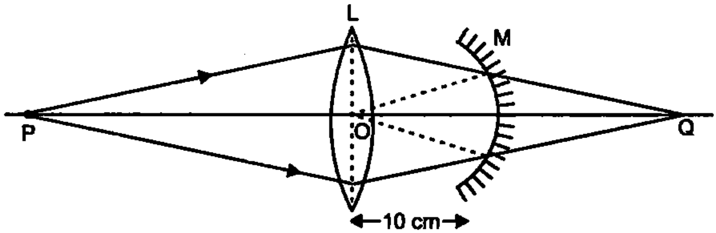

Fig. 9.20

Here $u = ?$

as there is no parallex between the object and the image

$\therefore \quad v = u$

$\therefore$ From Convex lens formula, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{+15} = \frac{1}{30} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{1}{30} - \frac{1}{15}$$

$$\frac{1}{u} = -\frac{1}{30}$$

$\therefore \quad u = -30$ cm

Now the image will form at Q which is the virtual object for concave mirror. The final image will form at the optical centre. Hence the distance of the object is 30 cm from the mirror.

❑❑❑